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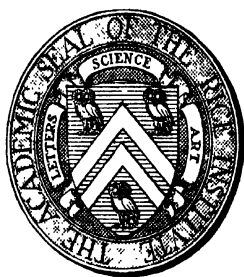






# THE RICE INSTITUTE PAMPHLET

VOLUME FOUR



Published by

THE RICE INSTITUTE

A university of liberal and technical learning  
founded by William Marsh Rice in the City of  
Houston, Texas, and dedicated by him to  
the advancement of Letters, Science, and Art



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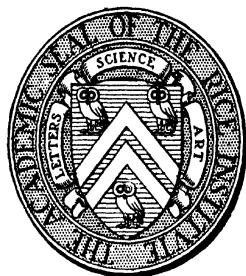
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# I

## AGGREGATES OF ZERO MEASURE<sup>1</sup>

### I

WE say that a linear aggregate  $E$  is of measure zero if, when we are given a number  $\epsilon$  arbitrarily small, we can inclose all the points of  $E$  within intervals whose sum is less than  $\epsilon$ . For an aggregate of two dimensions we have a similar definition, replacing the intervals by the rectangles. Moreover, we see that we may speak of squares instead of rectangles, because if we are given a rectangle we can find a finite number of squares of which the total area differs as little as we please from the area of the rectangle, and such that every point within the rectangle is also within one of these squares. We could also replace squares by circles without altering the generality of the definition.

Aggregates of measure zero play a very important part in the theory of functions of a real and of a complex variable. It is therefore useful to be able to compare the different aggregates of measure zero among themselves. This comparison is aided by the concept of regular aggregates. In the first place, then, we shall define regular aggregates and the fundamental points of these aggregates, and we shall show that every regular aggregate is equivalent to another regular aggregate of which the fundamental points are chosen in a special manner, for example, as the points with rational coördinates. Finally, we shall consider the classification of aggregates of measure zero, with given fundamental points. This classification will be based on the asymptotic decrease of the intervals (or squares) of exclusion.

<sup>1</sup> Translated from the French by Professor Griffith Conrad Evans, of the Rice Institute.

## Aggregates of Zero Measure

An aggregate of measure zero is said to be regular when it can be defined in the following manner:

*Let  $A_1, A_2, \dots, A_n, \dots$  be a denumerable infinity of points, said to be fundamental points. To each integral number  $h$  let us make correspond an infinity of squares  $C_1^{(h)}, C_2^{(h)}, \dots, C_n^{(h)}, \dots$ , of which the areas form a convergent series, such that the square  $C_n^{(h)}$  incloses in its interior  $C_n^{(h+1)}$  and approaches  $A_n$  when  $h$  increases indefinitely. Let  $E_n$  be the aggregate of points inside of the squares  $C_n^{(h)}$  ( $n = 1, 2, \dots$ ). The aggregate of points contained in all the  $E_n$  is a regular aggregate (which is evidently of zero measure).*

Every aggregate of zero measure can be considered as part of a regular aggregate. In other words, if  $A$  is any aggregate of measure zero, we can define a regular aggregate  $E$  of zero measure, such that every point of  $A$  belongs to  $E$ . To prove this proposition let us imagine a sequence of numbers  $\epsilon_1, \epsilon_2, \dots, \epsilon_n$ , decreasing and tending to zero, the series  $\Sigma \epsilon_n$  being supposed convergent. Since the aggregate  $A$  is of measure zero, we can define an aggregate  $A^{(h)}$  of squares (with sides parallel to the axes) the sum of whose area is less than  $\epsilon_n$ , and such that every point of  $A$  is inside one of these squares  $A^{(h)}$ . We define first the squares  $A^{(1)}$ , then the squares  $A^{(2)}$ ; if there are portions of these squares  $A^{(2)}$  which are outside all the squares  $A^{(1)}$ , we can suppress them as useless. In order to proceed in a perfectly definite manner, we consider the first of the squares  $A^{(1)}$ , say  $A_1^{(1)}$ , and operate successively on the portions of the successive squares  $A^{(2)}$  which are inside  $A_1^{(1)}$ ; we continue in the same way with  $A_2^{(1)}$ , being careful each time to omit the portions already considered, etc. These operations lead us to consider rectangles, each of which may be replaced by an enumerable infinity of squares (in particular cases a finite number). It is sufficient, in order to form the squares according to a definite law, to construct successively the

greatest possible square inside the rectangle, taking as the vertex nearest the origin of coördinates that vertex of the rectangle which is nearest the origin of coördinates. If among the squares so defined there are some which contain no point of the aggregate  $A$  we suppress them. We may assume the squares to be arranged in the order of decreasing size (if two of them happen to be equal in size we shall arrange them according to the relative values of the abscissas of their centers; and if these abscissas are equal, according to the value of their ordinates). In the same way we arrange the squares  $A^{(2)}$  (after the required transformations), and so on.

We define an aggregate  $B$  of squares which will consist of all the squares  $A^{(1)}$ , and besides a certain number of the squares  $A^{(2)}, A^{(3)}, \dots$ . In the same way  $B^{(2)}$  will include all the squares  $A^{(2)}$  and, besides, a certain number of the squares  $A^{(3)}, \dots$ . It is clear that the sum of the squares  $B^{(h)}$  is less than  $E_h + E_{h+1} + \dots$  is finite no matter what  $h$  may be and approaches zero when  $h$  increases indefinitely. Since all the squares  $A^{(h)}$  will be part of the  $B^{(h)}$ , every point of  $A$  is inside of one of the squares  $B^{(h)}$ . In order that the aggregate  $E$  defined by the  $B^{(h)}$  may be regular we must be able to number the  $B^{(h)}$ ,  $B_1^{(h)}, B_2^{(h)}, \dots, B_n^{(h)}, \dots$ , in such a way that  $B_n^{(h+1)}$  shall be less than  $B_n^{(h)}$ .

We achieve this result in the following manner. Consider first the squares  $A^{(1)}$ , if there are any, whose area is greater than  $\epsilon_2$  (we know that there are none whose area is greater than  $\epsilon_1$ , since the sum of all the  $A^{(1)}$  is less than  $\epsilon_1$ ). We designate these squares as  $B_1^{(1)}, B_2^{(1)}, \dots, B_{n_1}^{(1)}$ . Let us consider next those remaining squares  $A^{(1)}$  of which the area is greater than  $\epsilon_3$ , and let us denote them by  $B_{n_1+1}, B_{n_1+2}, \dots, B_{p_1}^{(1)}$ . Let us take now the squares  $A^{(2)}$  whose area is greater than  $\epsilon_3$ ; they are arranged in a definite order, as we have said. If the first of them is inside one of the  $A^{(1)}$  already numbered,

## Aggregates of Zero Measure

for example inside  $B_k^{(1)}$ , we shall denote it by  $B_k^{(2)}$ , otherwise we shall denote it at the same time by  $B_{p_1+1}^{(1)}$  and by  $B_{p_1+1}^{(2)}$ . In the same way, if the second of the  $A^{(2)}$  that we take is inside one of the  $A^{(1)}$  already numbered, different from  $B_k^{(1)}$ , say  $B_h^{(1)}$ , we shall denote it by  $B_h^{(2)}$ . If it is not inside any of the  $A^{(1)}$  (it cannot be inside an  $A^{(1)}$  without a number, since its area is greater than  $\epsilon_3$  and the  $A^{(1)}$  without numbers have areas less than  $\epsilon_2$ ), or if it is inside the particular  $B_k^{(1)}$  which has already been utilized, we shall denote it at the same time by  $B_{p_1+2}^{(1)}$  and by  $B_{p_1+2}^{(2)}$ . In this way we manage to define a certain number of new squares  $B^{(1)}$  which we will call  $B_{p_1+1}^{(1)}$ ,  $B_{p_1+2}^{(2)}$ ,  $\dots$ ,  $B_{n_2}^{(1)}$ , and a certain number of squares  $B^{(2)}$  which include all the  $A^{(2)}$  of area greater than  $\epsilon_3$ .

Let us consider now the squares  $A^{(1)}$  of area greater than  $\epsilon_4$ , and let us denote them by  $B_{n_1+1}^{(1)}$ ,  $B_{n_1+2}^{(1)}$ ,  $\dots$ ,  $B_{p_3}^{(1)}$ ; we can proceed in the same way as before for the  $A^{(2)}$  whose areas are greater than  $\epsilon_4$ , and we can then pass on to the  $A^{(3)}$  whose areas are greater than  $\epsilon$ . Those among them which are inside of the  $B^{(2)}$  already numbered will have the same numbers (each number being given of course but one time). The others will be denoted at the same time by  $B_3^{(1)}$ ,  $B_3^{(2)}$ ,  $B_3^{(3)}$ . We can continue indefinitely in the same way, the  $\epsilon_k$  approaching zero when  $k$  increases indefinitely and each operation involving only a finite number of squares. In this way every square belonging to  $A^{(n)}$  will appear in  $B^{(n)}$  in a determinate position. Moreover, it is obvious that  $B_q^{(n)}$  approaches zero no matter what  $q$  may be when  $h$  increases indefinitely. It is impossible that certain series  $B_q^{(1)}$ ,  $B_q^{(2)}$ ,  $\dots$ ,  $B_q^{(r)}$  should terminate, because that would mean that no one of the squares  $A^{(r+1)}$  is inside  $B_q^{(r)}$ ; that is to say, that  $B_q^{(r)}$  would inclose no point of the aggregate  $A$ , which is contrary to our hypothesis. The aggregates of squares  $B^{(n)}$  define, then, a regular aggregate which includes all the points of  $A$ , and our theorem is proved.

We notice that in the definition of the regular aggregate  $E$  there are certain series  $B_q^{(1)}, B_q^{(2)}, \dots$ , of which a certain number of the first terms denote squares that coincide among themselves. That, in fact, is no difficulty. We can, however, avoid this circumstance by slightly modifying the definitions of the first  $B_q$  of such a series; if, for instance,  $B_q^{(1)}, B_q^{(2)}, B_q^{(3)}$  coincide, we can replace  $B_q^{(2)}$  by  $(1 + \epsilon)B_q^{(2)}$ , and  $B_q^{(1)}$  by  $(1 + \epsilon_1)(1 + \epsilon)B_q^{(1)}$  (we designate by  $\alpha C$  a square similarly placed to  $C$ , with the ratio  $\alpha$  of similarity). These operations multiply the total extent of the squares  $B^{(h)}$  by a factor less than the convergent infinite product  $\prod(1 + \epsilon_k)$ .

We notice that the regular aggregate  $E$  which we have defined is not necessarily the most simple of the regular aggregates of measure zero which include the  $A$ , but it is not important that our demonstration should give us *the most simple*. The essential thing is to show that there exists *one*; it is then possible to consider without contradiction the collection of all the regular aggregates of measure zero which contain  $A$ , and we can choose from this collection if not the simplest (which may not exist, in the same way that the smallest number greater than  $\sqrt{2}$  does not exist), at least an aggregate  $E$  whose simplicity is as close as we please to the greatest possible.

From now on we shall consider especially the regular aggregates. Such an aggregate is defined by the fundamental points  $A_n$ , which are limits of the  $B_n^{(h)}$  when  $h$  increases indefinitely, and by the magnitudes of the excluding squares  $B_n^{(h)}$  corresponding to  $A_n$ .<sup>1</sup> The derived aggregate of the fundamental points is a closed set  $A'$ . In the general case this set is composed of a perfect aggregate and a reducible aggregate. The excluding intervals which correspond

<sup>1</sup> It might seem desirable to consider also the relative positions of the  $A_n$  in these squares; but by modifying slightly the definitions we can so arrange that every  $B_n^{(h)}$  has  $A_n$  for its center.

to the points of the reducible aggregate have only in common the points of this reducible aggregate itself. Their study therefore gives us nothing new. The really interesting part of a regular aggregate of zero measure is that which is attached to those points of  $A'$  which form a perfect aggregate. We shall have to distinguish cases according to the nature of this perfect aggregate. We shall limit ourselves, however, to the consideration of the case where the aggregate  $A'$  contains all the points of a certain area of simple form. The points  $A_n$  will then be dense within this area.<sup>1</sup> All the cases where the area is of a single piece and simply connected may be reduced by conformal representation to the case of the area bounded by a circle. We shall show that if we have two different systems of points  $A_n$  and  $B_n$ , dense within the interior of equal circles and also dense on their circumferences,<sup>2</sup> we can establish between these points a reciprocal continuous one-to-one correspondence in such a way that the ratio between the distance of any two points  $A_p, A_q$  and the distance of the corresponding points  $B_p, B_q$  will be included between two limits as close to unity as we please. It will follow from this theorem that we shall be able without loss of generality to suppose that the fundamental points of an aggregate of zero measure, when these points are dense within a certain region, coincide with a given dense aggregate in that region — for instance, with the points of rational coördinates.

<sup>1</sup> We shall thus leave aside those aggregates of zero measure which we obtain by assuming that  $A$  is a perfect linear aggregate which without being linear yet contains no area. For example, we could exclude certain fixed areas around the points with rational coördinates and take for the  $A_n$  the points with algebraic coördinates which did not belong to the excluded areas. We could also build up in some arrangement several similar constructions, or even a denumerable infinity of such constructions superposed, and thus obtain regions which would be quite complicated from the point of view of Analysis Situs.

<sup>2</sup> The case when neither aggregate has points on the circumference can be treated in the same way.

## II

THE theorem which we have in view can be expressed as follows: *Given two equal circles  $C$  and  $C'$ , and two enumerable aggregates  $A$  and  $B$ , of which the first is dense in  $C$  and on the circumference  $C$ , and the second is dense in  $C'$  and on the circumference  $C'$ , and given an arbitrarily small number  $\epsilon$ , then we can number the points of  $A$  and  $B$  in such a way that to a point on the contour we make correspond a point on the contour, and that we have, whatever  $p$  and  $q$  may be,*

$$1 - \epsilon < \frac{A_p A_q}{B_p B_q} < 1 + \epsilon.$$

We shall say that in this case the two aggregates are *similar* by  $\epsilon$ .

In order to prove this theorem we shall assume that the points of the two aggregates are arranged provisionally in a determinate order, and we shall consider successively the first point of  $A$ , then the first point of  $B$ , then the second point of  $A$ , then the second point of  $B$ , and so on. Thus we shall not miss any point belonging to either of the two aggregates. To each new point that we consider in one aggregate, we shall make correspond a determinate point in the other; and when the turn of this new point comes we shall omit it.

We shall suppose that the centers of the circles  $C$  and  $C'$  do not belong to the aggregates  $A$  and  $B$  (nothing would be changed if both of them should belong, for we could make them correspond; and if one of them belonged, but the other not, we could make a conformal transformation, differing little from the identical transformation, which would transform the second circle into an equal circle whose center could then be made to correspond to the center of the first



circle). In this way we can investigate the two circles by considering them superposed and yet distinct. It is possible now to choose two rectangular axes  $Ox$  and  $Oy$  in such a way that the diameters parallel to the axes contain no points of  $A$  or  $B$ , and every line parallel to either of the axes contains at most one point of  $A$  and one point of  $B$  (because the totality of directions of lines which connect the center with points of  $A$  or with points of  $B$ , or connect the points of  $A$  among themselves, or the points of  $B$  among themselves, or which are perpendicular to these directions, form an enumerable aggregate). Let us assume an infinite series of positive numbers  $\epsilon_1, \epsilon_2, \dots, \epsilon_n, \dots$  such that

$$1 - \epsilon < \Pi(1 - \epsilon_n) \quad , \quad \Pi(1 + \epsilon_n) < 1 + \epsilon.$$

The circle  $C$  is divided by the diameters parallel to the axes in four equal regions which provisionally we shall call 1, 2, 3, 4; and the circle  $C'$  is divided in homologous regions which we shall designate in the same manner.

Consider first  $A_1$ , which may be, for instance, in the region 3: since it cannot be on the diameters, it must be *inside* this region, unless it be on the circumference (a case which we are for the moment excluding). Let us now designate by  $A'_1$  the point of the region 3 of  $C'$  which coincides with  $A_1$  when  $C'$  is moved upon  $C$  by a translation. If  $A'_1$  happens to belong to  $B$ , which is not the general case, we shall call it  $B_1$ . Otherwise we shall define a square with center at  $A_1$ , such that the ratio of the greatest to the least of the shortest possible distances of all the points in the square to points on the boundary of the region 3 shall be less than  $1 + \epsilon_1$ . This shortest distance is parallel to the axes for the rectilinear portions of the boundary and coincides with the radius for the curvilinear portion, and, from our hypothesis in regard to  $A_1$ , is not zero. So the construction of the square is always possible. We now choose  $B_1$  arbitrarily from the points of

$B$  inside this square (if we wish to avoid having to make an arbitrary choice from among a denumerable infinity of points, we can take the point of  $B$  whose number is smallest in the provisional classification). Having chosen this point  $B_1$  we construct parallels to the axes passing through  $A_1$  and  $B_1$ , each set of which, with the diameters already drawn, will divide its circle into regions (nine in each) which will correspond two by two. Some of these regions will be rectangles (in this case only one), while the others will be quadrilaterals or triangles of which certain sides are parallels to the axes and one side is an arc of the circle. If we agree to consider as the dimensions of such regions the dimensions of the rectilinear sides, it follows from the construction that the ratio between homologous dimensions of two corresponding regions is included between  $1 + \epsilon$  and  $1 - \epsilon$ .<sup>1</sup> In the case which we have momentarily excepted, where  $A_1$  is on the circumference, we can take  $B_1$ , also on the circumference, in such a way that the same condition shall be verified with respect to the regions, a construction which is always possible.

Let us turn now to the second point  $B_2$ , taken from the second aggregate. We make correspond to it a point  $A_2$  situated in the homologous region, chosen in such a way that the new regions obtained by drawing parallels to the axes through  $A_2$  and  $B_2$  have homologous sides whose dimensions are included between  $(1 + \epsilon_1)(1 + \epsilon_2)$  and  $(1 - \epsilon_1)(1 - \epsilon_2)$ . This condition necessitates assigning to  $A_2$  a certain area inside this region, and  $A_2$  is chosen inside this region either arbitrarily, or according to some definite law, as has been explained for  $B_1$ , care being taken to have  $A_2$  on the circumference  $C$ , if  $B_2$  is on the circumference  $C'$ .

<sup>1</sup> We have, in fact,  $\frac{1}{1 + \epsilon_1} > 1 - \epsilon_1$ , and, according to our construction, the ratios of homologous sides are included between  $1 + \epsilon_1$  and  $\frac{1}{1 + \epsilon_1}$ .

We continue in the same way, taking alternately a point in  $A$  and a point in  $B$ , making it correspond to some point in the other aggregate. After  $n$  operations we shall have at most  $(n + 2)^2$  regions, and the ratio of two homologous dimensions of two regions which correspond will always be included between

$$(1 - \epsilon_1)(1 - \epsilon_2) \cdots (1 - \epsilon_n)$$

and

$$(1 + \epsilon_1)(1 + \epsilon_2) \cdots (1 + \epsilon_n)$$

and therefore between  $1 - \epsilon$  and  $1 + \epsilon$ . If we continue in this way indefinitely, every point of  $A$  and every point of  $B$  will have a number, after a finite number of operations, and this number will be at most double the number of the same point in the provisional classification.

This final classification satisfies completely the conditions of our theorem. For, if we consider any two points  $A_p, A_q$ , with their corresponding points  $B_p, B_q$ , the difference of the abscissa  $x_p$  and  $x_q$  of  $A_p$  and  $A_q$ , when the regional division has progressed far enough (that is, after a number of operations not greater than the larger of the two members  $p, q$ ), will be equal to the sum of the rectilinear sides of certain regions, and the abscissas  $x'_p, x'_q$  of  $B_p$  and  $B_q$  will be equal to the sum of rectilinear sides of the corresponding regions. We shall have then

$$(1) \quad 1 - \epsilon < \frac{x'_p - x'_q}{x_p - x_q} < 1 + \epsilon$$

and similarly

$$(2) \quad 1 - \epsilon < \frac{y'_p - y'_q}{y_p - y_q} < 1 + \epsilon,$$

from which follows immediately

$$1 - \epsilon < \frac{\sqrt{(x'_p - x'_q)^2 + (y'_p - y'_q)^2}}{\sqrt{(x_p - x_q)^2 + (y_p - y_q)^2}} < 1 + \epsilon.$$

But this last relation is the statement of our theorem.

We might show in the same way the analogous theorem about the angles  $\alpha$  and  $\beta$  which the lines  $A_p A_q$  and  $B_p B_q$  make with the axis  $Ox$ . In fact, we have

$$\tan \alpha = \frac{y_p - y_q}{x_p - x_q}, \quad \tan \beta = \frac{y'_p - y'_q}{x'_p - x'_q},$$

so that from equations (1) and (2) we deduce immediately

$$\frac{1 - \epsilon}{1 + \epsilon} < \frac{\tan \alpha}{\tan \beta} < \frac{1 + \epsilon}{1 - \epsilon}.$$

If we take the angles  $\alpha$  and  $\beta$  positive, since they are almost of the same value,  $\cot \beta + \tan \alpha$  is greater than or at least equal to 2, and therefore, neglecting  $\epsilon^2$ , we shall have

$$|\alpha - \beta| < |\tan(\alpha - \beta)| = \left| \frac{\frac{\tan \alpha}{\tan \beta} - 1}{\frac{1}{\tan \beta} + \tan \alpha} \right| < \frac{1}{2} \left| \frac{\tan \alpha}{\tan \beta} - 1 \right| < \epsilon.$$

The properties of the correspondence which we have shown to exist between two enumerable aggregates  $A$  and  $B$ , which are dense in equal circles  $C$  and  $C'$ , are worth studying more completely. Here follow some remarks that might be useful in such a study. In the first place we observe that if any partial arrangement of points  $A_{n_1}, A_{n_2}, \dots$  approach a limiting point  $P$ , there corresponds to it a partial series of points  $B_{n_1}, B_{n_2}, \dots$  which approaches a limit  $P'$ . The correspondence between  $P$  and  $P'$  is well defined, — that is, is independent of the partial series that may be chosen. We have in this way a one-to-one correspondence between the points of  $C$  and the points of  $C'$ .

Let us agree to call the parallels to the axes, drawn through the points of the aggregate, lines of discontinuity. To any point  $M$  not on a line of discontinuity corresponds an homologous point  $M'$ , and the transformation of the region in the

neighborhood of  $M$  into the region in the neighborhood of  $M'$  may be written in the form

$$\begin{aligned}x' &= (h + \eta)x \\ y' &= (h + \eta')y,\end{aligned}$$

where  $x, y$  are the coördinates of  $M$ ,  $x', y'$  are the coördinates of  $M'$ ,  $h, k$  are constants of value between  $1 - \epsilon$  and  $1 + \epsilon$ , and  $\eta$  and  $\eta'$  are functions of  $x$  and  $y$  which approach zero when  $x^2 + y^2$  approaches zero. The constants  $h$  and  $k$  are the two ratios of similitude (parallel to the two axes) of the neighborhoods of  $M'$  and  $M$ . If the points  $M'$  and  $M$  lie on a line of discontinuity, the ratio of similitude in the direction perpendicular to this line has not the same value on both sides of the line. At a point  $M$  which is the intersection of two lines of discontinuity, there are four values for each ratio of similitude, corresponding respectively to the positive and negative variations of the two coördinates. The ratio of similitude  $h$  is thus defined throughout  $C$ . It is discontinuous on the lines of discontinuity, but continuous at other points.

If we know nothing about the provisional numbering of the aggregates  $A$  and  $B$ , we can merely say this about the relation between the provisional numbering and the final numbering: that the final number  $n$  is at most twice the provisional number  $p$ ; for every point numbered provisionally  $A_p$  or  $B_p$  is chosen after at most  $2p$  operations. We cannot, however, give an upper limit to  $p$  as a function of  $n$ .

It will be possible to determine such a limit, provided that we take care to choose the system of provisional numbering from among those that are *sensibly homogeneous*. Let us make our meaning clear. By definition, in order to arrange a very large number  $p$  of points in a homogeneous manner in a circle  $C$ , we shall construct a square grating

such that  $p$  of its vertices are inside  $C$ ; if  $\alpha_p$  is the length of a segment of the grating, we shall put one point in each square of side  $\alpha_p$ , and  $l^2$  in each square of  $\lambda_p$ , exactly if  $l$  is an integral number, approximately if  $l$  is not. Let us write  $\lambda_p = \lambda$  and take  $\lambda$  as fixed and  $p$  variable. Then for every value of  $p$  we can calculate the approximate number of points inside the square of side  $\lambda$ , a number which may be given asymptotically as  $p\lambda^2/\pi r^2$ ,  $r$  being the radius of the circle  $C$ . We shall say that the arrangement of points of the enumerable aggregate  $A_1, A_2, \dots, A_p, \dots$  is asymptotically homogeneous if, for any square of side  $\lambda$ , the number  $\lambda_p$  of points of index less than  $p$  inside this square approaches this same symptotic value  $p\lambda^2/\pi r^2$  when  $p$  increases indefinitely; i.e., if the ratio  $\pi\lambda_p r^2/p\lambda^2$  between the numbers  $\lambda_p$  and the symptotic value  $p\lambda^2/\pi r^2$  approaches 1 as  $p$  increases indefinitely. We shall say that the arrangement is *sensibly homogeneous* if this ratio becomes and remains limited by two constants  $\alpha$  and  $\beta$  ( $\alpha < 1 < \beta$ ) independent of  $p$  and of the position of the square of side  $\lambda$ .

In the preceding definition of homogeneous arrangement, nothing was said about the points that happened to be situated on the boundary. If the boundary is a square of side  $a$ , the maximum number of points situated on this boundary for a grating of measure  $a/n$  is  $4n$ , the total number of points being  $n^2$ . Generally speaking, the number of points on the boundary will be said to be normal if it is of the order of magnitude of the square root of the total number of points. We must observe that this notion of normal depends on the assumption that *there are* points on the contour. If the points were arranged *arbitrarily*, in the general case there would be *no point* on the boundary, and this is indeed the simpler hypothesis. But if *there are* points on the contour, the case is probably that there is some sort of a relation between the way the contour is chosen and the way the

points are given. Hence it is natural to suppose that the probability that a point falls on an arc of the boundary of unit length is some finite proportion of the probability that a point falls in unit area. This hypothesis is verified, for instance, if the boundary is a circle and if the points of the aggregate are those with rational coördinates. Other such hypotheses might be conceived, related to the theory of numbers.

We must then, in the case where there are points on the boundary, add to the hypothesis that the arrangement is sensibly homogeneous inside, the hypothesis that it is sensibly homogeneous on the boundary.

In many questions, the preceding definition of sensibly homogeneous arrangements is inadequate; it is necessary to add a condition which may be called *intrinsic homogeneity*, because it introduces the relative positions of the points of the aggregate. If we consider the vertices of a grating, which we take as the type of homogeneity (or, say, a net of equilateral triangles), we see that the shortest distance between two vertices is proportional to the inverse square root of the total number of points. We say that a two-dimensional aggregate is intrinsically homogeneous if the shortest distance between any two of its points of number less than  $p$  is of the order of magnitude  $1/\sqrt{p}$ .<sup>1</sup> Homogeneity of arrangement and intrinsic homogeneity are thus seen to be independent conceptions, neither being a consequence of the other.

Given a denumerable aggregate, dense within a circle (or square), it is always possible to number its points in such a way as to satisfy the conditions of homogeneity. One of the simplest methods of doing this is as follows. After having numbered some of the points, we trace a grating

<sup>1</sup> An analogous condition should be verified for the shortest distance to the boundary of points very near to this boundary and not lying on it.

fine enough to make a few more squares than points already numbered, and such that one square includes at most one of these points. There will then be some squares that do not contain such points. In each of these we number one point of the aggregate, by choosing it inside a square concentric with the first, and twice smaller, taking the point of smallest subscript in the provisional numbering (thus we are sure of not omitting any point).

Any system of numbering that satisfies both conditions of homogeneity will be spoken of as *normal*. It is easy to verify the fact that the methods of numbering habitually used lead to normal arrangements.

When the two aggregates that are dense in  $C$  and  $C'$  are numbered normally, it is possible to arrange matters so that the one-to-one correspondence set up between their elements shall be itself normal; *i.e.*, there exist between the provisional numbering,  $p$ , and the final numbering,  $n$ , inequalities of the form

$$p^\alpha < n < p^\beta,$$

where the exponents  $\alpha$  and  $\beta$  are finite and depend only on the number of dimensions in the aggregate considered, and on the convergent series  $\Sigma \epsilon_n$  which has been used. (In order to be sure that  $\alpha$  and  $\beta$  are finite, there must be a finite quantity  $h$  such that  $\lim n^h \epsilon_n = 0$ .)

We divide the aggregate  $A$  into two others,  $A'$  and  $A''$ , still everywhere dense, and the aggregate  $B$ , similarly, into  $B'$  and  $B''$ . It is then easy to show that the correspondence can be set up in such a way that the points of  $A'$  correspond to those of  $B'$  and the points of  $A''$  to the points of  $B''$ . For that, it would not be sufficient of course to apply the general theorem first to  $A'$  and  $B'$  and then to  $A$  and  $B$ , because the correspondence thus set up between two points  $P$  and  $P'$  inside  $C$  and  $C'$ , respectively, would not in



general be the same by means of the two separate correspondences.

This procedure we can extend to the case where  $A$  and  $B$  each consists of a denumerable infinity of aliquot parts, everywhere dense. We can establish, for instance, a continuous one-to-one correspondence between the rational numbers in a certain interval, and the algebraic numbers in an equal interval, in such a way that to the rational numbers whose denominators consist of  $h$  and only  $h$  distinct prime factors, correspond the algebraic numbers which are the roots of an irreducible equation of degree  $h$  (for  $h = 1$  we get the rational numbers; if we wish to consider only the irrational algebraic numbers we must take irreducible equations of degree  $h + 1$ ).

### III

LET us consider now two regular aggregates of zero measure, of which the fundamental points are precisely the denumerable aggregates  $A$  and  $B$  inside the circles  $C$  and  $C'$ . If we suppose that the squares of exclusion belonging to the corresponding fundamental points have as their sides lines which correspond, it is evident that the two aggregates will correspond point by point in the one-to-one correspondence that we have established between the points  $P$  inside  $C$  and the points  $P'$  inside  $C'$ . In other words, *given a regular aggregate of zero measure of which the fundamental points  $B$  are dense in  $C'$ , we can define a regular aggregate of zero measure of which the fundamental points are the elements of an arbitrary aggregate  $A$ , dense in  $C$ , in such a way that the two aggregates correspond to each other continuously and in a one-to-one manner* (the ratio of similitude being contained between  $1 - \epsilon$  and  $1 + \epsilon$ ).

Hence in order to study regular aggregates of zero measure of which the fundamental points are dense within a certain

region, we can without loss of generality assume that the fundamental points are, for instance, the points with rational coördinates. In particular it is easy to prove this important proposition: *Every regular aggregate of zero measure of which the fundamental points are dense within a certain region has the order of the continuum.* In other words, if we arrange at pleasure the diminishing of the squares of exclusion in the neighborhood of fundamental points, it is not possible to make this diminution rapid enough so that the fundamental points shall be the only ones of the aggregate.

For simplification let us consider the case of a single dimension; the demonstration is in principle the same for any number of dimensions. Let  $A_n$  be the intervals of exclusion belonging to the points  $A_n$ . For each value of  $h$  we can define a positive function  $\phi_h(n)$  increasing with  $n$ , such that we shall have

$$\text{measure } (A_n^h) > \phi_h(n).$$

On the other hand, if we are given a denumerable succession of increasing functions  $\phi_h(n)$ , it is possible, according to a theorem of Paul du Bois-Reymond, to construct a function  $\phi(n)$  increasing more rapidly than any of the functions  $\phi_h(n)$ . After having found this function  $\phi(n)$ , the theory of continuous functions enables us to define an infinite number of irrational numbers  $x$  (an infinity which has the order of the continuum) such that there exists for each of them a denumerable infinity of relations of the form

$$x - \frac{m}{n} < \phi_h(n)$$

where  $m$  and  $n$  are integers. Such a number  $x$ , whatever  $h$  may be, belongs to at least one of the intervals  $A_n^{(h)}$ ; it is therefore an element of the aggregate defined by the points

$A_n$  and these intervals of exclusion. In order to define the numbers  $x$  and show that their aggregate is of the order of the continuum, it is sufficient to investigate a continuous fraction in which the incomplete quotients increase very rapidly. If we write

$$P_{n+1} = a_n P_n + P_{n-1}$$

$$Q_{n+1} = a_n Q_n + Q_{n-1}$$

and assume that

$$a_n > \phi(Q_n),$$

where  $\phi(n)$  is the function which we have just defined, we shall have, from the nature of the convergents,

$$\left| x - \frac{P_n}{Q_n} \right| < \left| \frac{P_{n+1}}{Q_{n+1}} - \frac{P_n}{Q_n} \right| = \frac{1}{Q_n Q_{n+1}} < \frac{1}{\phi(Q_n)}.$$

But the totality of systems of integers  $a_n$  which verify the relations  $a_n > \phi(Q_n)$  have, themselves, the order of the continuum.<sup>1</sup>

If we wished to have intervals of exclusion which should decrease rapidly enough so that the aggregate of points defined by them would be composed only of the fundamental points, the  $\phi_h(n)$  would have to contain functions increasing more rapidly than any  $\phi(n)$ . According to the theorem of Paul du Bois-Reymond, that is not possible if the indices  $h$  are denumerable. It would be necessary then to make belong to any fundamental point a transfinite infinity of intervals of exclusion, the corresponding functions  $\phi_\alpha(n)$  (where  $\alpha$  denotes a transfinite number) being such that every increasing function  $\phi(n)$  is surpassed by one of them. In this way, however, we get outside the domain of definitions expressible in a finite number of words.

In order to classify the regular aggregates of zero measure, it is better to consider rather than the functions  $\phi_h(n)$  which

<sup>1</sup> Each  $a_n$  may be odd or even; the aggregate of  $x$  then includes an aggregate of the same order as that of the numbers 0.1010110..., written in the binary scale.

we have defined, the functions  $\psi_h(n)$  determined by the relations

$$\sum_{p=n}^{\infty} \text{measure } A_p^{(h)} = \psi_h(n).$$

The convergence of the series formed by the intervals of exclusion of order  $h$  implies that the functions  $\psi_h(n)$  should increase indefinitely with  $n$ . After the theorem of Paul du Bois-Reymond, there exists a function  $\psi(n)$  increasing less rapidly than any of these, which nevertheless approaches  $+\infty$  as  $n$  approaches  $\infty$ . Hence whatever the value of  $h$ , if we take  $n$  large enough, we shall have

$$\sum_{p=n}^{\infty} \text{measure } A_p^h < \frac{1}{\psi(n)};$$

that is to say, that the different series formed by the intervals of exclusion all converge more rapidly than the series

$$\sum \left[ \frac{1}{\psi(n)} - \frac{1}{\psi(n+1)} \right].$$

The more rapid the increase of  $\psi(n)$ , the fewer points are included in the aggregate of measure zero, because the intervals of exclusion in that case decrease more rapidly. It is natural, then, to take the function  $\psi(n)$  as defining what we may call the asymptotic order of the regular aggregate of zero measure. These orders can be expressed by means of the notations used for orders of infinity.  $\psi(n) = n^p$  will be said to be of order  $p$ ,  $\psi(n) = \epsilon^n$  of order  $\omega$ ,  $\psi(n) = \omega^2$  of order  $\omega^2$ , etc. We meet the aggregates of order  $\omega^2$  in defining monogenic functions which are not analytic.

## IV

PERHAPS it is opportune to emphasize a little the general conclusions which follow from this rapid study of aggregates of zero measure.

Aggregates of zero measure have a fundamental position in the theory of functions. It is, in fact, always possible to inclose the singularities of finite functions in aggregates which are either of zero measure or of measure as small as we like. On the other hand, aggregates which are not of zero measure have a uniform quality, being formed of continuous aggregates either positive or negative. They are heterogeneous with regard to the continuum. Aggregates of zero measure can, however, be sensibly homogeneous with regard to the continuum, that is to say, identical with themselves in intervals as small as we like.

The concept of aggregate of zero measure is so general that we cannot hope to make a profound investigation of the properties of functions without studying minutely this general notion. That is to say, we must not regard all aggregates of zero measure as undifferentiable. The classification based on the asymptotic diminution of intervals of exclusion seems to me to be a first step in this study which faces the students of analysis.

With this question, as with all those where the general notion of increasing functions enters (as, for example, in the theory of the convergence of series with positive terms), difficulties of a transfinite nature are presented which we cannot hope entirely to surmount. But, on the other hand, the problems which are actually met with are generally if not always free of these difficulties (this is the case, for instance, with the usual criteria for the convergence of series of positive terms; for, although theoretically quite special, they are nevertheless practically sufficient for the treatment of the series which are presented in all researches in analysis). We can legitimately hope that it will be the same way with the classification of aggregates of measure zero. Theoretically the complexity of this classification surpasses that of the study of series of positive terms, a study which will never

be finished; but practically, a relatively restricted number of classes will suffice for the needs of analysis.

In closing I should like to direct attention to a notable consequence of the theorem about the correspondence between two denumerable aggregates which are everywhere dense. It might seem natural, passing from the finite to a denumerable infinitive, to suppose that the positions of equilibrium of the centers of gravity of the molecules of a solid body should form a denumerably dense aggregate. But *a priori* it would seem quite an arbitrary hypothesis to suppose that they should coincide with the points of rational coördinates. This simple arithmetic determination seems to have nothing to do with the physical conception. In fact, it evidently is not necessary. But it is as general as any other. The important point is that the hypothesis verifies the conditions of homogeneity of arrangement and intrinsic homogeneity, as we have stated them. The arithmetic treatment of the approximation of numbers by rational numbers is thus the reflection of the general properties of dense aggregates.

## II

# MONOGENIC UNIFORM NON-ANALYTIC FUNCTIONS<sup>1</sup>

I. THE THEORIES OF CAUCHY, WEIERSTRASS  
AND RIEMANN

THE integration by d'Alembert of the equation of vibrating strings led to a series of researches out of which the notion of an arbitrary function took shape. Among the geometers who contributed to clarify the new ideas, there should be mentioned Euler, in the front rank, and besides him Clairaut, Daniel Bernoulli and Lagrange. The question was that of the relation between the analytic and the physical definitions of a function: if a string is displaced *arbitrarily* from its position of equilibrium, does there exist a formula which represents exactly the initial state of the string? Fourier answered in the affirmative and set out the method of calculation of the coefficients of the trigonometric series which represents an arbitrary function. The views put forward by the genius of Fourier have been confirmed by the vigorous analysis of Lejeune-Dirichlet.

The discovery of Fourier revolutionized the notions prevalent up to that time; it was believed, with Euler, that to every analytic expression there corresponded a curve of which successive parts depended on each other: in order to express this interdependence, Euler created the expression 'continuous function': the sense of this expression has since been modified.

Under the influence of the same ideas Lagrange endeavored to prove that every continuous function can be developed

<sup>1</sup> Translated from the French by Professor Percy John Daniell, of the Rice Institute.

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in a Taylor series : this series would be the tangible form of the connection, so mysterious till then, between the different arcs of a continuous curve ; the knowledge of a small arc would have been sufficient to know the whole curve ; but Fourier proved exactly that the problem here was illusory, for the physicist who draws a curve remains at each instant free to modify its aspect ; the curve once drawn, it is always possible to represent it in its entirety by a unique analytic expression.

This led to the apparently paradoxical result that there existed no logical reason for regarding two segments of the same straight line, for example, as corresponding to the same function, since it was always permissible likewise to regard as a unique function the ordinate of the continuous curve formed of two different straight lines. At the most it could be said that, in the case of two segments of the same straight line, the formula is simpler than in the case of two segments of different straight lines, but this criterion of simplicity does not seem capable of precise definition, unless one is confined to algebraic functions. The paradox was cleared up by extension of the field of study of functions ; Cauchy showed that the properties of real functions could only be well understood if imaginary values of the variable were also studied ; the idea of a function of a complex variable became indispensable. Cauchy based this idea on the definition of monogeneity ; a function of the complex variable  $z = x + iy$  is called monogenic if it has a unique derivative. A function which is monogenic at every point of a region without any exception — that is, not allowing in the region any singular point — can be developed in a Taylor series in the neighborhood of any point in the region ; the radius of convergence of the series is equal to the distance from the center to the nearest singular point. From this fundamental theorem



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Cauchy deduces the calculation of the integrals of the proper differential equations along any path in the plane.

Cauchy's theory was systematized by Weierstrass and Riemann. Weierstrass defined an *analytic function*, in an exact manner, by means of *elements* and thus arrived at the idea of a region of natural existence, an idea which was contained implicitly in Cauchy's work, but which was not mentioned explicitly by him. Riemann conceived a monogenic function *a priori* independently of any analytic expression and showed the advantages of this geometrical conception.

In reality, the analytic point of view of Weierstrass and the geometric one of Riemann find their most perfect synthesis in Cauchy's fundamental theorem: monogeneity within a circle involves the existence of a Taylor series convergent within the circle. This theorem established a necessary connection between values of the same function as a simple consequence of monogeneity: it is sufficient to know that a function is monogenic within a circle, in order that its value at any interior point should be known by a knowledge of its values in the neighborhood of another point. Since our aim is to define monogenic functions in regions more general than those considered up to the present in the theory of analytic functions, it is necessary to make precise the definition of these new regions.

I shall call a region in which an analytic function can be defined in the sense of Weierstrass a *Weierstrassian region* or *W region*. I shall call regions more general than *W* regions, in which a uniform monogenic function can be defined, *Cauchy regions*, or *C regions*, in honor of the creator of the theory of monogenic functions. We shall see that the essential properties of monogenic functions in the *C* domains which we define are the same as in *W* regions; this does not

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exclude the possibility of defining  $C'$  regions more general again than our  $C$  regions. In other words, we cannot assert that our generalization covers all uniform monogenic functions: but it brings us to a definition of a more general class than the class  $W$  of the analytic functions of Weierstrass.

$W$  regions are characterized by the following properties. Let us call a  $\Gamma$  circle every circle such that all the points within  $\Gamma$  belong to  $W$ . Every point  $P$  of  $W$  is within a  $\Gamma$  circle: the  $\Gamma$  circles corresponding to two points  $P$  and  $Q$  of  $W$  can be reunited by a finite number of  $\Gamma$  circles cutting each other two by two. To every uniform analytic function there corresponds a  $W$  region; inversely, M. Runge has shown that to every  $W$  region corresponds an infinity of uniform analytic functions having precisely  $W$  as the region of existence.

If it is assumed that there is no other process of analytic continuation than the Taylor series, the boundary of the  $W$  region is a *natural limit* of existence of the analytic function, and those portions of the plane, if such exist, which do not belong to  $W$  ought to be considered as a lacunar space. On this point Weierstrass has insisted several times, and it has been made conspicuous in the clearest way by M. Henri Poincaré. Let us consider a region  $D$  of simple form, such as the interior of a circle, and let us define a function  $G(z)$  having  $D$  as its lacunar space and another function  $G_1(z)$  defined only within  $D$  and having consequently all the rest of the plane as its lacunar space. Let us divide the contour of  $D$  into two arcs  $D'$  and  $D''$ . M. Poincaré shows that it is possible to find two uniform functions  $F(z)$  and  $F_1(z)$  existing in the whole plane, except for the singular line  $D'$  for  $F$  and  $D''$  for  $F_1$ , and in such a way that

$$\begin{aligned} F + F_1 &= G \quad \text{outside } D, \\ F + F_1 &= G_1 \quad \text{within } D. \end{aligned}$$

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If then the functions  $F$  and  $F_1$  are regarded as uniform, the function  $G(z)$  has the continuation  $G_1(z)$  which has been chosen entirely arbitrarily; it is then proper to discard all ideas of a continuation within the lacunar space. This paradox is apparently cleared up if it is observed that, when a function such as  $F(z)$  possesses a singular line  $D'$ , supposed impassable, this function remains uniform in Weierstrass's sense when there is added a non-uniform function such as  $\log \frac{z - z_0}{z - z_1}$ ,  $z_0$  and  $z_1$  being two points of the line  $D'$ . The remarkable result due to M. Poincaré can then be interpreted by the hypothesis that  $F(z)$  and  $F_1(z)$  are not really uniform: but in order that this hypothesis should have a meaning, it is necessary to generalize the definition of continuation, in a way so as to be able to pass in certain cases the impassable cuts of Weierstrass; we shall see very soon how this result can be obtained.

But I wished before now to say some words concerning the ideas of Riemann, although it is specially in the study of non-uniform functions, of which I shall not speak here, that Riemann's theory has shown itself productive.

Cauchy has insisted several times on the importance of monogeneity. If an elementary function obtained by a simple calculation made on  $z$  is considered and if, for such a function  $G(z)$ , the ratio

$$\frac{G(z + \delta z) - G(z)}{\delta z}$$

is calculated, this ratio tends to a determinate limit when  $\delta z$  tends to zero, with any argument. Cauchy expresses this essential fact by calling the function monogenic.

If we put

$$G(z) = P(x, y) + iQ(x, y),$$

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the condition of monogeneity is translated into the two fundamental equations

$$\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y},$$
$$\frac{\partial P}{\partial y} = -\frac{\partial Q}{\partial x}.$$

Cauchy has shown that these equations, when they are verified in a region of the plane, involve the existence of the Taylor series; that is to say, of that which can be called analyticity in Weierstrass's sense. Cauchy's demonstration assumes the continuity of the derivative; M. Goursat, in a well-known piece of work, has shown that the existence of the first derivative is sufficient, and involves the continuity and existence of all the derivatives; M. Paul Montel has extended this result to cases where the existence of the derivative has not been assumed in a set of points of measure zero. The statement of these researches is outside my scope; I should mention them nevertheless, because they are in a way complementary to the results which I shall state further on. What is sufficient to remember is that, in the  $W$  regions, monogenic functions are analytic; for this reason the expression *monogenic function* is no longer in use by certain geometers, the expression *analytic function* being considered equivalent; as our aim is precisely to define monogenic functions which are not analytic, it is important to distinguish clearly between the two expressions.

It is difficult to find out if Cauchy conceived the existence of a monogenic function independently of any analytic expression. In fact, he always reasoned about functions which were defined, implicitly or explicitly through known functions, by means of ordinary or partial differential equations; but his reasoning applies without modification to a function defined in a purely ideal way as a correspondence

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between  $z$  and  $G(z)$ . This was the conception of Riemann, and has certainly rendered good service, as much in the field of real variables where it was introduced by Dirichlet, as in the field of complex variables, by accustoming mathematicians to very general methods of reasoning, made once for all and susceptible of application to cases not foreseen at the time when the reasoning was done. In fact, there is no real difference between Cauchy's and Riemann's point of view; to apply considerations like those of Riemann to *one* determinate function, this function must be defined, that is to say must be distinguishable from other functions; and if this definition is effective, it returns to the category of those which Cauchy admitted. This point belongs to the controversies concerning the axiom of Zermelo; Riemann's point of view is otherwise legitimate, whatever attitude is adopted in this controversy; for those who require a precise definition, it saves one from thinking of all the processes of definition which can be imagined; for those to whom an ideal definition is sufficient, it allows one to treat ideally even those functions which will never be defined practically.

It is by means of Cauchy's fundamental theorem

$$f(\delta) = \frac{1}{2\pi i} \int_c \frac{f(z)dz}{z - \xi}$$

that it can be shown that monogeneity in a  $W$  region involves analyticity in the region. We shall use this theorem also in studying monogenic functions in a region, not  $W$ ; it will be convenient in order to argue in a general manner about all the possible methods of definition of these functions, to consider them as defined in Riemann's way; that is, to assume that nothing is known about such a function except that it is monogenic. It is necessary to show afterward that a theory thus constructed is not empty, by giving actual examples of functions defined no longer ideally, but explicitly.

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I shall restrict myself to stating the definition of  $C$  regions in a particular case; if the properties of sets of zero measure studied in the previous lecture are used, it can be seen that this particular statement can be considerably generalized.

Let us consider a  $W$  region, and a region within  $W$  in which we define a denumerable infinity of fundamental points, everywhere dense; we shall assume that these fundamental points  $A_n$  are the points within the circle  $|z| = 1$  whose coördinates are rational. To each point  $A_n$  is attached a positive number  $r_n$ , and we shall assume that these numbers  $r_n$  tend very rapidly to zero as  $n$  increases indefinitely; we shall define later the manner of decrease; it is sufficient here to know that the remainder of the convergent series  $r_1 + r_2 + \dots + r_n + \dots$  is less than a quarter of the last term retained; we shall denote by  $C_q$  the region obtained by excluding from the  $W$  region the points within circles  $C_n^{(q)}$  defined as follows. Let us consider circles  $S_n^{(q)}$

having as their centers the points  $A_n$  and for their radii  $\frac{r_n}{2^q}$ ;

the circle  $C_1^{(q)}$  has its center at  $A_1$ , and its radius is the smallest of the numbers between  $r_1/2^q$  and  $r_1/2^{q+1}$  and such that it does not cut any of the circles  $S_n^{(q)} (n > 1)$ ; this is possible in virtue of the hypothesis  $r_1 > 4 \sum_2^{\infty} r_k$ , from which it follows

that  $\frac{r_1}{2^q} - \frac{r_1}{2^{q+1}} > 2 \sum_2^{\infty} \frac{r_k}{2^q}$ ; the  $S_n^{(q)}$  circles are then either inside

$C_1^{(q)}$  (including those which touch internally), or outside  $C_1^{(q)}$  (including those which touch externally). We shall take no account of the interior circles, and we shall denote by  $A_n$  the fundamental point of smallest index corresponding to the exterior circles; the circle  $C_2^{(q)}$  will have its center at  $A_n$  and its radius the smallest number contained

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between  $\frac{r_{n_2}}{2^q}$  and  $\frac{r_{n_2}}{2^{q+1}}$  such that it does not cut any of the circles  $S_n^{(q)} (n > n_2)$ ; it is exterior to the circle  $C_1^{(q)}$  since it is interior to the circle  $S_2^{(q)}$ , and at the same time exterior to the circles  $S_n^{(q)}$  of index less than  $n_2$ , for these circles are interior to  $C_1^{(q)}$  because of the method by which  $n_2$  was chosen. Similarly the circle  $C_3^{(q)}$  etc. is defined and one sees that if the region obtained by excluding the points inside circles  $C_n^{(q)}$  is denoted by  $C_q$ , and the region obtained by excluding the points inside circles  $S_n^{(q)}$  by  $C'_q$ , all the points of  $C_q$  belong to  $C'_{q+1}$ , while all the points of  $C'_{q+1}$  belong to  $C_{q+1}$ ; the consideration of the regions  $C_q$  is then equivalent to that of the regions  $C'_{q+1}$  and evades the difficulties which result from intersections of the circles.

The points of the circumference of  $C_n^{(q)}$  are said to constitute the frontier of  $C_q$ ; the points of  $C_q$  which do not belong to this frontier are called *interior* to  $C_q$ ; it is important to observe that we use the word *interior* here in a different sense from the usual one in the theory of  $W$  regions. The points of the set  $C_q$ , situated in the interior of the circle of radius 1, form a perfect set, which can be considered as the derived set of the set of its frontier points  $C_n^{(q)}$ .

The region  $C$  is defined as the set of all points such that each of them is interior to some  $C_q$ : the region  $C$  is not perfect, for it does not contain the points  $A_n$ , which are its limiting points. We know that the set (of zero measure) of points which do not belong to  $C$  has the power of the continuum. We shall say that a region  $D$  is *interior* to  $C$ , when all the points of  $D$  belong to one and the same  $C_q$ , of fixed index. Among the regions interior to  $C$ , we shall consider a little more exclusively the regions  $C_q$ : every point of  $C_q$  is interior to  $C_{q+1}$ .

The region  $C$  will be said to belong to the class  $(C)$  of

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Cauchy regions which we are studying here if the numbers  $r_n$  are such that, for  $n$  sufficiently large,

$$(1) \quad \log \log \log \frac{1}{r_n} > n;$$

if this condition is verified for two regions  $C$  and  $C'$ , it is verified for the part common to  $C$  and  $C'$ .

Together with the regions  $C_p$  and  $C$ , we shall consider *reduced* regions which we shall denote by  $\Gamma_p$  and  $\Gamma$ . To a region  $C$  corresponds a determinate system of regions  $C_p$ , and an infinity of systems of reduced regions; the following is the definition of one of these systems. Let us suppose numbers  $\rho_n$  given, tending to zero rapidly as  $n$  increases indefinitely, but much less rapidly than  $r_n$ ; more precisely, we shall suppose that

$$(2) \quad \frac{1}{\rho_n^2} < \log \log \frac{1}{r_n};$$

and, at the same time, whatever the fixed number  $\alpha$ , that, for  $n$  sufficiently large,

$$(3) \quad \frac{1}{\rho_n} > n^\alpha;$$

these two conditions (2) and (3) are quite consistent by virtue of (1).<sup>1</sup>

The regions  $\Gamma_p$  are defined by means of  $\rho_n$  as the  $C_p$ 's by means of  $r_n$ , that is, are limited by circles of radii between  $\frac{\rho_n}{2^p}$  and  $\frac{\rho_n}{2^{p+1}}$  exterior to each other. The region  $\Gamma$  is formed of the set of points *interior* (in the sense indicated above) to each  $\Gamma_p$ . The regions  $\Gamma_p$  are perfect,  $\Gamma$  is not perfect; the set complementary to  $\Gamma$  has zero measure and the power of the continuum.

The set  $C$  contains all points of  $\Gamma$  since  $C_p$  contains all

<sup>1</sup> (1) (2) and (3) could be replaced by wider conditions: my aim here is to simplify the statement.



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points of  $\Gamma_p$ : but  $C$  contains besides points which do not belong to  $\Gamma$ .

The following theorem is fundamental:

*If a function of the coördinates of a point  $P$  is defined in  $C$  and continuous in every  $C_p$ , the knowledge of its values at all points of  $\Gamma$  involves the knowledge of its values at all points of  $C$ .*

In other words, two functions continuous in  $C$  (that is, defined in all  $C$  and continuous in every region interior to  $C$ ) cannot coincide in all  $\Gamma$  without coinciding in all  $C$ ; or, finally, *a function continuous in  $C$  and zero in  $\Gamma$  is zero in  $C$ .*

In fact, let  $P$  be a point of  $C$ ; this point belonging to a set  $C_p$  interior to  $C$ , it is a limiting point of the set formed by the frontier<sup>1</sup> of  $C_p$ ; it is sufficient in order to prove that the function is zero at  $P$ , since it is continuous in  $C_p$ , to show that it is zero on each circumference which constitutes this frontier (the remark has already been made that each of these circumferences is interior to  $C_{p+1}$ ); then, on one of these circumferences (as on every rectifiable curve traced in the plane), the points which are part of  $\Gamma$  are everywhere dense; the function being continuous on this curve is then zero throughout this curve if it is zero at all points of  $\Gamma$ .

When we speak of a reduced region, we shall assume that we consider a determinate region, the  $\rho_n$ 's being chosen in a precise way, satisfying the inequalities (2) and (3). It might happen that we had to consider at the same time another region  $\Gamma'$  defined by numbers  $\rho'_n$ ; if

$$(4) \quad \rho_n'^{\beta} = \rho_n$$

we say that  $\Gamma'$  is of order  $\beta$  with respect to  $\Gamma$ ; if  $\beta$  is greater than one, the numbers  $\rho'_n$  satisfy the inequalities (2) and (3)

<sup>1</sup> We neglect points  $P$  which would be interior to  $C$  in Weierstrass's sense; for them the proposition is evident, since they are centers of circles inclosing no  $A_n$  in their interior, they are also interior to  $\Gamma$  in Weierstrass's sense.

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when the  $\rho_n$ 's satisfy them: in\* this case the set  $\Gamma'_p$  is interior to  $\Gamma_p$ , for the excluded circles of radii  $\frac{\rho'}{2^p}$  are larger than the circles of radii  $\frac{\rho_n}{2^p}$  (for  $\rho_n$  can always be supposed less than 1).

Let us remark finally that the points of  $C_p$  which lie on any curve whatever, a straight line for instance, form a perfect set, defined by contiguous intervals (in M. Baire's sense), which are the chords intercepted on the straight line by the circles. This set may or may not contain intervals: but in every case it is perfect, and consequently a function continuous in  $C_p$  and zero at all the points which limit the contiguous intervals is zero at all points of the set at the same time with all its derivatives in  $C_p$ .

### II. MONOGENIC FUNCTIONS IN $C$ REGIONS

WE shall say that a function  $F(z)$  is monogenic in a region such as  $C$  if:

1°. It is continuous *within*  $C$  (that is, as we have explained, continuous in every  $C_p$ , *interior* to  $C$ ; since the set  $C_p$  is perfect, this continuity in  $C_p$  is uniform);

2°. At every point  $P$  of  $C$ , it has a derivative with respect to  $z$ , unique and continuous within  $C$ . To define the derivative a set  $C_p$  of which  $P$  is a part is considered, and denoting by  $P'$  any other point of  $C$ , the limit of the ratio

$$(5) \quad \frac{F(\rho') - F(\rho)}{\overline{\rho\rho'}}$$

is found when the vector  $\overline{\rho\rho'} = z' - z$  tends to zero; if this limit exists for every value of  $\rho$ , it is evidently independent of the value of  $p$ , for all points of  $C_p$  belong to  $C_{p+q}$ ; for this reason this limit can be called the derivative of  $F(z)$

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*within*  $C$ , that is in every region *interior* to  $C$ . The continuity of the derivative within  $C$  is to be understood in the same way as the continuity of the function itself *within*  $C$ : continuity in each  $C_p$  interior to  $C$ . This hypothesis of the continuity of the derivative is doubtless superfluous; but it simplifies the argument.

Since the set  $C_p$  is perfect, every function continuous in  $C_p$  is *bounded* in  $C_p$ .

Let us mention at once an example of the simplest kind of a  $C$  region and of a function monogenic in this region.<sup>1</sup>

Let us form the series

$$F(z) = \sum_{n=1}^{\infty} \sum_{p=0}^n \sum_{q=0}^n \frac{e^{-\epsilon \frac{n^4}{n^2}}}{z - \frac{p+qi}{n}}.$$

Clearly this series is convergent outside the square  $T$  of which the vertices are the points  $z = 0, 1, i, 1 + i$ . Inside this square the series has an infinity of poles; in fact, all the points whose coördinates are rational numbers  $x = \frac{p}{n}, y = \frac{q}{n}$ .

But if circles having these poles as centers and radii  $\frac{\epsilon}{n^4}$  be considered, the series is absolutely and uniformly convergent at all points outside these circles, whatever the fixed number  $\epsilon$  may be. The same is true if circles  $\Gamma_n^{(h)}$  with centers at the points  $\frac{p}{n}, \frac{q}{n}$  and radii  $\frac{1}{h} e^{-\epsilon \frac{n^4}{n^2}}$  are considered, where  $h$  is a fixed integer which we are allowed to increase indefinitely. I shall call  $\Gamma_h$  the set of circles  $\Gamma_n^{(h)}$  and  $C_h$  the set of points which are not inside any of the circles  $\Gamma_n^{(h)}$ . There exists an infinity of curves which cross the circle and of which all the points belong to one same region  $C_h$ .

<sup>1</sup> The region  $C$  considered here is a little more general than the regions defined above, in the sense that the series  $\sum_n$  converges a little less rapidly.

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The function  $F(z)$  is evidently monogenic within the region  $C$  which is the limit of the  $C_n$ 's; it has in fact at each point of this region a determinate unique derivative, which is obtained by differentiating the series term by term. The value of this derivative is independent of the way in which the increment  $\delta z$  tends to zero, with the reservation, of course, that  $z$  and  $z + \delta z$  are inside  $C_n$ .

The study of monogenic functions within a region  $C$  requires the extension of Cauchy's fundamental theory to the contour which limits a perfect region  $C_n$ . To this end we shall establish at once the following fundamental property of a function  $F(z)$  monogenic in the region  $C$ . If we denote by  $p$  a fixed number

$$(6) \quad \int_K F(z) dz = \sum_n \int_{C_n^{(p)}} F(z) dz$$

the curve  $K$  being any simple curve all of whose points are inside  $C_p$ , the sum  $\Sigma$  referring to all the circles  $C_n^{(p)}$  which are inside  $C_p$ ; the integrals are all taken in the direct sense.

We shall set

$$(7) \quad F(z) = P(x, y) + i Q(x, y),$$

so that the equation (6) becomes two equations, of which it is sufficient to demonstrate one; for example,

$$(8) \quad \int_K P dx - Q dy = \sum_n \int_{C_n^{(p)}} P dx - Q dy.$$

To prove this relation, we define a function  $P_1(x, y)$ , finite and determinate at all points interior to  $K$ , and coinciding with  $P(x, y)$  at the points inside  $K$  which belong to  $C_p$ ; there remains the definition of  $P_1(x, y)$  inside the circles  $S_n^{(p)}$ ; on the circumference of these circles it coincides with  $P(x, y)$ . We shall define  $P_1(x, y)$  inside the circle by the condition that on chords of the circle parallel to  $Oy$  it varies

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linearly. (Its values at the extremities are known for they coincide with that of  $P(x, y)$ .) The function  $P_1(x, y)$  thus defined is continuous within  $K$  and has at every point a derivative  $\frac{\partial P_1}{\partial y}$ ; this derivative is bounded according to the hypothesis that the derivatives of  $P$  are bounded (which is involved by the existence and continuity of the derivatives of  $F(z)$ ); in fact, at points inside  $C_p$  the derivative of  $P_1$  coincides with the derivative of  $P$ ; at points inside  $C_n^{(p)}$ , the derivative of  $P_1$  is constant along a chord parallel to  $Oy$  and equal to the quotient of the difference of the values of  $P_1$  (that is, of  $P$ ) at the ends of the chord, divided by the length of this chord. The difference of the values of  $P$  is

$$(9) \quad \int_{MN} \frac{\partial P}{\partial x} dx + \frac{\partial P}{\partial y} dy$$

if  $MN$  denotes the arc subtended by the chord.

This integral is less than the product of the length of the arc  $MN$  and the sum  $\frac{\partial P}{\partial x} + \left| \frac{\partial P}{\partial y} \right|$  and its quotient when it is divided by the chord  $MN$  is at most equal to

$$\frac{\pi}{2} \left( \left| \frac{\partial P}{\partial x} \right| + \left| \frac{\partial P}{\partial y} \right| \right)$$

and is consequently bounded at the same time as the derivatives

$$\frac{\partial P}{\partial x} \text{ and } \frac{\partial P}{\partial y}.$$

Similarly the values of  $P_1$  lying between the values of  $P$ ,  $P_1$  have the same boundary as  $P$ .<sup>1</sup>

<sup>1</sup> The derivative  $\frac{\partial P_1}{\partial y}$  is discontinuous at points on a circumference. This produces no inconvenience; one can modify the definition of  $P_1$  by choosing other curves instead of straight lines. Sufficiently simple results can be obtained by taking the sum of a parabola and a sinusoidal curve.

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According to a classical result, denoting by  $^{(\kappa)}$  the area within  $K$

$$\iint_{^{(\kappa)}} \frac{\partial P_1}{\partial y} dx dy = - \int_K P_1 dx = - \int_K P dx$$

since on  $K$ ,  $P_1$  coincides with  $P$ .

Similarly  $Q_1(x, y)$  being defined by means of  $Q(x, y)$  in the same way as  $P_1$  by means of  $P$  (taking always parallels to  $Ox$  in place of parallels to  $Oy$ ):

$$\iint_{^{(\kappa)}} \frac{\partial Q_1}{\partial x} dx dy = \int_K Q_1 dy = \int_K Q dy$$

It follows that

$$(10) \quad \int_K P dx - Q dy = - \iint_{^{(\kappa)}} \left( \frac{\partial P_1}{\partial y} + \frac{\partial Q_1}{\partial x} \right) dx dy.$$

The double integral of the right-hand side reduces to zero for those portions of the area  $(K)$  which belong to  $C_p$ , for at a point inside  $C_p$

$$\frac{\partial P_1}{\partial y} + \frac{\partial Q_1}{\partial x} = \frac{\partial P}{\partial y} + \frac{\partial Q}{\partial x} = 0.$$

The formula (10) then reduces to

$$(11) \quad \int_K P dx - Q dy = - \sum \int_{C_n^{(p)}} \left( \frac{\partial P_1}{\partial y} + \frac{\partial Q_1}{\partial x} \right) dx dy.$$

But the area of  $C_n^{(p)}$  is equal to  $\frac{\pi r_n^2}{4^p}$ ; on the other hand, the moduli of  $\frac{\partial P_1}{\partial y}$  and  $\frac{\partial Q_1}{\partial x}$  are less than a fixed number independent of  $n$  (depending on  $p$ , but  $p$  is fixed); then

$$(12) \quad \left| \int_K P dx - Q dy \right| < M \sum r_n^2.$$

It is easy to obtain from this the formula (6); since the series  $\sum r_n^2$  is, in fact, convergent, we can choose  $n$  in such a way that the remainder of this series  $\sum_{n+1}^{\infty} r_n^2$  is less than  $\frac{\epsilon}{M}$ .

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When the number  $n$  has been thus chosen, let us denote by  $K'$  the contour formed of the contour  $K$  traversed in the direct sense and the circumferences  $C_1^{(p)}$ ,  $C_2^{(p)}$ , ...,  $C_n^{(p)}$  traversed in the retrograde sense; we can argue about  $K'$  as we have done about  $K$  (by completing it if we wish by rectilinear cuts to make it a simple contour); we shall obtain

$$\left| \int_{K'} P dx - Q dy \right| < M \sum_{n+1}^{\infty} r_n^2 < \epsilon$$

that is, the integrals being taken in the direct sense

$$\left| \int_K P dn - Q dy - \sum_{r=1}^n \int_{C_r^{(p)}} P dx - Q dy \right| < \epsilon.$$

If  $\epsilon$  is made to tend to zero,  $n$  increases indefinitely and from it we obtain the relation (8) from which the relation (6) follows.

We deduce now from (6) Cauchy's fundamental theorem; let  $x$  denote a point within a reduced region  $\Gamma_p$  and  $\gamma_q$  a circle with center  $x$  within  $Cp$ , and with radius between  $\frac{1}{2^q}$  and  $\frac{1}{2^{q-1}}$ .

There exists such a circle  $\gamma_q$ , whatever the number  $q$  (at least after a certain value of  $q$ ). In fact  $x$  being within  $\Gamma_p$ , whatever  $n$  may be,  $a_n$  being the affix of  $A_n$

$$|x - a_n| > \frac{1}{2^p} P_n$$

Consequently, the points  $a_n$  for which

$$|x - a_n| < \frac{1}{2^{q-1}}$$

are such that

$$(13) \quad \frac{1}{2^p} P_n < \frac{1}{2^{q-1}}$$

Let us denote by  $n_q$  the smallest value of  $n$  after which this inequality (13) is satisfied; all the  $a_n$ 's inside the circle

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of center  $x$  and radius  $\frac{1}{2^{q-1}}$  have indices greater or equal to  $n_q$ ; the sum  $\frac{1}{2^p} \sum r_n$  of the radii of the corresponding circles in  $C_p$  is then extremely small compared to  $\frac{1}{2^p} P_{n_q}$  since the  $r_n$ 's are much smaller than the corresponding  $P_n$ 's; since this sum is extremely small compared to  $\frac{1}{2^{q-1}}$ , there exist circles of center  $x$  and radius between  $\frac{1}{2^{q-1}}$  and  $\frac{1}{2^q}$  and which do not cut any of the circles  $C_p^{(p)}$ ; *a fortiori* they do not cut the circles  $C_n^{(p)}$  whose centers are more distant from  $x$ , for the radii  $\frac{r_n}{2^p}$  of these other circles are very small compared with  $\frac{P_n}{2^p}$  and their centers are further from  $x$  than  $\frac{P_n}{2^p}$ .

The circle  $\gamma_q$  being thus defined, let us consider the function

$$f(z) = \frac{F(z)}{z - x}$$

within the region contained between the contour  $K$  and  $\gamma_q$ ; clearly in this region this function is monogenic; we then obtain the relation

$$\int_K f(z) dz - \int_{\gamma_q} f(z) dz = \sum \int_{C_n^{(p)}} f(z) dz$$

the sum on the right-hand side referring to the  $C_n^{(p)}$ 's which are contained between  $\gamma_q$  and  $K$ .

If  $M$  denotes the maximum value of  $|F(z)|$  within  $C_p$ , the maximum value of  $f(z)$  on different  $C_n^{(p)}$ 's is evidently  $2^{q+1}M$ ; if  $q+1$  is put in the place of  $q$ , an infinity of new terms are introduced on the right-hand side, but it is easily seen that the lengths of the paths of integration (circumferences of the  $C_n^{(p)}$ 's contained between  $\gamma_q$  and  $\gamma_{q+1}$ ) have a



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sum of an order much less than  $\frac{1}{2^{q+1}}$ ; the right-hand side is then a convergent series and

$$\lim_{q \rightarrow \infty} \int_{\gamma_q} f(z) dz = \int_K f(z) dz - \sum \int_{C_n^{(p)}} f(z) dz$$

the sign  $\Sigma$  now referring to all the circumferences  $C_n^{(p)}$  which limit  $C_p$ . As for the left-hand side, it follows from the continuity of  $F(z)$  at the point  $x$  in  $C_p$ , all the  $\gamma_q$ 's being interior to  $C_p$ , that it is equal to  $2\pi i F(x)$ . The generalized Cauchy formula follows

$$(14) \quad 2\pi i F(x) = \int_K \frac{F(z) dz}{z - x} - \sum_n \int_{C_n^{(p)}} \frac{F(z) dz}{z - x}.$$

From this formula the classical consequences can be deduced and in particular the fact that *monogeneity (existence of the first derivative) within the region C involves the existence of the derivatives of all orders*. This formula (14) shows moreover that non-analytic monogenic functions can be put in the form of series whose terms are analytic functions. It is natural then to look for an associative method of continuation applicable to such sums. The problem is nothing else than the problem of divergent series: to each analytic function corresponds a Taylor development convergent in a circle, but divergent outside this circle; this development is determined by a knowledge of the values of the derivatives. If a series of analytic functions is indefinitely differentiable, its derivatives are expressed linearly by means of the derivations of the terms, and the Taylor series which corresponds to these derivatives is a linear function of the Taylor series corresponding to the different terms of the series. But if the function is not analytic at the point where the series is developed, this Taylor series will be the sum of series whose radii of convergence decrease indefinitely and, in the case we are studying, will have a zero radius of convergence.

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The problem of divergent series consists in transforming such a series into a convergent series in such a way that the result coincides with the analytic continuation in the case where this continuation is possible. Thanks to the fine researches of M. Mittag-Leffler, this problem has been resolved for the first time in an entirely satisfactory way; it should be observed that, if it is desired to use these results for the continuation of non-analytic monogenic functions, they must be interpreted either by the language of divergent series, or by an equivalent language if one prefers not to speak of divergent series; but in every case by a new language, specially adapted to the real novelty of the results, and not by the old language of Weierstrassian analytic continuation; that is the only language which may not be used, since it has an absolutely precise meaning, which cannot be modified; Weierstrass's theory is, in some way, so perfect that it can only be departed from by creating a new language: if, as M. Mittag-Leffler proposed, Weierstrass's language were adopted, M. Mittag-Leffler's series would be only a simplified method of calculation containing nothing more from the theoretical point of view than Weierstrass's theory contains.

### III. CONTINUATION BY SERIES ( $M$ )

IN order to study continuation by M. Mittag-Leffler's series, or series ( $M$ ), we suppose that the point is interior to a reduced region  $\Gamma'_p$ , of order equal to 2 with respect to  $\Gamma_p$  (the circles of exclusion are defined by numbers  $\rho'_n$  equal to  $\sqrt{\rho_n}$ ); evidently then an infinity of straight lines issuing from the point  $x$  can be drawn interior to  $\Gamma_p$ . More precisely, if  $x$  belongs to  $\Gamma'$ , within every given angle having its vertex  $x$ , a straight line interior to  $\Gamma_p$ , of convenient index, can be found; this index can increase in-

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definitely as the angle tends to zero, but is determinate when the angle is given (this follows from the fact that the sum of the angles subtended at  $x$  by the circles which limit  $\Gamma_{p'}$  is less than twice the sum of the convergent series  $\sum \left( \frac{\rho_n}{2^{p'}} : \frac{\rho'_n}{2^p} \right)$  and is consequently as small as we please if  $p'$  is sufficiently large). We shall suppose, so as not to complicate our notation, that  $p$  has been taken equal to  $p'$  in the preceding argument (the point  $x$  interior to  $\Gamma_p$  is *a fortiori* interior to  $\Gamma_{p'}$  if  $p' > p$ ).

We develop  $F(z)$  in a series on one of the straight lines which we are about to define, interior to  $\Gamma_p$ . Each of the terms of the right-hand side of (14) is an analytic function on this straight line and can therefore be developed in a series of Mittag-Leffler or ( $M$ ) polynomials; it is enough to show that the multiple series formed of the set of these series is absolutely convergent, in order to show that it represents  $2\pi i F(z)$ .

This series is then formed by means of the derivatives of  $F(z)$  at the point  $x$  (these derivatives exist, as we have remarked, according to (14) for every displacement on the straight line and in  $\Gamma_p$ ), in the same way as the ( $M$ ) development of an analytic function is formed by means of the derivatives of that function; we assume, to save writing, that  $x = 0$ .

I remind the reader of the properties of ( $M$ ) developments which I have demonstrated in my memoir on series of polynomials and rational fractions ("Acta Mathematica," I, xxiv). One finds that

$$(15) \quad \frac{1}{1-z} = \sum G_n(z),$$

$G_n(z)$ 's being polynomials which it is useless to write again and the series  $\sum |G_n(z)|$  being convergent in the 'star.' A

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region  $S(R, \rho)$  is defined as follows:  $R$  being  $> 1$  and  $\rho < 1$ , we consider the circle of center  $o$  and radius  $R$ , the circle of center  $1$  and radius  $\rho$  and the tangents to this last circle from  $o$ , the points of contact being  $M$  and  $N$ ; the region  $S(R, \rho)$  is bounded by the arc  $MN$  less than  $\pi$ , the continuations  $MM'$  and  $NN'$  of  $OM$  and  $ON$  as far as the circumference of radius  $R$  and the arc  $M'N'$  greater than  $\pi$ . In this region, putting  $\frac{32R}{\rho} = \lambda$ ,

$$(16) \quad \sum |G_i(z)| < R^{\lambda^{\lambda}}.$$

Consider an integral along one of the circumferences  $C_n^{(p)}$  of radius  $\frac{r_n}{2^p}$ .

$$(17) \quad \int_{C_n^{(p)}} f(z) dz$$

We develop it on a straight line interior to  $\Gamma_p$ , that is outside the circumference having the same center  $a_n$  as  $C_n^{(p)}$  and of radius  $\frac{\rho_n}{2^p}$ . The radius  $\frac{r_n}{2^p}$  being very small compared with  $\rho_n$ , we shall commit no appreciable error by replacing this integral by the majorant function  $\frac{Mr_n}{a_n - x}$ , denoting by  $M$  the maximum of  $|F(z)|$  in  $C_p$ ,  $2\pi r_n$  being the length of the path of integration (we suppress the factors  $2^p$  which have no influence since  $p$  is fixed). If one puts  $x = a_n x'$ ,

$$(18) \quad \frac{Mr_n}{a_n - x} = \frac{Mr_n}{a_n} \frac{1}{1 - x'}.$$

If the point  $x$  is inside the region  $S(R, \rho)$  defined by the circle of radius  $\frac{\rho_n}{2^p}$  and center  $A_n$  and by a circle of radius  $> 1$  (2 for example) which contains within it all the regions we are considering, the point  $x' = \frac{x}{a_n}$  will be within the region

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$S\left(\frac{2}{|a_n|}, \frac{\rho_n}{|a_n|}\right)$ , and by developing  $\frac{1}{1-x'}$  we get the inequality

$$\sum |G_n(x')| < \left[\frac{2}{|a_n|}\right]^{\lambda^\lambda}$$

putting  $\lambda = \frac{64}{\rho_n}$ ; since  $|a_n|$  is greater than  $\rho_n$  we can write

$$\sum |G_n(x')| < \lambda^{\lambda^\lambda}.$$

The development ( $M$ ) of (17) is, according to (18), when all the terms are replaced by their moduli, less than

$$\frac{Mr_n}{|a_n|} \lambda^{\lambda^\lambda}$$

But according to (2)

$$\frac{1}{r_n} > e^{\frac{1}{e\rho_n^2}}$$

and if  $n$  is large enough  $\frac{1}{\rho_n^2} > \lambda^{\frac{3}{2}}$  since  $\lambda = \frac{64}{\rho_n}$  and so  $|a_n|$  being  $> \rho_n$ ,

$$M \frac{r_n}{|a_n|} \lambda^{\lambda^\lambda} < \lambda M \lambda^{\lambda^\lambda} e^{-e^{\lambda^{\frac{3}{2}}}}.$$

This converges very rapidly to zero when  $n$ , and consequently  $\lambda$ , increases indefinitely. The absolute convergence of the ( $M$ ) series is then demonstrated.

Now consider two points  $x_1$  and  $x_2$  belonging to  $\Gamma'$ ; we can construct two angles  $A_1$  and  $A_2$  with vertices at  $x_1$  and  $x_2$ , and such that every half-straight line  $D_1$  within  $A_1$  meets every half-straight line  $D_2$  within  $A_2$  at a point  $x_3$  within the total region considered. We can choose  $D_1$  and  $D_2$  in such a way that these two straight lines belong to the same  $\Gamma_p$  ( $p$  being chosen large enough, but afterwards remaining fixed). It will then be possible to calculate the function at  $x_2$  by means of its values and the values of its derivatives at  $x_1$ , by forming only two ( $M$ ) developments, one with the

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origin  $x_1$  and the other with the origin  $x_3$ . If the function is zero at  $x$ , as well as all its derivatives, these  $(M)$  developments are identically zero and the function is zero at  $x_2$ . From what has been said further back it can be concluded that if a monogenic function is zero at every point of an arc, however small (at all points of this arc interior to  $C$ ), when there exists on this arc at least one point interior to  $\Gamma_p$ , a limit of points interior to  $\Gamma_p$ , the function being zero at all these points is zero, as well as its derivatives, at one point of  $\Gamma_p$  at least, and consequently identically zero in  $\Gamma_p$  (whatever  $p$  may be) and identically zero in  $C$ . These new monogenic functions possess then the fundamental property of analytic functions.

### IV. THE LOGARITHMIC POTENTIAL

IN the preceding work we have considered singular isolated points, corresponding in the physical point of view to the hypothesis of an infinite density at certain points; a statement can easily be given in which the density is everywhere finite.

Consider a regular uniform analytic function zero at infinity. If  $\Sigma$  is a circle such that all the singular points of the function are inside  $\Sigma$ , if  $\zeta$  is any point outside  $\Sigma$ ,

$$F(\zeta) = \frac{1}{2\pi i} \int_{\Sigma} \frac{F(z)}{\zeta - z} dz$$

the integration being taken in the direct sense.

Let  $\Sigma_1$  and  $\Sigma_2$  be two concentric circles outside  $\Sigma$ , let  $a$  be the center of these circles,  $\rho_1$  and  $\rho_2$  their radii.

Evidently, if  $\rho$  is contained between  $\rho_1$  and  $\rho_2$ ,

$$F(\zeta) = \frac{1}{2\pi} \int_0^{2\pi} \frac{F(a + \rho e^{i\alpha}) \rho e^{i\alpha} d\alpha}{\zeta - a - \rho e^{i\alpha}}.$$

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If we multiply this equality by  $(\rho_2 - \rho)^m (\rho - \rho_1)^n$  and integrate between the limits  $\rho_1$  and  $\rho_2$ , the expression becomes

$$F(\zeta) \int_{\rho_1}^{\rho_2} (\rho_2 - \rho)^m (\rho - \rho_1)^n d\rho \\ = \frac{1}{2\pi} \int_0^{2\pi} \frac{F(a + \rho e^{i\alpha}) (\rho_2 - \rho)^m (\rho - \rho_1)^n \rho e^{i\alpha} d\alpha d\zeta}{\zeta - a - \rho e^{i\alpha}}$$

Put

$$\int_{r_1}^{r_2} (\rho_2 - \rho)^m (\rho - \rho_1)^n d\rho = \frac{1}{2\pi} H$$

$$a + \rho e^{i\alpha} = x + iy.$$

Then

$$F(\zeta) = \frac{1}{H} \iint_{C_1 C_2} \frac{F(x + iy) (\rho_2 - \rho)^m (\rho - \rho_1)^n e^{i\alpha} dx dy}{\zeta - x - iy}$$

or putting

$$\zeta = \xi + i\eta$$

$$(19) \quad F(\zeta) = \theta(\xi, \eta) = \iint_{C_1 C_2} \frac{\phi(x, y) dx dy}{\xi + i\eta - x - iy}$$

the region of integration being the ring contained between the circles  $C_1$  and  $C_2$ .

We shall define the function  $\phi(x, y)$  outside this ring by giving it the value zero; the whole plane can then be taken as the region of integration. The function  $\phi(x, y)$  is bounded and continuous in the whole plane; its derivatives are also bounded, at least as far as order  $m$  on  $C_1$  and as far as order  $n$  on  $C_2$ ; by an artifice analogous to that which we are about to employ, it would be easy to arrange matters so that all the derivatives would be continuous; in general it is enough to know that the derivatives are continuous as far as some order, fixed beforehand.

If the function  $F(z)$  has a singular point  $a$ ,  $\rho_1$  can be made to tend to zero and if, further, the product  $\rho^m F(z)$  remains finite for  $z = a$ , the formula holds for  $\rho_1 = 0$ ; if this product does not remain finite, in the formula we replace  $(\rho - \rho_1)^m$

by  $e^{\bar{p}}$  or  $e^{-e''}$  etc. Further, in the case of a unique singular point, the circle  $C_2$  can be drawn with a radius as small as we please, after the circle  $C_1$  has been reduced to zero.

It is easy to deduce from this that every regular analytic uniform function, zero at infinity, can be represented in every region  $D$  interior to its region of existence  $W$ , and approaching  $W$  as nearly as we wish, by an expression of the form

$$(20) \quad F(\zeta) = \theta(\xi, \eta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\phi(x, y) dx dy}{\xi + i\eta - x - iy}$$

the function  $\phi(x, y)$  being bounded and, further, zero at all points of  $D$  (this hypothesis involves the fact that  $\phi(x, y)$  is zero at infinity, since the point at infinity belongs to  $D$ ).

Inversely every expression of the form (20) in which  $\phi(x, y)$  is a bounded function, zero at infinity, and continuous in the whole plane, as well as its derivatives (at least up to order  $m$ ), represents a function which is monogenic at every point where  $\phi(x, y)$  is zero; for by a simple calculation,

$$\frac{\partial \theta}{\partial \xi} + i \frac{\partial \theta}{\partial \eta} = 2 \pi \phi(\xi, \eta).$$

If the points where  $\phi(x, y)$  is zero form a  $W$  region, the theory of analytic functions shows us that the function  $\theta(\xi, \eta)$  is determined at every point of  $W$  by the knowledge of its values in the neighborhood of any particular point of  $W$ . The problem of the general determination of the region of existence of monogenic functions can then be set as follows: to find the conditions which  $\phi(x, y)$  should satisfy in order that this fundamental property of  $\theta(\xi, \eta)$  should hold; that is, that the knowledge of this function on an arc of a curve where it is monogenic allows the calculation of its value in the whole region of monogeneity.



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Consider, for example, the series of rational fractions with simple poles. Denoting by  $C_a$  a circle with center  $a$  and radius  $\rho$ , and taking a point  $\zeta$  outside this circle for which  $|z - a| = r$ ,

$$\frac{1}{\zeta - a} = \iint_{C_a} \frac{3(\rho - r)}{\pi \rho^3} \frac{dx dy}{\zeta - z}.$$

When the point  $\zeta$  is inside the circle  $C_a$ , the integral is easily calculated; putting  $\left| \frac{\zeta - a}{\rho} \right| = \lambda$  its value is

$$\frac{3 \lambda^2 - 2 \lambda^3}{\zeta - a}.$$

The function

$$\theta(\xi, \eta; a) = \iint_{C_a} \frac{3(\rho - r)}{\pi \rho^3} \frac{dx dy}{\xi + i\eta - x - iy}$$

is then bounded in the whole plane; outside  $C_a$  it is monogenic and coincides with the analytic function  $\frac{1}{\zeta - a}$ .

Evidently an infinity of functions  $\theta_n(\xi, \eta)$  can be defined in a similar way, such that the equation

$$\theta_n(\xi, \eta) = \frac{1}{\zeta - a_n}$$

holds for every point  $\zeta = \xi + i\eta$  outside the circle  $C_n$  with center  $a_n$  and radius  $\rho_n$ , these functions being moreover *bounded* and *continuous* in the whole plane; if the  $|a_n|$ 's are bounded and if the coefficients  $A_n$  are such that the series

$$\sum \frac{|A_n|}{\rho_n^3}$$

is convergent, the series

$$\theta(\xi, \eta) = \sum \theta_n(\xi, \eta)$$

will be absolutely and uniformly convergent in the whole plane, and will be represented by an integral of the form

$$(21) \quad \theta(\xi, \eta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\phi(x, y) dx dy}{\xi + i\eta - x - iy},$$

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the function  $\phi(x, y)$  being the sum of a series everywhere convergent whose respective terms are zero outside various circles  $C_n$ ; this function  $\phi(x, y)$  is then zero at all points exterior to all these circles and the function  $\theta(\xi, \eta)$  is monogenic at these points. If the radii  $\rho_n$  are replaced by  $\epsilon\rho_n$ ,  $\epsilon$  being as small as we please, the function  $\phi(x, y)$  is zero in a more and more extended region; it remains bounded, but its bound increases indefinitely as  $\epsilon$  tends to zero. We are thus led to consider *a priori* a function such as (21) and to study it in the region  $C$  where  $\phi(x, y)$  is zero. It is natural to suppose the region  $C$  to be simply connected; we limit ourselves to the case where this region  $C$  consists of  $W$  regions (these regions may reduce to a zero as a limiting case) and of a finite or infinite number of straight lines  $\Delta$ , in such a way that any two points can be reunited by a polygonal line with a finite number of sides.

An important idea is then that of the order of infinity of the function  $\phi(x, y)$  in the neighborhood of the straight lines. By a calculation\* analogous to that which has just been developed, the convergence of the  $(M)$  developments can be shown by making the hypothesis that  $\phi(x, y)$  is not only zero upon the straight lines (which is the necessary condition of monogeneity) but tends very rapidly to zero in the neighborhood of each straight line. More precisely, if  $\sigma$  denotes the distance of the point  $(x, y)$  from the straight line  $\Delta$  considered, it is assumed that the product

$$e^{\frac{1}{\sigma^2}} \phi(x, y)$$

tends uniformly to zero as  $\sigma$  tends to zero. By means of this hypothesis, it can be affirmed that the function  $\theta(\xi, \eta)$  is determined in the whole region of its existence by the knowledge of its values at any point of this region. This hypothesis contains as a special case the condition satisfied

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by analytic functions in  $W$  regions, for if a straight line is within  $W$ , the function  $\phi(x, y)$  is identically zero at all points whose distances from the straight line are less than a number  $\sigma$ , chosen conveniently.

The region  $C$  can be reduced to the real axis; that is the case of the function

$$\phi(\xi) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{e^{-\frac{1}{2}\pi y^2} dx dy}{(x^2 + y^2)(\xi - x - iy)}$$

The Taylor development

$$\theta(\xi) = \theta(\xi_0) + (\xi - \xi_0)\theta'(\xi_0) + \dots$$

diverges for any value of  $\xi_0$  but is summable ( $M$ ), whatever  $\xi_0$  may be, for every value of  $\xi$ , its sum being equal to the function  $\theta(\xi)$ . The function  $\theta(\xi)$  will be called quasi-analytic.

Calculations of double integrals of form (21) lead easily to expressions of the same form; similarly in differentiating, transforming the double integral by integration by parts, it is only necessary to assume the existence of the derivatives of  $\phi(x, y)$  exactly to the order of the derivatives of  $\theta(\xi, \eta)$  which it is desired to calculate. To calculate the product, if we put

$$(22) \quad \theta(\xi, \eta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\phi(x, y) dx dy}{\xi - z}$$

$$(23) \quad \theta_1(\xi, \eta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\phi_1(x, y) dx dy}{\xi - z},$$

the product becomes

$$\theta(\xi, \eta)\theta_1(\xi, \eta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\phi(x, y)\phi_1(x_1, y_1) dx dy dx_1 dy_1}{(\xi - z)(\xi - z_1)},$$

or since

$$\frac{1}{(\xi - z)(\xi - z_1)} = \frac{1}{z - z_1} \left( \frac{1}{\xi - z} - \frac{1}{\xi - z_1} \right),$$

if we put

$$\begin{aligned}\psi(x, y) &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\phi(x_1, y_1) dx_1 dy_1}{z - z_1}, \\ \psi_1(x, y) &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\phi_1(x_1, y_1) dx_1 dy_1}{z - z_1}, \\ (24) \quad \theta(\xi, \eta) \theta_1(\xi, \eta) &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{[\phi(x, y) \psi_1(x, y) + \psi(x, y) \phi_1(x, y)] dx dy}{\zeta - z}.\end{aligned}$$

We can then put in the form of a double integral (24) every polynomial  $P$  in terms of one or more functions of  $\theta(\xi, \eta)$  and their derivatives; if the regions of existence have a simply connected common region the differential equation obtained by equating  $P$  to zero cannot be satisfied in any portion of this region without being satisfied in the whole region  $C$ .

## V. CONCLUSION

THE results we are establishing suppress the absolutely sharp demarcation established by Weierstrass's theory between real analytic functions and real non-analytic functions. I do not wish to develop the consequences of this fact from the point of view of the theory of functions; I prefer to insist a little on its importance from the point of view of the relations between mathematics and physics. It is a necessary postulate in the application of mathematics to experimental sciences, that sufficiently slight variations in the data ought not to influence the results appreciably; for, if it were not so, since the experimental data are never known with vigorous precision, one could not foresee any phenomenon. But certain mathematical properties are at least apparently discontinuous, depending for example on

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the fact that some number is rational or irrational. Thus the solutions of the equation

$$\frac{d^2y}{dx^2} + m^2y = \cos nx$$

are of a different nature according as the ratio  $\frac{m}{n}$  is commensurable or incommensurable. Nevertheless, in this case, if  $\frac{m}{n}$  varies continuously, the solution  $y$  varies very little in an interval of variation of  $x$  large compared with the length of the periods. It is not always thus, certainly, but the cases in which there is no continuity have been little studied; the equation

$$\left(\frac{dx}{dt}\right)^2 + x^2 = a^2 + 2\lambda t$$

can be given as an example in which the solutions vary discontinuously as  $\lambda$  becomes equal to zero; but this equation does not come under the Hamiltonian type.

It is important to know whether the properties of harmonic functions (that is, of potentials) vary continuously when the definition of the functions itself varies continuously. This has no place in Weierstrass's theory; the introduction of quasi-analytic functions restores continuity; a distribution of attracting masses infinitely near to  $Ox$  leads, if the density is *sufficiently slight* in the neighborhood of  $Ox$ , to properties of the potential on  $Ox$  which are not dissimilar from the case where the density is zero in the neighborhood of  $Ox$ .

EMILE BOREL.

THE GENERALIZATION OF ANALYTIC  
FUNCTIONS  
ON THE THEORY OF WAVES AND GREEN'S  
METHOD \*

THE GENERALIZATION OF ANALYTIC  
FUNCTIONS†

INTRODUCTION

THE generalization which is treated in the following pages has already been the subject of several investigations of mine, in the first place in several notes, published in the "Rendiconti" of the Reale Accademia dei Lincei, then in an extended memoir which appeared in the "Acta Mathematica." Several of the lectures which I read at Stockholm were also devoted to this subject. And it is now my purpose, in returning to it, to consider the general case in some detail, beginning with the first foundations. In treating the general case it is necessary to consider certain elements, which I have called functions of hyperspaces, and which represent extensions of the functions of curves that I have already treated several times, in particular, in a recent course at the Sorbonne.

A space of  $n$  dimensions contains spaces of  $0, 1, 2, \dots, n-1$  dimensions, and for that reason we consider functions of

\*Three lectures presented at the inauguration of the Rice Institute, by Senator Vito Volterra, Professor of Mathematical Physics and Celestial Mechanics in the University of Rome.

†Translated from the Italian by Professor Griffith Conrad Evans, of the Rice Institute.

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these spaces. We shall begin by extending to these functions the fundamental concepts of continuity and differentiation, and we shall consider the condition that a function be of the first degree. This condition depends upon an extension of Stokes's theorem. We shall then consider a relation between these functions analogous to that of monogeneity, which for functions in the ordinary sense was established by Cauchy. This leads to new types of equations with functional derivatives, which present analogies with the equation of Laplace.

We can separate the functions with which we are dealing into elementary and otherwise. The former have interesting properties and applications. A certain operation of composition turns out to possess quite curious arithmetical properties.

We shall finally develop the operations of differentiation and integration, and the extension of Cauchy's theorem in complete generality.

### THE GENERALIZATION OF ANALYTIC FUNCTIONS

#### *First Lecture*

GENERAL OBSERVATIONS ON HYPERSPACES — GENERAL FORMULÆ FOR MATRICES, AND RELATIONS BETWEEN THE DIRECTION COSINES OF HYPERSPACES — FUNCTIONS OF HYPERSPACES AND THEIR DERIVATIVES — EXTENSION OF STOKES'S THEOREM — CONDITIONS WHICH THE DERIVATIVES OF FUNCTIONS OF HYPERSPACES MUST SATISFY, AND FORMULÆ FOR THE TRANSFORMATION OF COÖRDINATES — ISOGENEITY — CONDITIONS FOR ISOGENEITY.

#### *1. General observations on hyperspaces*

1. A hyperspace (space of  $n$  dimensions) will be characterized by the multiplicity of values of  $n$  independent variables  $x_1, x_2, \dots, x_n$ . A hyperspace  $S_r$  of  $r$  dimensions ( $r < n$ ), contained in it, will correspond to the multiplicity of values which the  $x_1, x_2, \dots, x_n$  assume when they are constrained

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by  $n - r$  independent relations, or in other words, when they depend on  $r$  independent variables  $\omega_1, \omega_2, \dots, \omega_r$  to which they are bound by the  $n$  relations

$$(I) \quad \begin{cases} x_1 = x_1(\omega_1, \omega_2, \dots, \omega_r) \\ x_2 = x_2(\omega_1, \omega_2, \dots, \omega_r) \\ \vdots \\ x_n = x_n(\omega_1, \omega_2, \dots, \omega_r) \end{cases}$$

We assume the differentiability of the preceding relations, and obtain

$$(2) \quad dx_i = \sum_{s=1}^r \frac{\partial x_i}{\partial \omega_s} d\omega_s \quad (i = 1, 2, \dots, n).$$

2. Let us consider the matrix

$$(3) \quad \begin{vmatrix} \frac{\partial x_1}{\partial \omega_1} & \frac{\partial x_2}{\partial \omega_1} & \dots & \frac{\partial x_n}{\partial \omega_1} \\ \frac{\partial x_1}{\partial \omega_2} & \frac{\partial x_2}{\partial \omega_2} & \dots & \frac{\partial x_n}{\partial \omega_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial x_1}{\partial \omega_r} & \frac{\partial x_2}{\partial \omega_r} & \dots & \frac{\partial x_n}{\partial \omega_r} \end{vmatrix}$$

Let  $\Delta^2$  be the square of this matrix, and let us assume that if the sign of  $\Delta$  is given at one point, it is fixed by continuity at all other points. When the sign of  $\Delta$  is given we shall say that the *direction* of the hyperspace  $S_r$  is fixed. The quantity

$$dS_r = \Delta d\omega_1 d\omega_2 \dots d\omega_r$$

will be called the *element of the hyperspace*.

Let us take a minor determinant of the matrix (3)

$$\Delta_{i_1 i_2 \dots i_r} = \begin{vmatrix} \frac{\partial x_{i_1}}{\partial \omega_1} & \frac{\partial x_{i_2}}{\partial \omega_1} & \dots & \frac{\partial x_{i_r}}{\partial \omega_1} \\ \frac{\partial x_{i_1}}{\partial \omega_2} & \frac{\partial x_{i_2}}{\partial \omega_2} & \dots & \frac{\partial x_{i_r}}{\partial \omega_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial x_{i_1}}{\partial \omega_r} & \frac{\partial x_{i_2}}{\partial \omega_r} & \dots & \frac{\partial x_{i_r}}{\partial \omega_r} \end{vmatrix}$$



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and write

$$(4) \quad \alpha_{i_1 i_2 \dots i_r} = \frac{\Delta_{i_1 i_2 \dots i_r}}{\Delta}.$$

The  $\alpha_{i_1 i_2 \dots i_r}$  will not change if we substitute for the  $\omega_1, \omega_2, \dots, \omega_r$  other variables bound by arbitrary relations to the first, and their signs will change only if we change the sign of the hyperspace; we shall call them the *direction cosines* of the hyperspace. We see at once that they must satisfy the relation

$$(A) \quad \sum_i \alpha_{i i_2 \dots i_r}^2 = 1,$$

in which  $\sum_i$  denotes summation extended over all the combinations of the indices  $i_1, i_2, i_n$ .

3. If a space  $S_{n-r}$  has a point in common with  $S_r$ , and the direction cosines of  $S_{n-r}$  are denoted by  $\beta_{h_1 \dots h_{n-r}}$ , we shall say that the two hyperspaces are normal to each other when we have the relation

$$\alpha_{i_1 i_2 \dots i_r} = \beta_{h_1 h_2 \dots h_{n-r}},$$

where all the  $i$ 's are different from the  $h$ 's, and the series of numbers  $i_1, i_2, \dots, i_r, h_1, h_2, \dots, h_{n-r}$  is a permutation of the numbers  $1, 2, \dots, n$ , which is always odd or always even.

4. Whatever  $l$  may be, we can write

$$(5) \quad d\omega_l = \sum_i A_{i_1 i_2 \dots i_{r-1}} \frac{d(x_{i_1}, x_{i_2}, \dots, x_{i_{r-1}})}{d(\omega_1, \omega_2, \dots, \omega_{l-1}, \omega_{l+1}, \dots, \omega_r)} \quad (l = 1, 2, \dots, r)$$

in which the sum is extended over all the combinations of the indices  $i_1, i_2, \dots, i_{r-1}$ , and the  $A$ 's are certain, in part indeterminate, infinitesimal parameters. In fact if we form the matrix of the coefficients of the  $A$ 's, among its minors will be found the  $r - 1$ th powers of the minors of the matrix (3), and so not all the minors of that matrix can be zero. If we substitute the values (5) in

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the equations (2) we obtain

$$(6) \quad dx_s = -\sum_i \frac{d(x_s, x_{i_1} \dots x_{i_{r-1}})}{d(\omega_1, \omega_2 \dots \omega_r)} A_{i_1 i_2 \dots i_{r-1}}.$$

Hence if  $a_{i_1, i_2, \dots, i_{r-1}} = -\Delta A_{i_1 i_2 \dots i_{r-1}}$  we shall have

$$(7) \quad dx_s = \sum_i a_{i_1 i_2 \dots i_{r-1}} \alpha_{i_1 i_2 \dots i_{r-1}}.$$

5. Besides the equations ( $\Lambda$ ) the  $\alpha$  satisfy other relations, which we shall find in the next section.

## 2. General formulæ about matrices. Relations between the direction cosines of a hyperspace

1. We shall establish in this section several fundamental formulæ regarding the minors of matrices, which we shall often have occasion to use. Let us consider the two matrices

$$(1) \quad \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{r1} & a_{r2} & \dots & a_{rn} \end{vmatrix} \quad (2) \quad \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{p1} & a_{p2} & \dots & a_{pn} \end{vmatrix}$$

the first with  $r$  rows, and the second with  $p$  rows, ( $n > r \geq p$ ), both however with the same elements. Let us write

$$\begin{vmatrix} a_{1i_1} & a_{1i_2} & \dots & a_{1i_r} \\ \cdot & \cdot & \cdot & \cdot \\ a_{ri_1} & a_{ri_2} & \dots & a_{ri_r} \end{vmatrix} = A_{i_1 i_2 \dots i_r}, \quad \begin{vmatrix} a_{1h_1} & a_{1h_2} & \dots & a_{1h_p} \\ \cdot & \cdot & \cdot & \cdot \\ a_{ph_1} & a_{ph_2} & \dots & a_{ph_p} \end{vmatrix} = B_{h_1 h_2 \dots h_p}$$

and consider

$$\Delta_s = \begin{vmatrix} a_{si_1} & a_{si_2} & \dots & a_{si_{r+1}} \\ a_{1i_1} & a_{1i_2} & \dots & a_{1i_{r+1}} \\ \cdot & \cdot & \cdot & \cdot \\ a_{ri_1} & a_{ri_2} & \dots & a_{ri_{r+1}} \end{vmatrix} = 0 \quad (s = 1, 2, \dots, p).$$

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We shall have

$$\begin{aligned} 0 &= \sum_{1 \leq s}^p \frac{\partial B_{h_1 h_2 \dots h_p}}{\partial a_{s h_1}} \Delta_s \\ &= - \sum_{1 \leq s}^p \frac{\partial B_{h_1 h_2 \dots h_p}}{\partial a_{s h_1}} \sum_{1 \leq t}^{r+1} (-1)^t a_{s t} A_{t_1 \dots t_{t-1} t_{t+1} \dots t_{r+1}} \\ &= - \sum_{1 \leq t}^r A_{t_1 \dots t_{t-1} t_{t+1} \dots t_{r+1}} \sum_{1 \leq s}^p (-1)^s a_{s t} \frac{\partial B_{h_1 \dots h_p}}{\partial a_{s h_1}}. \end{aligned}$$

From this it follows that

$$(3) \quad \sum_{1 \leq t}^{r+1} (-1)^t A_{t_1 \dots t_{t-1} t_{t+1} \dots t_{r+1}} B_{t_1 h_2 \dots h_p} = 0.$$

2. This is the formula which we wished to obtain. In particular, if we take as identical the two matrices (1) and (2), we shall have

$$(3') \quad \sum_{1 \leq t}^{r+1} (-1)^t A_{t_1 \dots t_{t-1} t_{t+1} \dots t_{r+1}} A_{t_1 h_2 \dots h_r} = 0.*$$

Among these equations let us notice specially the following, from which the others all follow:

$$(4) \quad 0 = A_{t_1 t_2 h_1 \dots h_{r-2}} A_{t_2 t_3 h_1 \dots h_{r-2}} + A_{t_1 t_2 h_1 \dots h_{r-2}} A_{t_1 t_3 h_1 \dots h_{r-2}} \\ + A_{t_1 t_2 h_1 \dots h_{r-2}} A_{t_1 t_2 h_1 \dots h_{r-2}}.^\dagger$$

3. From the preceding formulæ we see that the direction cosines of a hyperspace must satisfy the relations

$$(B) \quad \sum_{1 \leq s}^{r+1} (-1)^s \alpha_{t_1 t_2 \dots t_{s-1} t_{s+1} \dots t_{r+1}} \alpha_{s h_2 \dots h_r} = 0.$$

\* Vedi Antonelli: "Nota sulle relazioni indipendenti" ecc. ("Ann. d. Scuola Normale sup. di Pisa," Vol. III, page 71 e seg.)

† *Ibid.*, page 73.

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## 3. Functions of hyperspaces and their derivatives\*

1. A variable  $\phi$  will be said to be a *function of the hyperspace*  $S_r$  (of  $r$  dimensions) or a function of *order*  $r$ , if to every possible hyperspace with fixed direction corresponds a value of  $\phi$ . This correspondence will be denoted by means of the symbol  $\phi = \phi | [S_r] |$ . We shall assume that we are dealing only with closed hyperspaces  $S_r$ .†

Let us take a point  $P$  of  $S_r$  and through it draw a hyperspace  $S_{n-r}$  normal to  $S_r$ , taking in  $S_{n-r}$  a small neighborhood  $s$  of  $P$ . If we make  $P$  describe all the points of  $S_r$  we shall generate a portion of  $n$ -dimensional space, which we shall call a neighborhood of  $S_r$ . While  $P$  is describing  $S_r$  any other point  $P'$  of  $s$  describes a new hyperspace  $S'_r$ , which we shall say belongs to the neighborhood of  $S_r$ . The function  $\phi | [S_r] |$  will be said to be continuous if, when we take a quantity  $\sigma$  arbitrarily small, we can find a neighborhood of  $S_r$  such that

$$\text{mod } [\phi | [S'_r] | - \phi | [S_r] |] < \sigma,$$

where  $S'_r$  belongs to that neighborhood.

Besides the continuity of  $\phi | [S_r] |$  let us admit also the following property. Let us pass from the hyperspace  $S_r$  to the hyperspace  $S'_r$  by giving to each point of  $S_r$  a displacement  $\epsilon$  which varies continuously from point to point. The displacement  $\epsilon$  generates a hyperspace  $S_{r+1}$  of  $r+1$  dimensions, of amplitude say,  $\sigma$ . We shall assume that we can make  $\{ \phi | [S'_r] | - \phi | [S_r] | \}$  less than a number chosen arbitrarily small, provided  $\sigma$  be less than some value  $\sigma_0$ .

2. With this understood, take in  $S_r$  a neighborhood  $s$  of a point  $P$ , and give to  $s$  a displacement  $\delta x_i$  parallel to  $x_i$ .

\* Vedi la mia Nota I: "Sulle funzioni dipendenti da linee." ("Atti d. R. Acc. d. Lincei," Vol. III, fasc. 9.)

† Vedi: Betti: "Sopra gli spazii di un numero qualunque di dimensioni." (Annali di Mat., T. IV.)

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Let us denote by  $\delta\phi$  the corresponding variation of  $\phi$ , and let us suppose that the value

$$\lim_{\substack{s=0 \\ \delta x_i=0}} \frac{\partial\phi}{s \cdot \partial x_i} = \phi'_{x_i} \quad (i = 1, 2, \dots, n)$$

exists. We shall call this *the derivative of  $\phi$  with respect to  $x_i$  at the point  $P$* . With the assumption that the ratio which appears in the left-hand member approaches its limit uniformly, with respect to all possible points  $P$  and hyperspaces  $S_r$ , and that this limit is continuous, we can easily verify the fact that if we give to every point of  $S_r$  a displacement of components  $\delta x_1, \delta x_2, \dots, \delta x_n$ , the corresponding variation of  $\phi$  is given, except for infinitesimals of higher order, by the formula

$$(I) \quad \delta\phi = \int_{S_r} \sum_{i=1}^n \phi'_{x_i} \delta x_i dS_r.$$

3. Let us find out now what conditions the  $\phi'_{x_i}$  must satisfy. If the displacements are such as to carry the space  $S_r$  into itself, the quantity  $\delta\phi$  must vanish. Hence we must have  $\delta\phi = 0$  if we take (see § 1, form 7)

$$\delta x_i = \sum_h a_{h_1 h_2 \dots h_{r-1}} \alpha_{ih_1} \dots \alpha_{ih_{r-1}}$$

whatever the quantities  $a$  may be. Hence

$$0 = \int_{S_r} \sum_h \alpha_{h_1 h_2 \dots h_{r-1}} \sum_{i=1}^n \phi'_{x_i} \alpha_{ih_1} \dots \alpha_{ih_{r-1}} dS_r$$

and from this we have

$$(2) \quad \sum_{i=1}^n \phi'_{x_i} \alpha_{ih_1} \dots \alpha_{ih_{r-1}} = 0$$

for every possible combination of the indices  $h_2 \dots h_{r-1}$ .

4. Since now the  $\alpha$  satisfy the relations § 2, (B), we have

$$\sum_{i=1}^{r+1} (-1)^i \alpha_{q_i h_1 \dots h_{r-1}} \alpha_{q_1 \dots q_{i-1} q_{i+1} \dots q_{r+1}} = 0.$$

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If we multiply this by an undetermined parameter  $\lambda_{q_1 q_2 \dots q_{r+1}}$  which satisfies the condition that it changes sign for every transposition of the indices, we shall have

$$\begin{aligned} 0 &= \sum_q \lambda_{q_1 q_2 \dots q_{r+1}} \sum_{t=1}^{r+1} (-1)^t \alpha_{q_1 h_1 \dots h_{r-1}} \alpha_{q_1 \dots q_{t-1} q_{t+1} \dots q_{r+1}} \\ &= \sum_{t=1}^n \sum_q \lambda_{t q_1 \dots q_r} \alpha_{q_1 \dots q_r} \alpha_{t h_1 \dots h_{r-1}} \end{aligned}$$

and subtracting this from equation (2),

$$0 = \sum_{t=1}^n \left\{ \phi'_{x_t} - \sum_q \lambda_{t q_1 \dots q_r} \alpha_{q_1 q_2 \dots q_r} \right\} \alpha_{t h_1 \dots h_{r-1}}$$

whence

$$(3) \quad \phi'_{x_t} = \sum_q \lambda_{t q_1 \dots q_r} \alpha_{q_1 q_2 \dots q_r}^*$$

From this it follows that

$$\begin{aligned} \delta\phi &= \int_{S_r} \sum_{t=1}^n \sum_q \lambda_{t q_1 \dots q_r} \alpha_{q_1 q_2 \dots q_r} \delta x_t dS_r \\ &= \int_{S_r} \sum_q \lambda_{q_1 q_2 \dots q_{r+1}} \left\{ \sum_{t=1}^{r+1} (-1)^{t-1} \alpha_{q_1 q_2 \dots q_{t-1} q_{t+1} \dots q_{r+1}} \delta x_{q_t} \right\} dS_r. \end{aligned}$$

Consider now the elements  $dS_r$  and suppose drawn through every point of it a segment of components  $\delta x_1 \dots \delta x_n$ . The locus of these segments will be a space  $S_{r+1}$  of  $r+1$  dimensions. If the equations of the hyperspace  $S_r$  are

$$x_i = x_i(\omega_1, \omega_2, \dots, \omega_r) \quad (i = 1, 2, \dots, n)$$

the equations of the hyperspace  $S_{r+1}$  will be

$$x_i = x_i(\omega_1, \omega_2, \dots, \omega_r) + \omega_{r+1} \delta x_i \quad (i = 1, 2, \dots, n).$$

\* See my Note II: "Sulle funzioni dipendenti da linee." ("Atti della R. Acc. dei Lincei," Vol. III, fasc. 10.)

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Let us form the matrix

$$(4) \quad \begin{array}{ccccccc} & \frac{\partial x_1}{\partial \omega_1}, & \frac{\partial x_2}{\partial \omega_1} & \dots & \frac{\partial x_n}{\partial \omega_1} & & \\ & \cdot & \cdot & & \cdot & \cdot & \\ & \frac{\partial x_1}{\partial \omega_r}, & \frac{\partial x_2}{\partial \omega_r} & \dots & \frac{\partial x_n}{\partial \omega_r} & & \\ & \partial x_1, & \partial x_2 & \dots & \partial x_n & & \end{array}$$

Let us denote its square by  $\Delta_{r+1}^2$ , and the square of the matrix obtained from it by taking away the last line by  $\Delta_r^2$ . We shall have

$$\Delta_{r+1}^2 = \Delta_r^2 \left\{ \sum_q \sum_1^{r+1} (-1)^{t-1} \alpha_{q_1 \dots q_{t-1} q_{t+1} \dots q_{r+1}} \delta x_{q_t} \right\}^2$$

We can fix the direction of  $S_{r+1}$  with respect to  $S_r$  in such a way that

$$\Delta_{r+1} = (-1)^r \Delta_r \sqrt{\sum_q \sum_1^{r+1} (-1)^{t-1} \alpha_{q_1 \dots q_{t-1} q_{t+1} \dots q_{r+1}} \delta x_{q_t}}$$

where the sign of the radical is taken as positive. If now we denote the direction cosines of  $S_{r+1}$  by  $\beta_{q_1 q_2 \dots q_{r+1}}$ , which are calculated from the matrix (4), we shall have finally

$$\delta \phi = \int_{S_{r+1}} \sum_q \lambda_{q_1 q_2 \dots q_{r+1}} \beta_{q_1 q_2 \dots q_{r+1}} dS_r$$

Hence if  $S_r$  is a movable hyperspace which passes from  $S'_r$  to  $S''_r$ , thus generating a  $S_{r+1}$ , we shall have

$$(5) \quad \phi|[S''_r] - \phi|[S'_r] = \int_{S_{r+1}} \sum_q \lambda_{q_1 q_2 \dots q_{r+1}} \beta_{q_1 q_2 \dots q_{r+1}} dS_{r+1}.$$

It is well to note explicitly that besides varying from point to point of the total hyperspace (of  $n$  dimensions), the parameters  $\lambda$  may also vary for one and the same point according to the hyperspace to which they refer, and even

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for the same hyperspace one set of  $\lambda$ 's may be substituted for another provided the relations (3) are always satisfied.

5. A function  $\phi |[S_r]|$  will be said to be *regular* (or *simple*) when the following condition is satisfied. Let  $S'_r$  and  $S''_r$  be two hyperspaces having a common portion  $s$ , whose direction is different according as it is considered as belonging to the first or the second hyperspace. Denote by  $S'''_r$  the hyperspace which we get by taking away  $s$  from the combination of  $S'_r$  and  $S''_r$  and fix as its direction the direction of those two hyperspaces. We impose the condition

$$\phi |[S'''_r]| = \phi |[S'_r]| + \phi |[S''_r]|.$$

When  $\phi$  is regular it follows immediately that if  $S_r$  decreases indefinitely in amplitude

$$(C) \quad \lim \phi |[S_r]| = 0.$$

We have then immediately the further property that if  $S_r$  and  $S'_r$  are two hyperspaces with a common point  $P$ , whose elements at  $P$  are contained in a single  $S_{r+1}$ , of  $r+1$  dimensions,

$$(6) \quad \sum_a (\lambda_{a_1 a_2 \dots a_{r+1}} - \lambda'_{a_1 a_2 \dots a_{r+1}}) \beta_{a_1 a_2 \dots a_{r+1}} = 0$$

where  $\lambda$  and  $\lambda'$  are the parameters which correspond to  $\phi |[S_r]|$  and  $\phi |[S'_r]|$  at the point  $P$ , and the  $\beta$ 's are the direction cosines of  $S_{r+1}$ .

Upon this basis let us consider a hyperspace  $S_r$  passing through the point  $P$ , whose element at  $P$  is defined by the equations

$$dx_i = \sum_1^r a_{is} d\omega_s \quad (i = 1, 2, \dots, n)$$

and let  $S_r^{(i_1 \dots i_r)(h_1 \dots h_p)}$  denote hyperspaces passing through  $P$  defined by the equations

$$\begin{aligned} dx_{i_s} &= a_{i_s s} d\omega_s + \sum_1^p a_{i_s h_t} d\omega_{h_t} & \begin{cases} s = 1, 2, \dots, r \\ s \neq h_1, h_2, \dots, h_p. \end{cases} \\ dx_{i_v} &= \sum_1^p a_{i_v h_t} d\omega_{h_t} & (v = h_1, h_2, \dots, h_p, r+1, \dots, n.) \end{aligned}$$



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In particular let us consider the hyperspaces  $S^{(i_1 \dots i_{s-1} i_{s+1} \dots i_{r+1})}$  and  $S_r^{(i_1 \dots i_{r-1} i_{r+1} \dots i_{r+1})}$  whose elements at  $P$  are contained in a hyperspace of  $r+1$  dimensions, of which the direction cosines  $\beta$  are zero, except  $\beta_{i_1 i_2 \dots i_{r+1}} = 1$ . By means of (6), we have

$$\lambda_{i_1 i_2 \dots i_{r+1}}^{(i_1 \dots i_{s-1} i_{s+1} \dots i_{r+1})} = \lambda_{i_1 i_2 \dots i_{r+1}}^{(i_1 \dots i_{r-1} i_{r+1} \dots i_{r+1})}$$

where the indices  $i_1 i_2 \dots i_r$  denote the parameters  $\lambda$  corresponding to the hyperspace  $S_r^{(i_1 \dots i_r)}$ . Therefore we can suppress the indices and write simply

$$(7) \quad \lambda_{i_1 i_2 \dots i_{r+1}}^{(i_1 \dots i_{s-1} i_{s+1} \dots i_{r+1})} = \Lambda_{i_1 i_2 \dots i_{r+1}}.$$

6. Two hyperspaces  $S_r^{(i_1 \dots i_r)(h_1 \dots h_{p-1})}$  and  $S_r^{(i_1 \dots i_r)(h_1 \dots h_p)}$  have elements at  $P$  which are contained in a  $S_{r+1}$ , whose element at  $P$  is defined by the equations

$$\begin{aligned} dx_{i_s} &= a_{i_s s} d\omega_s + \sum_1^p a_{i_s h_t} d\omega_{h_t} & \begin{cases} s = 1, 2, \dots, r \\ s \neq h_1, h_2, \dots, h_p \end{cases} \\ dx_{i_{h_p}} &= a_{i_{h_p} h_p} d\omega_{r+1} + \sum_1^p a_{i_{h_p} h_t} d\omega_{h_t} \\ dx_{i_v} &= \sum_1^p a_{i_v h_t} d\omega_{h_t}, & (v = h_1, h_2, \dots, h_{p-1}, r+1, \dots, n.) \end{aligned}$$

Hence, if we denote by  $\beta$  the direction cosines of  $S_{r+1}$  and by  $\alpha^{(i_1 \dots i_r)(h_1 \dots h_p)}$  the direction cosines of  $S_r^{(i_1 \dots i_r)(h_1 \dots h_p)}$ , we shall have

$$\frac{\beta_{i_{h_p} m_1 m_2 \dots m_r}}{\alpha_{m_1 m_2 \dots m_r}^{(i_1 \dots i_r)(h_1 \dots h_p)}} = \kappa,$$

where  $\kappa$  is independent of the indices  $m_1, m_2, \dots, m_r$ , and all the  $\beta$ 's are zero, in the indices of which  $i_{h_p}$  is missing. From this it follows by reason of (6) that

$$\sum_m \left( \lambda_{i_{h_p} m_1 \dots m_r}^{(i_1 \dots i_r)(h_1 \dots h_p)} - \lambda_{i_{h_p} m_1 \dots m_r}^{(i_1 \dots i_r)(h_1 \dots h_{p-1})} \right) \alpha_{m_1 \dots m_r}^{(i_1 \dots i_r)(h_1 \dots h_p)} = 0,$$

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where the index  $(i_1 \dots i_r)(h_1 \dots h_p)$ , affixed to the  $\lambda$ , means that refers to the hyperspace having the same index. We have then

$$\sum_m \lambda_{i_{h_p} m_1 \dots m_r}^{(i_1 \dots i_r)(h_1 \dots h_p)} \alpha_{m_1 \dots m_r}^{(i_1 \dots i_r)(h_1 \dots h_p)} = \sum_m \lambda_{i_{h_p} m_1 \dots m_r}^{(i_1 \dots i_r)(h_1 \dots h_{p-1})} \alpha_{m_1 \dots m_r}^{(i_1 \dots i_r)(h_1 \dots h_p)},$$

in which, by means of (3), we can substitute for the  $\lambda_{i_{h_p} m_1 \dots m_r}^{(i_1 \dots i_r)(h_1 \dots h_p)}$  the  $\lambda_{i_{h_p} m_1 \dots m_r}^{(i_1 \dots i_r)(h_1 \dots h_{p-1})}$ , and consequently, the  $\Lambda_{i_{h_p} m_1 \dots m_r}$  of formula (7).

We observe however that the hyperspace  $S_r^{(i_1 \dots i_r)(i_1 \dots i_p)}$  is nothing but the hyperspace  $S_r$ , and therefore we can take for the  $\lambda$ 's belonging to this space, at the point  $P$ , the  $\lambda$ 's without index of formula (6). We have then the theorem

*If  $\phi$  is a regular function of the hyperspace  $S_r$ , contained in a hyperspace  $S_n$ , there exist for every point of  $S_n$  a system of values which can be considered as the parameters  $\lambda_{i_1 i_2 \dots i_{r+1}}$  for all the hyperspaces  $S_r$  which pass through that point.*

7. From the equations (5) (C), assuming that  $\phi|[S_r]|$  is regular we get,

$$(5') \quad \phi|[S_r]| = \int_{S_r} \sum_a \Lambda_{a_1 a_2 \dots a_{r+1}} \beta_{a_1 a_2 \dots a_{r+1}} dS_{r+1}.$$

Here  $S_{r+1}$  is an arbitrary hyperspace of  $r+1$  dimensions, whose boundary is  $S_r$ . If  $S_{r+1}$  grows indefinitely smaller about a point  $P$ , by writing

$$S_{r+1} = \int_{S_{r+1}} dS_{r+1}$$

we shall have

$$\lim \frac{\phi|[S_r]|}{S_{r+1}} = \sum_a \Lambda_{a_1 a_2 \dots a_{r+1}} \beta_{a_1 a_2 \dots a_{r+1}} = \frac{d\phi}{dS_{r+1}}$$

where the  $\beta$  are the direction cosines of  $S_{r+1}$  at  $P$ .

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Let us take  $S_{r+1} = S_{r+1}^{(i_1 i_2 \dots i_{r+1})}$  such that at  $P$  all the direction cosines  $\beta$  shall be zero except  $\beta_{i_1 i_2 \dots i_{r+1}} = 1$ . We shall have

$$\lim_{S_{r+1}^{(i_1 i_2 \dots i_{r+1})}} \frac{\phi | [S_r] |}{S_{r+1}^{(i_1 i_2 \dots i_{r+1})}} = \Lambda_{i_1 i_2 \dots i_{r+1}}.$$

Therefore we shall write

$$\Lambda_{i_1 i_2 \dots i_{r+1}} = \frac{\partial \phi}{\partial (x_{i_1} x_{i_2} \dots x_{i_{r+1}})},$$

and define this quantity as the *derivative of  $\phi$  with respect to  $x_{i_1} x_{i_2} \dots x_{i_{r+1}}$* . What relations must these derivatives satisfy? Before proceeding to the search for these relations, it will be necessary to give an extension of Stokes's theorem, a subject which is dealt with in the next section.

### 4. Extension of Stokes's theorem

1. Let  $L_{i_1 i_2 \dots i_r}$  be functions of the points of the hyperspace  $S_n$ , such that every transposition of the indices creates a change of sign, and form the expression

$$(I) \quad M_{i_1 i_2 \dots i_{r+1}} = \sum_1^{r+1} (-1)^{s-1} \frac{\partial L_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+1}}}{\partial x_{i_s}}.$$

Let  $S_{r+1}$  be a hyperspace, of  $r+1$  dimensions, bounded by a set of hyperspaces  $S_r$ , let its direction cosines be  $a_{i_1 i_2 \dots i_{r+1}}$ , and form the expression

$$\int_{S_{r+1}} \Omega dS_{r+1},$$

putting  $\Omega = \Sigma_i M_{i_1 i_2 \dots i_{r+1}} a_{i_1 i_2 \dots i_{r+1}}$ .

If the equations of  $S_{r+1}$  are

$$x_i = x_i(\omega_1, \omega_2 \dots \omega_{r+1}) \quad (i = 1, 2, \dots n),$$

we shall have

$$\begin{aligned}
 \Omega dS_{r+1} &= \sum_i M_{i_1, i_2, \dots, i_{r+1}} \frac{d(x_{i_1} x_{i_2} \dots x_{i_{r+1}})}{d(\omega_1 \omega_2 \dots \omega_{r+1})} d\omega_1 d\omega_2 \dots d\omega_{r+1} \\
 &= \sum_i \sum_1^n \frac{\partial L_{i_1, \dots, i_{s-1} i_{s+1} \dots i_{r+1}}}{\partial x_{i_s}} \frac{d(x_{i_1} x_{i_2} \dots x_{i_{s-1}} x_{i_{s+1}} \dots x_{i_{r+1}})}{d(\omega_1 \omega_2 \dots \omega_{r+1})} d\omega_1 d\omega_2 \dots d\omega_{r+1} \\
 &= \sum_i \frac{d(L_{i_1, \dots, i_r}, x_{i_1} x_{i_2} \dots x_{i_r})}{d(\omega_1, \omega_2, \dots \omega_{r+1})} d\omega_1 d\omega_2 \dots d\omega_{r+1} \\
 &= \sum_i \sum_1^{r+1} (-1)^{i-1} \frac{\partial L_{i_1, \dots, i_r}}{\partial \omega_i} \frac{d(x_1 \dots x_r)}{d(\omega_1 \dots \omega_{i-1} \omega_{i+1} \dots \omega_{r-1})} d\omega_1 d\omega_2 \dots d\omega_{r+1}.
 \end{aligned}$$

Hence

$$\begin{aligned}
 \int_{S_{r+1}} \Omega dS_{r+1} &= \int_{S_r} \sum_i \sum_1^{r+1} L_{i_1, i_2, \dots, i_r} \frac{d(x_{i_1} x_{i_2} \dots x_{i_r})}{d(\omega_1 \dots \omega_{i-1} \omega_{i+1} \dots \omega_{r+1})} d\omega_1 \dots d\omega_{i-1} d\omega_{i+1} \dots d\omega_{r+1}.
 \end{aligned}$$

We can make the hyperspace  $S$  depend on  $r$  independent parameters  $\bar{\omega}_1, \bar{\omega}_2 \dots \bar{\omega}_r$ , whence we shall have

$$\int_{S_{r+1}} \Omega dS_{r+1} = \int_{S_r} \sum_i L_{i_1, i_2, \dots, i_r} \frac{d(x_{i_1} x_{i_2} \dots x_{i_r})}{d(\bar{\omega}_1 \bar{\omega}_2 \dots \bar{\omega}_r)} d\bar{\omega}_1 d\bar{\omega}_2 \dots d\bar{\omega}_r.$$

From this comes the formula

$$(2) \quad \int_{S_{r+1}} \sum_i M_{i_1, i_2, \dots, i_{r+1}} \alpha_{i_1, i_2, \dots, i_{r+1}} dS_{r+1} = \int_{S_r} \sum_i L_{i_1, i_2, \dots, i_r} \beta_{i_1, i_2, \dots, i_r} dS_r,$$

where the  $\beta$ 's are the direction cosines of the hyperspace  $S_r$ .

2. From these formulæ it follows that if

$$\int_S \sum_i L_{i_1, i_2, \dots, i_r} \beta_{i_1, i_2, \dots, i_r} dS = 0,$$

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for every closed hyperspace  $S_r$  in the region  $S_n$ , the necessary and sufficient conditions that must be satisfied are

$$(3) \quad M_{i_1 i_2 \dots i_{r+1}} = \sum_1^{r+1} (-1)^{s-1} \frac{\partial L_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+1}}}{\partial x_{i_s}} = 0$$

for every combination of the indices  $i_1 i_2 \dots i_{r+1}$ .

5. *Conditions which the derivatives of functions of hyperspaces must satisfy. Formulæ for the change of coördinates*

1. Let  $\phi|[S_r]|$  be regular, and return to formula (5') of section 3. Since the integral which appears in the right-hand member does not change when  $S_{r+1}$  changes, provided the boundary  $S_r$  does not change, we must have

$$\int_{S_{r+1}} \sum_a \Lambda_{a_1 a_2 \dots a_{r+1}} \beta_{a_1 a_2 \dots a_{r+1}} dS_{r+1} = 0$$

when the integration is extended over any closed hyperspace  $S_{r+1}$ . Hence the necessary and sufficient conditions which the  $\Lambda$  must satisfy in order to be the derivatives of a regular function of hyperspaces  $S_r$  (see section 4, article 2) is

$$(D) \quad \sum_1^{r+2} (-1)^{s-1} \frac{\partial \Lambda_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}}}{\partial x_{i_s}} = 0$$

for every possible combination of the indices  $i_1 i_2 \dots i_{r+2}$ . We can write these equations, making use of the symbols of section 3, article 7, in the form

$$(D') \quad \sum_1^{r+2} (-1)^{s-1} \frac{\partial}{\partial x_{i_s}} \frac{\partial \phi}{\partial (x_{i_1} \dots x_{i_{s-1}} x_{i_{s+1}} \dots x_{i_{r+2}})} = 0.$$

We shall call these conditions the *conditions of integrability*.

2. Consider now the formulæ for change of variable, transforming the variables  $x_1, x_2 \dots x_n$  into  $x'_1, x'_2 \dots x'_n$  by

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means of the relations

$$x'_i = x'_i(x_1, x_2, \dots, x_n) \quad (i = 1, 2, \dots, n)$$

such that

$$\frac{d(x'_1, x'_2, \dots, x'_n)}{d(x_1, x_2, \dots, x_n)}$$

is always finite and different from zero. Let us consider two regions which correspond in a one-to-one manner,  $S_n$  and  $S'_n$ , one belonging to the first set of variables, the other to the second. Let  $S_{r+1}$  be a hyperspace, bounded by  $S_r$  and contained in  $S_n$ , and let  $S'_{r+1}$ , bounded by  $S'_r$ , correspond to it in  $S'_n$ . If we suppose that  $S_{r+1}$  is given by the equations

$$x_i = x_i(\omega_1 \omega_2 \dots \omega_{r+1}) \quad (i = 1, 2 \dots n),$$

we shall have

$$\begin{aligned} \phi | [S_r] | &= \int_{S_{r+1}} \sum_i \frac{\partial \phi}{\partial (x_{i_1} x_{i_2} \dots x_{i_{r+1}})} \frac{d(x_{i_1} \dots x_{i_{r+1}})}{d(\omega_1 \dots \omega_{r+1})} d\omega_1 d\omega_2 \dots d\omega_{r+1} \\ &= \int_{S_{r+1}} \sum_i \frac{\partial \phi}{\partial (x_{i_1} \dots x_{i_{r+1}})} \sum_h \frac{d(x_{i_1} \dots x_{i_{r+1}})}{d(x'_{h_1} \dots x'_{h_{r+1}})} \frac{d(x'_{h_1} \dots x'_{h_{r+1}})}{d(\omega_1 \dots \omega_{r+1})} d\omega_1 \dots d\omega_{r+1} \\ &= \int_{S_{r+1}} \sum_h \frac{d(x'_{h_1} \dots x'_{h_{r+1}})}{d(\omega_1 \dots \omega_{r+1})} \sum_i \frac{\partial \phi}{\partial (x_{i_1} \dots x_{i_{r+1}})} \frac{d(x_{i_1} \dots x_{i_{r+1}})}{d(x'_{h_1} \dots x'_{h_{r+1}})} d\omega_1 \dots d\omega_{r+1} \\ &= \int_{S'_{r+1}} \sum_h \beta'_{h_1 \dots h_{r+1}} \left( \sum_i \frac{\partial \phi}{\partial (x_{i_1} \dots x_{i_{r+1}})} \frac{d(x_{i_1} \dots x_{i_{r+1}})}{d(x'_{h_1} \dots x'_{h_{r+1}})} \right) dS'_{r+1} \end{aligned}$$

where the  $\beta'$  denote the direction cosines of  $S'_{r+1}$ .

If we write

$$\Lambda'_{h_1 h_2 \dots h_{r+1}} = \sum_i \frac{\partial \phi}{\partial (x_{i_1} \dots x_{i_{r+1}})} \frac{d(x_{i_1} \dots x_{i_{r+1}})}{d(x'_{h_1} \dots x'_{h_{r+1}})}$$

we shall have

$$\phi | [S_r] | = \phi | [S'_r] | = \int_{S'_{r+1}} \sum_h \Lambda'_{h_1 h_2 \dots h_{r+1}} \beta'_{h_1 h_2 \dots h_{r+1}} dS'_{r+1},$$

whence

$$\Lambda'_{h_1 \dots h_{r+1}} = \frac{\partial \phi}{\partial (x'_{h_1} x'_{h_2} \dots x'_{h_{r+1}})}.$$

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The desired formulæ for the transformation of coördinates become then

$$(1) \quad \frac{\partial \phi}{\partial (x'_{h_1} x'_{h_2} \cdots x'_{h_{r+1}})} = \sum_i \frac{\partial \phi}{\partial (x_i x_{i_1} \cdots x_{i_{r+1}})} \frac{d(x_i x_{i_1} \cdots x_{i_{r+1}})}{d(x'_{h_1} x'_{h_2} \cdots x'_{h_{r+1}})}.$$

3. If we multiply the preceding equations by

$$\frac{d(x_{s_{r+2}} x_{s_{r+3}} \cdots x_{s_n})}{d(x'_{h_{r+2}} x'_{h_{r+3}} \cdots x'_{h_n})}$$

and add them, for all values of the  $h$ 's, we shall have

$$\begin{aligned} \sum_h \frac{\partial \phi}{\partial (x'_{h_1} x'_{h_2} \cdots x'_{h_{r+1}})} \frac{d(x_{s_{r+2}} x_{s_{r+3}} \cdots x_{s_n})}{d(x'_{h_{r+2}} x'_{h_{r+3}} \cdots x'_{h_n})} \\ = \frac{\partial \phi}{\partial (x_{s_1} x_{s_2} \cdots x_{s_{r+1}})} \frac{d(x_1 x_2 \cdots x_n)}{d(x'_1 x'_2 \cdots x'_n)} \end{aligned}$$

where

$$(h_1, h_2 \cdots h_{r+1}, h_{r+2} \cdots h_n) \equiv (s_1, s_2 \cdots s_{r+1}, s_{r+2} \cdots s_n) \equiv (1, 2, \cdots n),$$

the notation being used to denote the fact that the groups of the  $h$ 's and of the  $s$ 's are two even permutations of the first  $m$  integers. Hence

$$\begin{aligned} (2) \quad \frac{\partial \phi}{\partial (x_{s_1} x_{s_2} \cdots x_{s_{r+1}})} \\ = \frac{I}{\frac{d(x_1 x_2 \cdots x_n)}{d(x'_1 x'_2 \cdots x'_n)}} \sum_h \frac{\partial \phi}{\partial (x'_{h_1} x'_{h_2} \cdots x'_{h_{r+1}})} \frac{d(x_{s_{r+2}} x_{s_{r+3}} \cdots x_{s_n})}{d(x'_{h_{r+2}} x'_{h_{r+3}} \cdots x'_{h_n})}. \end{aligned}$$

4. By means of the equations (D'), which are satisfied by the functions  $\frac{\partial \phi}{\partial (x_{s_1} \cdots x_{s_{r+1}})}$ , and the analogous equations satisfied by the functions  $\frac{\partial \phi}{\partial (x'_{h_1} \cdots x'_{h_{r+1}})}$ , we obtain the theorem :

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If the quantities  $a_{i_1 i_2 \dots i_{r+1}}$  (which change sign for every transposition in the indices) satisfy the equations

$$(3) \quad \sum_1^{r+2} (-1)^{s-1} \frac{\partial a_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}}}{\partial x_{i_s}} = 0$$

the quantities  $a'_{h_1 h_2 \dots h_{r+1}}$  given by the formulæ

$$a'_{h_1 h_2 \dots h_{r+1}} = \frac{1}{\frac{d(x'_1 \dots x'_n)}{d(x_1 \dots x_n)}} \sum_1 a_{i_1 i_2 \dots i_{r+1}} \frac{d(x'_{h_{r+2}} x'_{h_{r+3}} \dots x'_{h_n})}{d(x_{i_{r+2}} x_{i_{r+3}} \dots x_{i_n})}$$

$$(i_1, i_2 \dots i_n) \equiv (h_1, h_2 \dots h_n) \equiv (1, 2, \dots n)$$

will satisfy the analogous equations

$$(3') \quad \sum_1^{r+2} (-1)^{s-1} \frac{\partial a'_{h_1 \dots h_{s-1} h_{s+1} \dots h_{r+2}}}{\partial x'_{h_s}} = 0.$$

5. Let us write  $\frac{\partial \phi}{\partial (x_{i_1} \dots x_{i_{r+1}})} = a_{i_1 \dots i_{r+1}}$ .

We wish to show that if the following conditions are satisfied

$$(4) \quad \sum_1^{r+2} (-1)^s a_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}} a_{i_s h_1 \dots h_r} = 0$$

and we make a change of variables from the  $x_1, x_2 \dots x_n$  to the  $x'_1, x'_2, \dots x'_n$ , we shall obtain the result that the quantities

$$a'_{h_1 \dots h_{r+1}} = \frac{\partial \phi}{\partial (x'_{h_1} \dots x'_{h_{r+1}})}$$

will satisfy the analogous equations

$$(4') \quad \sum_1^{r+2} (-1)^s a'_{h_1 \dots h_{s-1} h_{s+1} \dots h_{r+2}} a'_{h_s i_1 \dots i_r} = 0.$$



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In fact if we have the relations (4), the  $a_{t_1 \dots t_{r+1}}$  will be minor determinants of a matrix

$$\begin{vmatrix} A_{1,1} & A_{1,2} & \dots & A_{1,n} \\ . & . & . & . \\ A_{r+1,1} & A_{r+1,2} & \dots & A_{r+1,n} \end{vmatrix}$$

that is, we can write

$$a_{t_1 \dots t_{r+1}} = \begin{vmatrix} A_{1,t_1} & \dots & A_{1,t_{r+1}} \\ A_{r+1,t_1} & \dots & A_{r+1,t_{r+1}} \end{vmatrix}$$

If we write  $\frac{\partial x_s}{\partial x'_i} = B_{is}$

we shall have, by means of (1), the equations

$$a'_{h_1 \dots h_{r+1}} = \sum_t \begin{vmatrix} A_{1,t_1} & \dots & A_{1,t_{r+1}} \\ . & . & . \\ A_{r+1,t_1} & \dots & A_{r+1,t_{r+1}} \end{vmatrix} B_{h_1 t_1} \dots B_{h_{r+1} t_{r+1}}$$

that is, if we define  $\sum_s A_{ts} B_{hs} = C_{th}$ , the relations

$$a'_{h_1 \dots h_{r+1}} = \begin{vmatrix} C_{1,h_1} & \dots & C_{1,h_{r+1}} \\ C_{r+1,h_1} & \dots & C_{r+1,h_{r+1}} \end{vmatrix}$$

In other words, the quantities  $a'_{h_1 \dots h_{r+1}}$  are minor determinants of the matrix

$$\begin{vmatrix} C_{11} & C_{12} & \dots & C_{1n} \\ C_{21} & C_{22} & \dots & C_{2n} \end{vmatrix}$$

$$C_{r+1,1} C_{r+1,2} \dots C_{r+1,n}$$

and so the equations (4') will be satisfied.

When the equations (4) are satisfied, the function  $\phi | [S_r]$  is said to be *elementary* (see §§ 10, 14).

## 6. Isogeneity\*

1. Two complex functions  $f, \phi$ , of hyperspaces  $S_r$ , which are *regular*, are said to be *isogenous* if in every point of the total hyperspace  $S_n$ , the ratio

$$\frac{\frac{d\phi}{dS_{r+1}}}{\frac{df}{dS_{r+1}}}$$

is independent of the hyperspace  $S$ .

Separating the real and imaginary parts, let us write

$$\frac{\partial f}{\partial(x_{i_1}x_{i_2} \dots x_{i_{r+1}})} = p_{i_1 \dots i_{r+1}} + iq_{i_1 \dots i_{r+1}} = p_I + iq_I$$

$$\frac{\partial \phi}{\partial(x_{i_1}x_{i_2} \dots x_{i_{r+1}})} = \omega_{i_1 \dots i_{r+1}} + i\chi_{i_1 \dots i_{r+1}} = \bar{\omega}_I + i\chi_I$$

where  $I$  denotes the set of indices  $i_1 i_2 \dots i_{r+1}$ , that is,  $I \equiv (i_1 \dots i_{r+1})$ . The necessary and sufficient condition in order that  $f$  and  $\phi$  be isogenous may be written

$$(1) \quad \frac{\omega_I + \chi_I}{p_I + q_I} = \frac{\bar{\omega}_H + i\chi_H}{p_H + iq_H},$$

where  $H \equiv (h_1 h_2 \dots h_{r+1})$  is another arbitrary combination of the indices. From the preceding equations we find

$$(2) \quad \begin{aligned} \bar{\omega}_I p_H - \bar{\omega}_H p_I &= \chi_I q_H - \chi_H q_I, \\ \bar{\omega}_I q_H - \bar{\omega}_H q_I &= \chi_I p_H - \chi_H p_I. \end{aligned}$$

$$\begin{aligned} 2. \text{ Let us write } p_I p_H + q_I q_H &= E_{I, H}, \\ p_I q_H - p_H q_I &= D_{I, H}. \end{aligned}$$

\* See my note: "Sopra una estensione della teoria di Riemann sulle funzioni di variabile complessa." ("Atti della R. Acc. dei Lincei," Vol. III, fasc. 10.)

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Among the  $E$ 's and  $D$ 's we shall have the relations

$$(3) \quad D_{IH}E_{LK} + D_{HK}E_{LI} + D_{KI}E_{LH} = 0,$$

$$\begin{array}{l} E_{IH}E_{IL} \\ E_{KH}E_{KL} \end{array} \left| \begin{array}{lll} p_I p_H + q_I q_H & p_I p_L + q_I q_L & p_I q_I \| p_H q_H \\ p_K p_H + q_K q_H & p_K p_L + q_K q_L & p_K q_K \| p_L q_L \end{array} \right| \quad D_{IK}D_{HL}$$

whence

$$(4) \quad E_{IH}E_{KL} - E_{KH}E_{IL} = D_{IK}D_{HL}.$$

3. If we solve the equations (2) for  $\bar{\omega}_I$  and  $\chi_I$ , we shall have

$$\bar{\omega}_I = \frac{E_{IH}\chi_I - E_{II}\chi_H}{D_{HI}}, \quad \chi_I = \frac{E_{IH}\bar{\omega}_I - E_{II}\bar{\omega}_H}{D_{IH}}$$

Since, however, the first member of these equations does not depend on  $H$ , we must have

$$\begin{aligned} \bar{\omega} &= \frac{E_{IH}\chi_I - E_{II}\chi_H}{D_H} = \frac{E_{IK}\chi_I - E_{II}\chi_K}{D_K} \\ &= \frac{(E_{IH}\chi_I - E_{II}\chi_H)E_{IK} - (E_{IK}\chi_I - E_{II}\chi_K)E_{IH}}{D_{HI}E_{IK} - D_{KI}E_{IH}} \\ &= \frac{E_{IH}\chi_K - E_{IK}\chi_H}{D_{HK}}. \end{aligned}$$

In a similar way we can operate on the expression for  $\chi_I$ , and therefore whatever  $H$  and  $K$  may be we have the formulæ

$$(E) \quad \omega_I = \frac{E_{IH}\chi_K - E_{IK}\chi_H}{D_H}, \quad \chi_I = \frac{E_{IH}\bar{\omega}_K - E_{IK}\bar{\omega}_H}{D_{KH}}$$

4. From the preceding formulæ it follows that

$$D_{HK}\bar{\omega}_I = E_{IH}\chi_K - E_{IK}\chi_H,$$

$$D_{KI}\bar{\omega}_H = E_{HK}\chi_I - E_{HI}\chi_K,$$

$$D_{IH}\bar{\omega}_K = E_{KI}\chi_H - E_{KH}\chi_I,$$

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hence, whatever  $I, H, K$  may be, we have the formula

$$(F) \quad D_{HK}\bar{\omega}_I + D_{KI}\bar{\omega}_H + D_{IH}\bar{\omega}_K = 0,$$

and similarly,  $D_{HK}\chi_I + D_{KI}\chi_H + D_{IH}\chi_K = 0$ .

5. Let us return to the equations (E); from them it follows that

$$(5) \quad \Theta_{IL} = \frac{1}{D_{IL}} \begin{vmatrix} \chi_I & \chi_K E_{IH} - \chi_H E_{IK} \\ \chi_L & D_{IL} D_{HK} \end{vmatrix} \chi_L - \frac{\chi_K E_{LH} - \chi_H E_{LK}}{D_{IL} D_{HK}} \chi_I \\ = \frac{E_{IH}\chi_K\chi_L - E_{IK}\chi_H\chi_L + E_{LK}\chi_H\chi_I - E_{LH}\chi_K\chi_I}{D_{IL} D_{HK}}.$$

If we interchange  $I$  with  $H$  and  $L$  with  $K$  the last member of this equation will not change. Hence we shall have

$$(6) \quad \frac{\bar{\omega}_I}{D_{IL}} \begin{vmatrix} \chi_I & \chi_L \\ \bar{\omega}_L & \chi_L \end{vmatrix} = \frac{1}{D_{HK}} \begin{vmatrix} \bar{\omega}_H & \chi_H \\ \bar{\omega}_K & \chi_K \end{vmatrix}$$

In other words, the quantities  $\Theta_{IL}$  are independent of  $I$  and  $L$ , and so we can denote them all by  $\Theta$ .

If in (5) we put  $I = H, L = K$ , we shall have

$$(7) \quad \Theta = \frac{E_{HH}\chi_L^2 - 2 E_{HL}\chi_I\chi_L + E_{LL}\chi_I^2}{D_{IL}^2} \\ = \frac{(p_I\chi_L - p_L\chi_I)^2 + (q_I\chi_L - q_L\chi_I)^2}{D_{IL}^2}$$

formulæ which show that  $\Theta$  is a *positive* quantity. If in (5) we interchange  $\bar{\omega}$  and  $\chi$ , and  $p$  and  $q$ , the  $\Theta$  will not change, and we shall have for  $\Theta$  the alternative expression

$$(5') \quad \frac{E_{IH}\bar{\omega}_K\bar{\omega}_L - E_{IK}\bar{\omega}_H\bar{\omega}_L + E_{LK}\bar{\omega}_H\bar{\omega}_I - E_{LH}\bar{\omega}_K\bar{\omega}_I}{D_{IL} D_{HK}}.$$

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If we write  $\phi = \phi_1 + i\phi_2$  and make use of our symbols  $I, H \dots$ , we can write

$$\omega_I = \omega_{i_1 \dots i_{r+1}} \cdot \frac{\partial \phi_1}{\partial (x_{i_1} x_{i_2} \dots x_{i_{r+1}})} = \frac{\partial \phi_1}{\partial (x_I)},$$

$$\chi_I = \chi_{i_1 \dots i_{r+1}} \cdot \frac{\partial \phi_2}{\partial (x_{i_1} x_{i_2} \dots x_{i_{r+1}})} = \frac{\partial \phi_2}{\partial (x_I)}$$

where  $(x_I)$  is a substitute for  $(x_{i_1} x_{i_2} \dots x_{i_{r+1}})$ , *i.e.*

$$(x_I) \equiv (x_{i_1} x_{i_2} \dots x_{i_{r+1}}).$$

The expression for  $\Theta$  can now be written

(G)  $\Theta =$

$$\frac{E_{IH} \frac{\partial \psi}{\partial (x_I)} \frac{\partial \psi}{\partial (x_L)} - E_{IK} \frac{\partial \psi}{\partial (x_H)} \frac{\partial \psi}{\partial (x_K)} + E_{LK} \frac{\partial \psi}{\partial (x_H)} \frac{\partial \psi}{\partial (x_I)} - E_{LH} \frac{\partial \psi}{\partial (x_K)} \frac{\partial \psi}{\partial (x_I)}}{D_{IL} D_{HK}}$$

where in place of  $\psi$  we can put either  $\phi_1$  or  $\phi_2$ .

6. We know that the quantities  $\bar{\omega}$  and  $\chi$  must satisfy the following equations (see section 5, article 1)

$$\sum_1^{r+2} (-1)^{s-1} \frac{\partial}{\partial x_s} \bar{\omega}_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}} = 0, \quad \sum_1^{r+2} (-1)^{s-1} \frac{\partial}{\partial x_s} \chi_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}},$$

and therefore, from (E), we have the following equations

$$(H) \quad \left\{ \frac{\sum_1^{r+2} (-1)^{s-1} \frac{\partial}{\partial x_s} \chi_K E_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}, H} - \chi_H E_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}, K}}{D_H} \right\} = 0$$

$$\left\{ \frac{\sum_1^{r+2} (-1)^{s-1} \frac{\partial}{\partial x_s} \bar{\omega}_K E_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}, H} - \bar{\omega}_H E_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}, K}}{D_{HK}} \right\} =$$

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or, by reason of (H) and (F),  $\phi_1$  and  $\phi_2$  must satisfy the equations

$$(H') \quad \sum_1^{r+2} (-1)^{s-1} \frac{\partial}{\partial x_s} \\ E_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}} \frac{\partial \psi}{\partial (x_K)} - E_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}} \frac{\partial \psi}{\partial (x_H)} \\ D_{HK}$$

$$(F') \quad D_{HK} \frac{\partial \psi}{\partial (x_I)} + D_{KI} \frac{\partial \psi}{\partial (x_H)} + D_{IH} \frac{\partial \psi}{\partial (x_K)} = 0.$$

7. Conversely it can be shown that if  $\psi[|S_i|]$  is a real regular function and satisfies the preceding equations, it may be considered as the real part of a function  $\psi + i\theta$  isogenous to  $f$ . In fact, by means of (H') we can write

$$\frac{E_{I,H} \frac{\partial \psi}{\partial (x_K)} - E_{I,K} \frac{\partial \psi}{\partial (x_H)}}{D_H} = \frac{\partial \theta_{H,K}}{\partial (x_I)}$$

where  $(x_I) = (x_{i_1} \dots x_{i_{s-1}} x_{i_{s+1}} \dots x_{i_{r+2}})$ . But from (F') and (3) it follows that the first member of the preceding equations is independent of  $H$  and  $K$ , hence we can take the  $\theta_{HK}$  as independent of their subscripts and write them all equal to  $\theta$ , so that

$$\frac{E_{I,H} \frac{\partial \psi}{\partial (x_K)} - E_{I,K} \frac{\partial \psi}{\partial (x_H)}}{D_{HK}} = \frac{\partial \theta}{\partial (x_I)}$$

And now if from these equations we follow the inverse procedure to that of articles 1, 2, 3, we find that the ratio

$$\frac{\frac{\partial (\psi + i\theta)}{\partial (x_I)}}{p_I + iq_I}$$

is independent of the indices (I), so that  $\psi + i\theta$  is isogenous

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to  $f$ . The equations  $(H')$  and  $(F')$  operate in our case in the same way as the equation  $\Delta^2 = 0$  in the theory of Riemann.

### 7. Conditions for isogeneity.

1. If we take arbitrarily a regular function of hyperspaces  $S_r$ , it will not always be possible to associate with it an isogenous function. In order for that it is necessary that certain conditions be satisfied. In fact if  $F|[S_r]|$  is a regular function to which  $\Phi|[S_r]|$  is isogenous, and we write

$$\frac{\partial F}{\partial(x_{i_1} \dots x_{i_{r+1}})} = p_{i_1 \dots i_{r+1}}, \quad \frac{\partial \Phi}{\partial(x_{i_1} \dots x_{i_{r+1}})} = \bar{\omega}_{i_1 \dots i_{r+1}},$$

we must have

$$\frac{\bar{\omega}_{i_1 \dots i_{r+1}}}{p_{i_1 \dots i_{r+1}}} = \phi$$

where  $\phi$  is independent of the indices  $i_1 \dots i_{r+1}$ . Hence it follows that

$$\bar{\omega}_{i_1 \dots i_{r+1}} = \phi p_{i_1 \dots i_{r+1}}$$

so that

$$\begin{aligned} 0 &= \sum_{i_1}^{r+2} (-1)^{i_1} \frac{\partial \bar{\omega}_{i_1 \dots i_{r-1} i_{r+1} \dots i_{r+2}}}{\partial x_{i_1}} = \sum_{i_1}^{r+2} (-1)^{i_1} \frac{\partial (\phi p_{i_1 \dots i_{r-1} i_{r+1} \dots i_{r+2}})}{\partial x_{i_1}} \\ &= \sum_{i_1}^{r+2} (-1)^{i_1} p_{i_1 \dots i_{r-1} i_{r+1} \dots i_{r+2}} \frac{\partial \phi}{\partial x_{i_1}}. \end{aligned}$$

From this we conclude that *it is necessary and sufficient in order that there may exist a function isogenous to  $F|[S_r]|$  that the system of simultaneous linear differential equations*

$$(I) \quad \sum_{i_1}^{r+2} (-1)^{i_1} p_{i_1 \dots i_{r-1} i_{r+1} \dots i_{r+2}} \frac{\partial \phi}{\partial x_{i_1}} = 0$$

*admit solutions.*

It is for this reason that in § 9 we shall study systems of differential equations of this form. In the meantime let us observe that the equations (I) may in some cases be incompatible. Thus, if we have in four dimensions the regular function  $F|[S_1]|$ , the equations (I) become

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$$-p_{23} \frac{\partial \phi}{\partial x_1} + p_{13} \frac{\partial \phi}{\partial x_2} - p_{12} \frac{\partial \phi}{\partial x_3} = 0,$$

$$-p_{34} \frac{\partial \phi}{\partial x_2} + p_{24} \frac{\partial \phi}{\partial x_3} - p_{23} \frac{\partial \phi}{\partial x_4} = 0,$$

$$-p_{41} \frac{\partial \phi}{\partial x_3} + p_{31} \frac{\partial \phi}{\partial x_4} - p_{34} \frac{\partial \phi}{\partial x_1} = 0,$$

$$-p_{12} \frac{\partial \phi}{\partial x_4} + p_{42} \frac{\partial \phi}{\partial x_1} - p_{41} \frac{\partial \phi}{\partial x_2} = 0,$$

and these equations will be incompatible unless

$$p_{12}p_{34} + p_{13}p_{42} + p_{14}p_{23} = 0$$

2. We now proceed to prove the following theorem :

*The necessary and sufficient condition in order that equations (1) admit a common solution  $\phi$  is that we can write*

$$(2) \quad p_{i_1 \dots i_{r+2}} = \sum_{s=1}^{r+1} (-1)^s \frac{\partial \phi}{\partial x_{i_s}} \frac{\partial \psi}{\partial (x_{i_1} \dots x_{i_{s-1}} x_{i_{s+1}} \dots x_{i_{r+1}})}$$

where  $\psi$  is a regular function of hyperspaces.

Let us write 
$$\frac{\partial \psi}{\partial (x_{i_1} \dots x_{i_r})} = q_{i_1 \dots i_r}.$$

It is easy to show that if the equations

$$(2') \quad p_{i_1 \dots i_{r+1}} = \sum_{s=1}^{r+1} (-1)^s \frac{\partial \phi}{\partial x_{i_s}} q_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+1}}$$

are satisfied, the equations (1) will also be satisfied. In fact, we shall have

$$\begin{aligned} & \sum_{t=1}^{r+2} (-1)^t p_{i_1 \dots i_{t-1} i_{t+1} \dots i_{r+2}} \frac{\partial \phi}{\partial x_{i_t}} \\ &= \sum_{t=1}^{r+2} \sum_{s=1}^{r+2} (-1)^{s+t} \frac{\partial \phi}{\partial x_{i_s}} \frac{\partial \phi}{\partial x_{i_t}} q_{i_1 \dots i_{s-1} i_{s+1} \dots i_{t-1} i_{t+1} \dots i_{r+1}}, \end{aligned}$$



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in which  $\sum_1^{r+2}{}^{(t)}$  is extended over all the values of the index  $s$  from 1 to  $r+2$ , the value  $t$  excepted, and  $s'$  should be taken equal to  $s$  or to  $s-1$  according as  $s < t$  or  $s > t$ . Hence the left-hand member of the equation is zero, and the equations (1) are satisfied. From (2') it also follows easily that

$$\sum_1^{r+2} (-1)^s \frac{\partial p_{i_1 \dots i_{s-1} t_{s+1} \dots t_{r+2}}}{\partial x_{i_s}} = 0.$$

Thus we have shown that our condition is sufficient. To show that it is also necessary, let us execute a change of variables, instead of  $x_1, x_2 \dots x_n$  taking  $x'_1 = \phi, x'_2 = x_2 \dots x'_n = x_n$ . If we prime the letters which refer to the new variables, we shall have

1st) if  $i_1, i_2, \dots, i_r \neq 1$

$$q_{i_1 \dots i_r} = q'_{i_1 \dots i_r} + \sum_1^r (-1)^{t-1} q'_{i_1 \dots i_{t-1} t_{t+1} \dots i_r} \frac{\partial \phi}{\partial x_{i_t}}$$

2d) if  $i_h = 1$

$$q_{i_1 \dots i_r} = \sum_1^r (-1)^{s-1} q'_{i_1 \dots i_{s-1} t_{s+1} \dots i_r} \frac{\partial \phi}{\partial x_{i_s}} = (-1)^{h-1} q'_{i_1 \dots i_{h-1} t_{h+1} \dots i_r} \frac{\partial \phi}{\partial x_{i_h}}.$$

Supposing momentarily that  $i_1 \dots i_{r+1} \neq 1$  we shall have

$$\begin{aligned} p_{i_1 \dots i_{r+1}} &= \sum_1^{r+1} (-1)^s \frac{\partial \phi}{\partial x_{i_s}} q'_{i_1 \dots i_{s-1} t_{s+1} \dots i_{r+1}} \\ &+ \sum_1^{r+1} (-1)^s \frac{\partial \phi}{\partial x_{i_s}} \sum_1^{r+1}{}^{(s)} (-1)^{t'} q'_{i_1 \dots i_{t-1} t_{t+1} \dots i_s \dots i_{s+1} \dots i_{r+1}} \frac{\partial \phi}{\partial x_{i_t}} \end{aligned}$$

where  $t' = \begin{cases} t-1 \\ t \end{cases}$  according as  $\begin{cases} t < s \\ t > s. \end{cases}$

Hence

$$(3) \quad p_{i_1 \dots i_{r+1}} = \sum_1^{r+1} (-1)^s q'_{i_1 \dots i_{s-1} t_{s+1} \dots i_{r+1}} \frac{\partial \phi}{\partial x_{i_s}}.$$

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If we suppose instead that some one of the indices of  $p$  is equal to 1, say  $i_1 = 1$ , we shall have

$$(3') \quad p_{1i_2 \dots i_{r+1}} = -\frac{\partial \phi}{\partial x_1} q'_{i_2 \dots i_{r+1}} - \frac{\partial \phi}{\partial x_1} \sum_{s=2}^{r+1} (-1)^s q'_{i_2 \dots i_{s-1} i_{s+1} \dots i_{r+1}} \frac{\partial \phi}{\partial x_{i_s}} \\ + \sum_{s=2}^{r+1} (-1)^s q'_{i_2 \dots i_{s-1} i_{s+1} \dots i_{r+1}} \frac{\partial \phi}{\partial x_1} \frac{\partial \phi}{\partial x_{i_s}} = -\frac{\partial \phi}{\partial x_1} q'_{i_2 \dots i_{r+1}}.$$

We shall show that (3) is a consequence of (3'). In fact, from (3') we have

$$q'_{i_2 \dots i_{r+1}} = -\frac{p_{1i_2 \dots i_{r+1}}}{\left(\frac{\partial \phi}{\partial x_1}\right)}$$

so that (3) becomes

$$p_{i_1 \dots i_{r+1}} = -\sum_{s=1}^{r+1} (-1)^s \frac{p_{1i_2 \dots i_{s-1} i_{s+1} \dots i_{r+1}}}{\frac{\partial \phi}{\partial x_1}} \frac{\partial \phi}{\partial x_{i_s}}$$

and if we put  $i_0 = 1$ , this gives us

$$\sum_{s=0}^{r+1} (-1)^s p_{i_0 i_1 \dots i_{s-1} i_{s+1} \dots i_r} \frac{\partial \phi}{\partial x_{i_s}} = 0,$$

an equation which is identically true.

We must now prove that the functions

$$q'_{i_2 \dots i_{r+1}} = -\frac{p_{1i_2 \dots i_{r+1}}}{\frac{\partial \phi}{\partial x_1}}$$

satisfy the conditions of integrability (see section 5, article 1), assuming therein that  $\phi$  is constant.

We have in fact (see section 5, article 3)

$$p'_{1i_2 \dots i_{r+1}} = \frac{1}{\frac{d(\phi x_2 \dots x_n)}{d(x_1 x_2 \dots x_n)}} \sum_{h_1 h_2 \dots h_{r+1}} \frac{d(x_{i_{r+2}} \dots x_{i_n})}{d(x_{h_{r+2}} \dots x_{h_n})}$$

where

$$(h_1 h_2 \dots h_{r+1} h_{r+2} \dots h_n) \equiv (i_1 i_2 \dots i_{r+1} i_{r+2} \dots i_n) \equiv (1, 2, \dots, n)$$

so that

$$p'_{1i_2 \dots i_{r+1}} = \frac{p_{1i_2 \dots i_{r+1}}}{\left(\frac{\partial \phi}{\partial x_1}\right)}.$$

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If  $i_1, i_2 \dots i_{r+1} \neq 1$ , we have (see section 5, article 3)

$$p'_{i_1 \dots i_{r+1}} = \frac{1}{\frac{d(\phi x_2 \dots x_n)}{d(x_1 x_2 \dots x_n)}} \sum_{h_1 \dots h_{r+1}} \frac{d(x_{i_{r+2}} \dots x_{i_n})}{d(x_{h_{r+2}} \dots x_{h_n})}$$

$$\frac{1}{\frac{d\phi}{dx_1}} \sum_0^{r+1} (-1)^s p_{i_0 i_1 \dots i_{s-1} i_{s+1} \dots i_{r+1}} \frac{\partial \phi}{\partial x_{i_s}} = 0.$$

And so if we apply the theorem of section 5, article 3, we shall have

$$0 = \sum_0^{r+1} (-1)^s \frac{\partial}{\partial x_{i_s}} \frac{p_{i_1 i_2 \dots i_{s-1} i_{s+1} \dots i_{r+1}}}{\frac{\partial \phi}{\partial x_1}}$$

$$= \sum_2^{r+1} (-1)^s \frac{\partial}{\partial x_{i_s}} q'_{i_2 \dots i_{s-1} i_{s+1} \dots i_{r+1}}.$$

The functions  $q'$  then satisfy the conditions of integrability, and it will be possible to determine a function  $\psi$  which satisfies equations (2). Thus it is shown that the given condition is necessary.

3. Given the  $F$  for which (1) is satisfied, the  $\psi$  which satisfies (2) is not determined. We shall see how all the  $\psi$ 's which satisfy (2) may be found when one of them,  $\psi_1$ , is known. If  $\psi_1$  and  $\psi$  satisfy (2), and we write

$$\psi - \psi_1 = \psi_2, \quad \frac{\partial \psi_2}{\partial (x_{i_1} \dots x_{i_r})} = q_{i_1 \dots i_r}^{(2)},$$

we shall have

$$0 = \sum_1^{r+1} (-1)^s \frac{\partial \phi}{\partial x_{i_s}} q_{i_1 \dots i_r}^{(2)}$$

and therefore

$$q_{i_1 \dots i_r}^{(2)} = \sum_1^r (-1)^s \frac{\partial \phi}{\partial x_{i_s}} \frac{\partial \Theta}{\partial (x_{i_1} \dots x_{i_{s-1}} x_{i_{s+1}} \dots x_{i_r})}.$$

in which  $\Theta | [S_{r-1}]$  is arbitrary.

# THE GENERALIZATION OF ANALYTIC FUNCTIONS

## Second Lecture

EXPRESSIONS FOR ISOGENOUS FUNCTIONS — AUXILIARY REMARKS ON SYSTEMS OF SIMULTANEOUS DIFFERENTIAL EQUATIONS — ON THE ELEMENTARY FUNCTIONS — COMPOSITION OF FUNCTIONS OF HYPERSPACES — NEW CONSIDERATIONS WITH REFERENCE TO THE RELATION OF ISOGENEITY — DIFFERENTIATION AND INTEGRATION — ISOGENEITY OF ORDER  $r$ .

### 8. Expressions for isogenous functions

1. If  $F|[S_r]|$  and  $\Phi|[S_r]|$  are isogenous, it follows from what has been shown in the preceding section that we can write

$$\frac{\partial F}{\partial(x_{t_1} \cdots x_{t_{r+1}})} = p_{t_1 \cdots t_{r+1}} = \sum_1^{r+1} (-1)^s \frac{\partial f}{\partial x_{t_s}} \frac{\partial \psi}{\partial(x_{t_1} \cdots x_{t_{s-1}} x_{t_{s+1}} \cdots x_{t_{r+1}})}$$

$$\frac{\partial \Phi}{\partial(x_{t_1} \cdots x_{t_{r+1}})} = \bar{\omega}_{t_1 \cdots t_{r+1}} = \sum_1^{r+1} (-1)^s \frac{\partial \phi}{\partial x_{t_s}} \frac{\partial \psi}{\partial(x_{t_1} \cdots x_{t_{s-1}} x_{t_{s+1}} \cdots x_{t_{r+1}})},$$

where  $\psi|[S_{r-1}]|$  is regular and  $\phi$  is a function of  $f$ ; and we know that the ratio  $\frac{\bar{\omega}_{t_1 \cdots t_{r+1}}}{p_{t_1 \cdots t_{r+1}}}$  (independent of the indices) is equal to  $\frac{d\phi}{df}$ .

2. Let us write

$$L_{t_1 \cdots t_{r+1}} = f \frac{\partial \psi}{\partial(x_{t_1} \cdots x_{t_r})}.$$

It follows that

$$(I) \quad p_{t_1 \cdots t_{r+1}} = \sum_1^{r+1} (-1)^s \frac{\partial L_{t_1 \cdots t_{s-1} t_{s+1} \cdots t_{r+1}}}{\partial x_{t_s}}.$$

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If now  $S_{r+1}$  is a space of  $r+1$  dimensions whose boundary is  $S_r$ , we shall have

$$F|[S_r]| = \int_{S_{r+1}} \sum_i p_{i_1 \dots i_{r+1}} \alpha_{i_1 \dots i_{r+1}} dS_{r+1},$$

where the  $\alpha_{i_1 \dots i_{r+1}}$  are the direction cosines of  $S_{r+1}$ . And if we substitute for the  $p$ 's their values (1) and apply the extension of Stokes's theorem (see Section 4), we shall have

$$(2) \quad F|[S_r]| = \int_{S_r} f \frac{d\psi}{dS_r} dS_r,$$

and similarly,

$$(2') \quad \Phi|[S_r]| = \int_{S_r} \phi \frac{d\psi}{dS_r} dS_r.$$

3. Conversely, if  $F$  and  $\Phi$  are given by the preceding formulae, with  $\phi = \phi(f)$ , the  $F$  and  $\Phi$  must be isogenous.

### 9. Auxiliary remarks on systems of simultaneous differential equations

1. Consider the system of differential equations

$$(1) \quad H_{i_1 i_2 \dots i_{r+2}} = \sum_s^{r+2} (-1)^s A_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}} \frac{\partial \phi}{\partial x_{i_s}} = 0$$

whose coefficients satisfy the conditions

$$(2) \quad \sum_s^{r+2} (-1)^s A_{i_s i_{s+1} \dots i_{r+1}} A_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}} = 0$$

and are such that they change sign with every transposition of the indices. With this convention, if we have an  $H$  with two of its indices equal, its value must be zero.

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2. Among the  $A$ 's, one at least must be different from zero. If  $A_{i_1 i_2 \dots i_{r+1}}$  is such a one, all the equations (1) will follow from the equations (independent among themselves).

$$(3) \quad H_{i_1 \dots i_{r+1} h_1} = 0, \quad H_{i_1 \dots i_{r+1} h_2} = 0, \dots H_{i_1 \dots i_{r+1} h_{n-r-1}} = 0,$$

in which none of the  $h_1, h_2 \dots h_{n-r-1}$  is equal to another, or to an  $i$ .

Let us take, in fact, the system

$$(4) \quad H_{i_1 \dots i_{r+1} k_1} = 0, \quad H_{i_1 \dots i_{r+1} k_2} = 0, \dots H_{i_1 \dots i_{r+1} k_{r+2}} = 0,$$

where the  $k_s$  are arbitrary. If a  $k_s$  is equal to one of the  $i$ , the corresponding equation will be an identity; otherwise, it will be one of the equations (3). The equations (4) can be written in the form

$$A_{i_1 \dots i_{r+1}} \frac{\partial \phi}{\partial x_{k_s}} + \sum_1^{r+1} (-1)^i A_{k_s i_1 \dots i_{l-1} i_{l+1} \dots i_{r+1}} \frac{\partial \phi}{\partial x_{i_l}} = 0.$$

If we multiply each one by  $(-1)^s A_{k_1 \dots k_{s-1} k_{s+1} \dots k_{r+2}}$  and add them together for all values of the subscript  $s$  from 1 to  $r+2$ , we shall have

$$A_{i_1 \dots i_{r+1}} \sum_1^{r+2} (-1)^s A_{k_1 k_2 \dots k_{s-1} k_{s+1} \dots k_{r+2}} \frac{\partial \phi}{\partial x_{k_s}} \\ + \sum_1^{r+1} (-1)^i \frac{\partial \phi}{\partial x_{i_l}} \sum_1^{r+2} A_{k_s i_1 \dots i_{l-1} i_{l+1} \dots i_{r+2}} A_{k_1 \dots k_{s-1} k_{s+1} \dots k_{r+2}} = 0,$$

$$\text{whence } \sum_1^{r+2} (-1)^s A_{k_1 k_2 \dots k_{s-1} k_{s+1} \dots k_{r+2}} \frac{\partial \phi}{\partial x_{k_s}} = H_{k_1 \dots k_{r+2}} = 0$$

so that the theorem is proved.

3. Now let us form the alternating function of Poisson

$$(H_{i_1 i_2 \dots i_{r+2}}, H_{h_1 h_2 \dots h_{r+2}})$$

taking  $i_1 = h_1, i_2 = h_2, \dots i_{r+1} = h_{r+1}$  and writing  $h_{r+2} = i_{r+3}$ ,

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we shall have

$$\begin{aligned}
 & (H_{h_1 \dots h_{r+2}}, H_{t_1 \dots t_{r+2}}) \\
 &= \sum_1^{r+2} \sum_1^{r+2} (-1)^{s+t} \left( A_{h_1 \dots h_{s-1} h_{s+1} \dots h_{r+2}} \frac{\partial A_{t_1 \dots t_{t-1} t_{t+1} \dots t_{r+2}}}{\partial x_{h_s}} \frac{\partial \phi}{\partial x_{t_t}} \right) \\
 &- \sum_1^{r+2} (-1)^{s+t} \left( A_{t_1 \dots t_{s-1} t_{s+1} \dots t_{r+2}} \frac{\partial A_{h_1 \dots h_{t-1} h_{t+1} \dots h_{r+2}}}{\partial x_{t_s}} \frac{\partial \phi}{\partial x_{h_t}} \right) \\
 &= \sum_1^{r+1} \sum_1^{r+1} (-1)^{s+t} \frac{\partial (A_{t_1 \dots t_{r+1}} A_{h_1 \dots h_{s-1} h_{s+1} \dots t_{t-1} t_{t+1} \dots t_{r+3}})}{\partial x_{t_s}} \frac{\partial \phi}{\partial x_{t_t}} \\
 &- \sum_1^{r+2} (-1)^s \frac{\partial A_{h_1 \dots h_{s-1} h_{s+1} \dots h_{r+2}}}{\partial x_{t_s}} \sum_1^{r+2} (-1)^t A_{t_1 \dots t_{t-1} t_{t+1} \dots t_{r+3}} \frac{\partial \phi}{\partial x_{t_t}} \\
 &+ \sum_1^{r+2} (-1)^s \frac{\partial A_{t_1 \dots t_{s-1} t_{s+1} \dots t_{r+2}}}{\partial x_{t_s}} \sum_1^{r+2} (-1)^t A_{h_1 \dots h_{t-1} h_{t+1} \dots h_{r+2}} \frac{\partial \phi}{\partial x_{h_t}} \\
 &+ \sum_1^{r+2} (-1)^{t+r+2} \frac{\partial (A_{t_1 \dots t_{r+1}} A_{h_1 \dots h_{t-1} h_{t+1} \dots h_{r+2}})}{\partial x_{t_{r+2}}} \frac{\partial \phi}{\partial x_{t_t}} \\
 &- \sum_1^{r+2} (-1)^{t+r+2} \frac{\partial (A_{t_1 \dots t_{r+1}} A_{h_1 \dots h_{t-1} h_{t+1} \dots h_{r+2}})}{\partial x_{t_{r+2}}} \frac{\partial \phi}{\partial x_{h_t}} \\
 &+ \sum_1^{r+2} (-1)^{s+r+2} \frac{\partial (A_{t_1 \dots t_{r+1}} A_{h_1 \dots h_{s-1} h_{s+1} \dots h_{r+2}})}{\partial x_{h_s}} \frac{\partial \phi}{\partial x_{t_{r+2}}} \\
 &- \sum_1^{r+2} (-1)^{s+r+2} \frac{\partial (A_{t_1 \dots t_{r+1}} A_{h_1 \dots h_{s-1} h_{s+1} \dots h_{r+2}})}{\partial x_{t_s}} \frac{\partial \phi}{\partial x_{h_{r+2}}}.
 \end{aligned}$$

If we write

$$\sum_1^{r+2} (-1)^s \frac{\partial A_{h_1 \dots h_{s-1} h_{s+1} \dots h_{r+2}}}{\partial x_{h_s}} = L_{h_1 \dots h_{r+2}},$$

we shall have

$$\begin{aligned}
 & (H_{h_1 \dots h_{r+2}}, H_{t_1 \dots t_{r+2}}) \\
 &= A_{t_1 \dots t_{r+1}} \sum_1^{r+3} (-1)^t L_{t_1 \dots t_{s-1} t_{s+1} \dots t_{r+3}} \frac{\partial \phi}{\partial x_{t_t}} \\
 &+ \sum_1^{r+3} (-1)^s \frac{\partial A_{t_1 \dots t_{r+1}}}{\partial x_{t_s}} H_{t_1 \dots t_{s-1} t_{s+1} \dots t_{r+3}} + L_{t_1 \dots t_{r+2}} H_{t_1 \dots t_{r+1} t_{r+3}} \\
 &- L_{t_1 \dots t_{r+1} t_{r+3}} H_{t_1 \dots t_{r+2}}.
 \end{aligned}$$

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Hence to the system (1) we must add the equations

$$\sum_{s=1}^{r+1} (-1)^s L_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+3}} \frac{\partial \phi}{\partial x_{i_s}}$$

so that if the conditions

$$L_{i_1 \dots i_{r+2}} = \sum_{s=1}^{r+2} (-1)^s \frac{\partial A_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}}}{\partial x_{i_s}} = 0$$

are satisfied for every combination of the indices  $i_1 \dots i_{r+2}$ , the system (1) will be complete.

4. From this it follows that the equations (1) of section 7 will form a complete system whenever, in addition to the conditions of integrability (see section 5, article 1), the functions  $p$  satisfy also the following conditions:

$$(5) \quad \sum_{s=1}^{r+2} (-1)^s p_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}} p_{i_s i_1 \dots i_r} = 0.$$

Hence for *elementary* functions (see section 5, article 5) the system of equations (1) of section 7 is *complete*.

### 10. The elementary functions

1. Let us suppose that the function  $F | [S_r] |$  is regular and elementary, so that the system (1) of section 7, or the equivalent system (3) of section 9, is complete. There will exist then  $r + 1$  independent integrals

$$\phi, \phi_1, \dots, \phi_r.$$

Hence the ratio

$$\theta = \frac{p_{i_1 \dots i_r}}{\left( \frac{d(\phi, \phi_1, \dots, \phi_r)}{d(x_{i_1} \dots x_{i_{r+1}})} \right)} = \frac{\left( \frac{dF}{d(x_{i_1} \dots x_{i_{r+1}})} \right)}{\left( \frac{d(\phi, \phi_1 \dots \phi_r)}{d(x_{i_1} \dots x_{i_{r+1}})} \right)}$$

will be independent of the subscripts  $i_1 \dots i_r$ , and we shall have

$$p_{i_1 \dots i_{r+1}} = \theta \frac{d(\phi, \phi_1 \dots \phi_r)}{d(x_{i_1} \dots x_{i_{r+1}})}.$$



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But we must have  $\sum_1^{r+2} (-1)^s \frac{\partial p_{t_1 \dots t_{s-1} t_{s+1} \dots t_{r+2}}}{\partial x_{t_s}} = 0$ ,

so that  $\sum_1^{r+2} (-1)^s \frac{\partial \theta}{\partial x_{t_s}} \frac{d(\phi_0, \phi_1 \dots \phi_r)}{d(x_{t_1} \dots x_{t_{s-1}} x_{t_{s+1}} \dots x_{t_{r+2}})} = 0$ ,

and consequently  $\sum_1^{r+2} (-1)^s p_{t_1 \dots t_{s-1} t_{s+1} \dots t_{r+2}} \frac{\partial \theta}{\partial x_{t_s}} = 0$ .

The quantity  $\theta$  will therefore be a function of  $\phi_0, \phi_1, \dots, \phi_r$ ,

and if we write  $\frac{\partial \phi_0}{\partial \phi} = \theta$ , we shall have

$$p_{t_1 \dots t_{r+1}} = \frac{\partial \phi_0}{\partial \phi} \frac{d(\phi_0, \phi_1 \dots \phi_r)}{d(x_{t_1} \dots x_{t_{r+1}})} = \frac{d(\phi_0, \phi_1 \dots \phi_r)}{d(x_{t_1} \dots x_{t_{r+1}})}.$$

We have therefore the following theorem :

*If  $F$  is an elementary function, it follows that*

$$\frac{\partial F}{\partial (x_{t_1} \dots x_{t_{r+1}})} = \frac{d(\phi_0, \phi_1 \dots \phi_r)}{d(x_{t_1} \dots x_{t_{r+1}})} = p_{t_1 \dots t_{r+1}}$$

*where  $\phi_0, \phi_1, \dots, \phi_r$  are independent integrals of the complete system*

$$(I) \quad \sum_1^{r+2} (-1)^s p_{t_1 \dots t_{s-1} t_{s+1} \dots t_{r+2}} \frac{\partial \phi}{\partial x_{t_s}} = 0.$$

2. Conversely, if we take  $r+1$  functions  $\phi_0, \phi_1, \dots, \phi_r$  and write

$$\frac{d(\phi_0, \phi_1, \dots, \phi_r)}{d(x_{t_1} \dots x_{t_{r+1}})} = p_{t_1 \dots t_{r+1}},$$

*the quantities  $p_{t_1 \dots t_{r+1}}$  will be the derivatives of an elementary function. In fact, they will satisfy the conditions of integrability, and also the conditions (5) of the preceding section (see section 5, article 5).*

We shall say that the functions  $\phi_0, \phi_1, \dots, \phi_r$  are *conjugate* to the function  $F$ , and that  $F$  is *conjugate* to them.

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3. If  $\Phi$  is isogenous to  $F$ , and we write

$$\frac{\partial \Phi}{\partial (x_{t_1} \dots x_{t_{r+1}})} = \bar{\omega}_{t_1 \dots t_{r+1}},$$

we must have

$$\frac{\bar{\omega}_{t_1 \dots t_{r+1}}}{p_{t_1 \dots t_{r+1}}} = \psi,$$

$\psi$  being an integral of equation (1). Hence  $\psi$  must be a function of  $\phi_0, \phi_1, \dots \phi_r$ . If we take  $\psi = \frac{\partial \lambda}{\partial \phi}$ , we shall have

$$\bar{\omega}_{t_1 \dots t_{r+1}} = \frac{\partial \lambda}{\partial \phi} \frac{d(\phi, \phi_1, \dots \phi_r)}{d(x_{t_1} \dots x_{t_{r+1}})} = \frac{d(\lambda, \phi_1, \dots \phi_r)}{d(x_{t_1} \dots x_{t_{r+1}})},$$

from which we deduce the theorem :

*All the functions isogenous to an elementary function are themselves elementary.*

4. If we apply to the elementary functions the formula (2), section 8, relative to the possibility of defining isogenous functions, we have

$$(2) \quad F|[S_r]| = \int_{S_r} \phi \frac{d(\phi_0, \phi_1 \dots \phi_r)}{d(\omega_1 \dots \omega_r)} d\omega_1 \dots d\omega_r,$$

where

$$x_1 = x_1(\omega_1, \dots \omega_r), \quad x_2 = x_2(\omega_1, \dots \omega_r), \quad \dots \quad x_n = x_n(\omega_1, \dots \omega_r),$$

the equations of the hyperspace  $S_r$ .

### II. The composition of functions of hyperspaces

I. The results which we have obtained in the preceding section can be expressed in a different form by means of special symbols. That is what we shall do in this section, after having proved a fundamental theorem.

Let  $F|[S_r]|$  and  $\Phi|[S_{t-r}]|$  be two regular functions of

## 90 The Generalization of Analytic Functions hyperspaces, and write

$$\frac{dF}{d(x_{h_1} \dots x_{h_{r+1}})} = p_{h_1 \dots h_{r+1}}, \quad \frac{\partial \phi}{\partial (x_{h_{r+2}} \dots x_{h_{t+2}})} = q_{h_{r+2} \dots h_{t+2}},$$

$$(I) \quad m_{i_1 \dots i_t} = \sum_h (-1)^{\binom{h_1 \dots h_{t+2}}{i_1 \dots i_{t+2}}} p_{h_1 \dots h_{r+1}} q_{h_{r+2} \dots h_{t+2}},$$

in which  $h_1 \dots h_{t+2}$  is a permutation of  $i_1 \dots i_{t+2}$ ; the sum  $\sum_h$  is extended over all the combinations of the  $t+2$  subscripts  $i_1 \dots i_{t+2}$ ,  $r+1$  at a time; and the symbol  $(-1)^{\binom{h_1 \dots h_{t+2}}{i_1 \dots i_{t+2}}}$  represents  $+1$  or  $-1$ , according as the substitution which appears in the exponent is even or odd.

2. We shall show that *there exists a regular function*  $\Psi[[S_{t+1}]]$ , *such that*

$$\frac{\partial \Psi}{\partial (x_{i_1} \dots x_{i_{t+2}})} = m_{i_1 \dots i_{t+2}}.$$

In fact, the quantities  $m$  satisfy the conditions of integrability (section 5, article I); that is,

$$\sum_{s=1}^{t+3} (-1)^s \frac{\partial m_{i_1 \dots i_{s-1} i_{s+1} \dots i_{t+3}}}{\partial x_{i_s}} \\ \sum_{s=1}^{t+3} (-1)^s \sum_h (-1)^{\binom{h_1 \dots h_{t+2}}{i_1 \dots i_{s-1} i_{s+1} \dots i_{t+3}}} \frac{\partial}{\partial x_{i_s}} (p_{h_1 \dots h_{r+1}} q_{h_{r+2} \dots h_{t+3}}) = 0.$$

3. To represent the fact that the relation (I) holds among the three functions  $F$ ,  $\Phi$ ,  $\Psi$  we shall write

$$\Psi \equiv (F, \Phi).$$

We have immediately

$$(F, \Phi) \equiv (-1)^{(r+1)(t-r+1)} (\Phi, F).$$

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If  $\Theta | [S_{v+1}] |$  is a regular function, and we write

$$\begin{aligned} \frac{\partial \Theta}{\partial (x_{h_{t+3}} \dots x_{h_{v+3}})} &= n_{h_{t+3} \dots h_{v+3}} \\ l_{i_1 \dots i_{v+3}} &= \sum_h (-1)^{\binom{h_1 \dots h_{v+3}}{i_1 \dots i_{v+3}}} p_{h_1 \dots h_{r+1}} q_{h_{r+2} \dots h_{t+2}} n_{h_{t+3} \dots h_{v+3}} \\ &= \sum_h (-1)^{\binom{h_1 \dots h_{v+3}}{i_1 \dots i_{v+3}}} m_{h_1 \dots h_{t+2}} n_{h_{t+3} \dots h_{v+3}}, \end{aligned}$$

it follows that there exists a function  $\Lambda | [S_{v+2}] |$  which is regular, and such that

$$\frac{\partial \Lambda}{\partial (x_{i_1} \dots x_{i_{v+3}})} = l_{i_1 \dots i_{v+3}}.$$

We shall write  $\Lambda = (F, \Phi, \Theta)$ .

And in general if the functions  $F^{(i)} | [S_r] |$  are regular, we shall understand by

$$(2) \quad M \equiv (F^{(1)}, F^{(2)}, \dots F^{(k)})$$

a regular function of hyperspaces  $S_R, R = \sum_1^k r_i + k$ , obtained as follows :

$$\Phi_2 \equiv (F^{(1)}, F^{(2)}), \Phi_3 \equiv (\Phi_2, F^{(3)}), \dots M = (\Phi_{k-1}, F^{(k)}).$$

We shall say that  $M$  is *composed* of the functions  $F^{(1)}, F^{(2)}, \dots F^{(k)}$  and we shall call the operation denoted by (2) the *composition* of the functions  $F^{(1)}, F^{(2)}, \dots F^{(k)}$ . The operation of *composition* of the functions  $F^{(i)}$  evidently possesses the *associative* property. Inversion of the elements of  $M$  can only produce changes in sign in the result.

The  $F^{(i)}$  will be spoken of as the *divisors* of  $M$ . If  $M$  has no other divisors but itself, it will be spoken of as *prime*. If two functions have no *divisor* in common, they will be said to be *mutually prime*.

4. Without stopping to develop the theory of divisibility in the present sense, we can give directly a few of its proper-

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ties and apply them to the results of the preceding sections. Thus, every regular function, which is not prime, can be decomposed into prime divisors, and this decomposition can be effected in more than one way. If a function divides one of the divisors of a function, it divides the function itself.

Two functions  $F$  and  $\Phi$  will be isogenous when

$$F \equiv (\Psi, f), \quad \Phi \equiv (\Psi, \phi),$$

where  $f, \phi$  are point functions and  $f$  is a function of  $\phi$ . If  $F$  and  $\Phi$  are isogenous, so will be also the functions

$$(F, \Theta) \text{ and } (\Phi, \Theta).$$

No function is isogenous to a prime function; in order that a function may be found isogenous to a given function it is necessary and sufficient that the given function should admit a divisor which is a point function. That is, it is necessary for it to have the form  $F \equiv (\Psi, f)$  with  $f$  a point function.

An elementary function is obtained by the composition of point functions, etc., etc.

### 12. *New considerations with reference to the relation of isogeneity*

1. So far we have been considering isogeneity between functions of hyperspaces of the same number of dimensions. We are now to generalize this relation so that it will apply to hyperspaces of different dimensions. Let us consider the two regular functions  $\Phi[[S_r]], \Psi[[S_t]]$ , with  $r > t$ , and write

$$\frac{\partial \Phi}{\partial (x_{t_1} \dots x_{t_{r+1}})} = a_{t_1 \dots t_{r+1}}, \quad \frac{\partial \Psi}{\partial (x_{t_1} \dots x_{t_{t+1}})} = b_{t_1 \dots t_{t+1}}.$$

We shall say that  $\Phi$  and  $\Psi$  are isogenous when the following conditions are satisfied:

$$(I) \quad \sum_s^{r+2} (-1)^s a_{t_1 \dots t_{s-1} t_{s+1} \dots t_{r+2}} b_{t_s t_1 \dots t_t} = 0.$$

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In the case where  $r$  is equal to  $t$ , these equations imply that the functions not only are isogenous in our first sense, but also that they are elementary. Conversely, if two elementary functions of hyperspaces of the same number of dimensions are isogenous in the sense of section 6, they are also in the present sense.

2. It is easy to show that *every function which admits  $\Phi$  as divisor is isogenous to  $\Psi$* . In fact, if we take

$$c_{t_1 \dots t_{v+2}} = \sum (-1)^{\binom{h_1 \dots h_{v+2}}{t_1 \dots t_{v+2}}} a_{h_1 \dots h_{r+1}} a'_{r+2, \dots, v+2},$$

we shall have 
$$\sum_s^{v+3} (-1)^s c_{t_1 \dots t_{s-1} t_{s+1} \dots t_{v+3}} b_{t_s h_1 \dots h_t} = c,$$

which proves the theorem.

3. We can now generalize a theorem given in section 7, article 2. We have:

*The necessary and sufficient condition that  $\phi \mid [S_r] \mid$  shall be isogenous to the elementary function  $\Psi \mid [S_{r-t}] \mid$ , is that*

$$(2) \quad \Phi \mid [S_r] \mid = (\Psi, \Theta).$$

That the condition is sufficient can be shown without any difficulty. In order to show that it is also necessary, let us write

$$\begin{aligned} \frac{d\Phi}{d(x_{t_1} \dots x_{t_{r+1}})} &= a_{t_1 \dots t_{r+1}}, \quad \frac{d\Psi}{d(x_{t_1} \dots x_{t_{r-t+1}})} = b_{t_1 \dots t_{r-t+1}}, \\ \frac{d\Theta}{d(x_{t_1} \dots x_{t_t})} &= c_{t_1 \dots t_t}, \\ b_{t_1 \dots t_{r-t+1}} &= \frac{d(\phi_1, \phi_2 \dots \phi_{r-t+1})}{d(x_{t_1} x_{t_2} \dots x_{t_{r-t+1}})}. \end{aligned}$$

We shall show that if (1) is true, (2) is also true; that is, that

$$(2') \quad a_{t_1 \dots t_{r+1}} = \sum (-1)^{\binom{h_1 \dots h_{r+1}}{t_1 \dots t_{r+1}}} b_{h_1 \dots h_{r-t+1}} c_{h_{r-t+2} \dots h_{r+1}}.$$

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For this purpose let us make a change of variable, taking instead of  $x_1, x_2, \dots, x_n$  the new variables  $\phi_1, \phi_2, \dots, \phi_{t+1}, x_{t+2}, \dots, x_n$ . If we indicate with a prime the symbols that belong with the new variables, we shall have

(i) If  $h_{r-t+2}, h_{r-t+3}, \dots, h_r \neq \phi_1, \phi_2, \dots, \phi_{r-t+1}$ , then

$$c_{h_{r-t+2} \dots h_r} = c'_{h_{r-t+2} \dots h_r} + \sum (-1)^{\binom{h_{p_1} \dots h_{p_t}}{h_{r-t+2} \dots h_{r+1}}} c'_{h_{p_1} \dots h_{p_t} l_1 \dots l_s} \frac{d(\phi_{l_1} \dots \phi_{l_s})}{d(x_{h_{p_{t+1}}} \dots x_{h_{p_{t-1}}})},$$

in which  $l_1, \dots, l_s$  are  $s$  of the numbers  $1, 2, \dots, r-t+1$ , and  $h_{p_1} \dots h_{p_t}$  is a permutation of the numbers  $h_{r-t+2}, \dots, h_{r+1}$ .

(ii) If one of the numbers  $h_{r-t+2} \dots h_r$  is equal to one of the numbers  $1, 2, \dots, t+1$ , then

$$c_{h_{r-t+2} \dots h_r} = \sum (-1)^{\binom{h_{p_1} \dots h_{p_{t-1}}}{h_{r-t+2} \dots h_r}} c'_{h_{p_1} \dots h_{p_{t-1}} l_1 \dots l_s} \frac{d(\phi_{l_1} \dots \phi_{l_s})}{d(x_{h_{p_{t+1}}} \dots x_{h_{p_{t-1}}})}$$

Equation (2') will then become

$$\begin{aligned} (2'') \quad a_{t_1 \dots t_{r+1}} &= \sum (-1)^{\binom{h_1 \dots h_{r+1}}{t_1 \dots t_{r+1}}} b_{h_1 \dots h_{r-t+1}} c'_{h_{r-t+2} \dots h_{r+1}} \\ &+ \sum (-1)^{\binom{h_1 \dots h_{r+1}}{t_1 \dots t_{r+1}}} \frac{d(\phi_1 \dots \phi_{r-t+1})}{d(x_{h_1} \dots x_{h_{r-t+1}})} \\ &\quad \sum (-1)^{\binom{h_{p_1} \dots h_{p_t}}{h_{r-t+2} \dots h_{r+1}}} c'_{h_{p_1} \dots h_{p_t} l_1 \dots l_s} \frac{d(\phi_{l_1} \dots \phi_{l_s})}{d(x_{h_{p_{t+1}}} \dots x_{h_{p_{t-1}}})}, \end{aligned}$$

in which the first sum is extended over all the possible combinations of the indices  $h_{r-t+2} \dots h_{r+1}$  which do not contain any of the numbers  $1, 2, \dots, r-t+1$ . The second sum may be rewritten in the form

$$\begin{aligned} &\sum (-1)^{\binom{h_{p_1} \dots h_{p_{t+1}}}{t_1 \dots t_{r+1}}} c'_{h_{p_1} \dots h_{p_t} l_1 \dots l_s} \\ &\sum (-1)^{h_{p_t}} \frac{d(\phi_1 \dots \phi_{r-t+1})}{d(x_{h_1} \dots x_{h_{r-t+1}})} \frac{d(\phi_{l_1} \dots \phi_{l_s})}{d(x_{h_{p_{t+1}}} \dots x_{h_{p_{t-1}}})}, \end{aligned}$$

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whence it vanishes. The equation (2'') reduces then to

$$(2''') \quad a_{t_1 \dots t_{r+1}} = \sum (-1)^{\binom{h_1 \dots h_{r+1}}{t_1 \dots t_{r+1}}} b_{h_1 \dots h_{r-t+1}} c'_{h_{r-t+2} \dots h_{r+1}}.$$

In particular we have

$$a_{1, 2, \dots, t+1, t_{t+2} \dots t_{r+1}} = b_{1, 2, \dots, t+1} c'_{t_{t+2} \dots t_{r+1}}$$

so that

$$(3) \quad c'_{t_{t+2} \dots t_{r+1}} = \frac{a_{1, 2, \dots, t+1, t_{t+2} \dots t_{r+1}}}{\left\{ \frac{d(\phi_1 \dots \phi_{t+1})}{d(x_1 \dots x_{t+1})} \right\}}.$$

Now by following a process analogous to that of section 7, article 2, it is easy to show that all the equations (2''') are a consequence of these last equations (3). And so it is sufficient for us to show that the quantities  $c'$ , obtained from (3), satisfy the conditions of integrability. We have in fact

$$a'_{1, \dots, t+1, t_{t+2}, \dots, t_{r+1}} = \frac{a_{1, \dots, t+1, t_{t+2}, \dots, t_{r+1}}}{\frac{d(\phi_1 \dots \phi_{t+1})}{d(x_1 \dots x_{t+1})}},$$

while  $a'$  will be zero if it has less than  $t+1$  of its subscripts taken from the numbers  $1, 2, \dots, t+1$ . If we apply then a process of reasoning analogous to that of section 7, article 2, we find that the conditions of integrability will be satisfied for the quantities  $c'$ .

### 13. *Differentiation and integration*

1. If two functions  $F | [S_n] |$ ,  $\Phi | [S_r] |$  are regular and isogenous, we know that the ratio

$$\phi : \frac{\left( \frac{d\Phi}{dS_{r+1}} \right)}{\left( \frac{dF}{dS_{r+1}} \right)}$$



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will be independent of the hyperspace  $S_{r+1}$ , and will depend merely upon the point of the space at which the derivative is taken. The quantity  $\phi$  will then be a point function for the total space of  $n$  dimensions. We shall denote it with the symbol  $\frac{d\Phi}{dF}$  and call it the *derivative of  $\Phi$  with respect to  $F$* .

As a fundamental theorem it can be shown that the *derivative of  $\Phi$  with respect to  $F$  is isogenous to both of the functions  $\Phi$  and  $F$* . The proof of this theorem comes immediately from formula (1) of section 7, with reference to the definition given in the preceding section.

2. Consider now a point function  $f$  isogenous to a regular function  $F | [S_r] |$ . By fixing the direction of the hyperspace  $S_{r+1}$  (see section 1, article 2) the quantity  $\frac{dF}{dS_{r+1}}$  will be defined (see section 3, article 7), and hence the quantity

$$\int_{S_{r+1}} f \frac{dF}{dS_{r+1}} dS_{r+1}$$

will also be defined. This integral we shall represent by the symbol

$$\int_{S_{r+1}} f dF.$$

Changing the direction of the hyperspace will change the sign of the integral.

We shall suppose that the hyperspace  $S_{r+1}$  is closed and forms the boundary of a hyperspace  $S_{r+2}$  immersed in a portion of the total hyperspace  $S_n$  throughout which  $f$  and  $F$  have no singularities. It follows that

$$\begin{aligned} \int_{S_{r+1}} f dF &= \int_{S_{r+1}} f \sum \frac{dF}{d(x_{t_1} \dots x_{t_{r+1}})} \alpha_{t_1 \dots t_{r+1}} dS_{r+1} \\ &= \int_{S_{r+1}} f \sum p_{t_1 \dots t_{r+1}} \alpha_{t_1 \dots t_{r+1}} dS_{r+1}, \end{aligned}$$

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where the  $\alpha_{i_1 \dots i_{r+1}}$  are the direction cosines of the hyperspace  $S_{r+1}$ . If we choose properly the direction of the hyperspace  $S_{r+2}$  and apply the generalization of Stokes's theorem (see section 4) we shall have

$$\begin{aligned} \int_{S_{r+1}} f dF &= \int_{S_{r+2}} \sum \beta_{i_1 \dots i_{r+2}} \sum (-1)^{s-1} \frac{\partial (f p_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}})}{\partial x_{i_s}} dS_{r+2} \\ &= \int_{S_{r+2}} \sum \beta_{i_1 \dots i_{r+2}} \left\{ \sum (-1)^s p_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}} \frac{\partial f}{\partial x_{i_s}} \right. \\ &\quad \left. + f \sum (-1)^{s-1} \frac{\partial p_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+2}}}{\partial x_{i_s}} \right\} dS_{r+2} = 0. \end{aligned}$$

Hence we have the theorem expressed by the formula

$$(I) \quad \int_{S_{r+1}} f dF = 0.$$

If, instead of a single hyperspace  $S_{r+1}$  we have the hyperspaces  $S_{r+1}^{(i)}$  ( $i=1, 2, \dots, n$ ) which bound a space  $S_{r+2}$  within which there are no singularities for  $f$  or  $F$ , we shall have the formula:

$$(I') \quad \sum_1^n \int_{S_{r+1}^{(i)}} f dF = 0,$$

in which the directions of the hyperspaces  $S_{r+1}^{(i)}$  are all to be chosen with reference to the conventions adopted for the generalization of Stokes's theorem.

*The theorem enunciated in the formulæ (I) and (I') is the direct extension of Cauchy's theorem.*

3. Let us take away from the total hyperspace all those portions in which either  $f$  or  $F$  have singularities, and then introduce cuts in such a way that every closed hyperspace  $S_{r+1}$  may be taken as the complete boundary of a hyperspace  $S_{r+2}$ .

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Take two hyperspaces  $S_r^0, S_r'$  such that a hyperspace  $S_{r+1}$  can be drawn to have them for its boundary, and choose the positive direction of  $S_r^0$  and the negative direction of  $S_r'$  so as to correspond by the theorem of Stokes to one direction of the hyperspace  $S_{r+1}$ . With the direction of  $S_{r+1}$  fixed in this way, the integral

$$(2) \quad \int_{S_{r+1}} f dF$$

will be determined.

It is easy to show that the value of the integral (2) will not depend on the hyperspace  $S_{r+1}$ , but merely on  $S_r^0$  and  $S_r'$ . In fact if  $S'_{r+1}$  is another hyperspace which has these same two spaces for its boundary, the totality of  $S_{r+1}$  and  $S'_{r+1}$  will form a closed hyperspace, and from the hypotheses that we have made, we shall have

$$\int_{S_{r+1}+S'_{r+1}} f dF = 0,$$

from which the desired property follows.

Therefore the integral (2) can be indicated by the expression

$$(2') \quad \int_{S_r^0}^{S_r'} f dF.$$

By changing the direction of  $S_{r+1}$  we change the sign of the integral; hence we may write

$$(3) \quad \int_{S_r^0}^{S_r'} f dF = - \int_{S_r'}^{S_r^0} f dF.$$

4. If we keep fixed the hyperspace  $S_r^0$  and vary  $S_r'$ , the integral (2') may be regarded as a function (regular) of  $S_r'$ , and we can write

$$(4) \quad \int_{S_r^0}^{S_r'} f dF = \Phi | [S_r'] |.$$

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*The function  $\Phi$  will be isogenous to  $F$  and we shall have*

$$(5) \quad \frac{d\Phi}{dF} = f,$$

*that is to say, the two operations of integration and differentiation are mutually inverse.*

### 14. Isogeneity of order $r$

1. A system of elementary functions will be said to have isogeneity of order  $r$  when all the functions of order greater than or equal to  $r$ , which are obtained from the system by means of composition (see section 11), vanish, while there is at least one function of order  $r - 1$  which does not vanish. All the elementary functions  $\Phi | [S_i] |$  of the system must depend on certain functions  $\phi_1, \phi_2, \dots, \phi_k, \dots$  in such a way (see section 10) that

$$\frac{\partial \Phi}{\partial (x_{i_1} \dots x_{i_{r+1}})} = \frac{d(\phi_{i_1} \dots \phi_{i_{r+1}})}{d(x_{i_1} \dots x_{i_{r+1}})}, \quad \Phi \equiv (\phi_{i_1}, \phi_{i_2}, \dots, \phi_{i_{r+1}}).$$

2. We have immediately the following theorems :

*The necessary and sufficient condition for isogeneity of order  $r$  that is*

$$(1) \quad \frac{d(\phi_{i_1} \dots \phi_{i_{r+1}})}{d(x_{i_1} \dots x_{i_{r+1}})} = 0$$

*for every possible combination of the numbers  $i_1, \dots, i_{r+1}, i_1, \dots, i_{r+1}$ .*

*A function of order  $r - 1$  is always isogenous to any other function of the system.*

In fact from (1) it follows that every function of order  $r - 1$  is isogenous to the functions of order zero of the system, that is, to the functions  $\phi_i$ . We shall have

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then

$$\begin{aligned} q_{i_s, h_1, \dots, h_i} &= \frac{\partial \Phi}{\partial (x_{i_s} x_{h_1} \dots x_{h_i})} = \sum_1^{i+1} (-1)^{u-1} \frac{\partial \phi_{i_u}}{\partial x_{i_s}} \frac{d(\phi_{i_1} \dots \phi_{i_{u-1}} \phi_{i_{u+1}} \phi_{i_{i+1}})}{d(x_{h_1} \dots x_{h_i})} \\ &= \sum_1^{i+1} (-1)^{u-1} \frac{\partial \phi_{i_u}}{\partial x_{i_s}} N_u. \end{aligned}$$

And if we let  $\psi \mid [S_{r-1}] \mid$  represent one of the functions of order  $r - 1$  of the system, and write

$$\frac{\partial \psi}{\partial (x_{i_1} \dots x_{i_r})} = p_{i_1 \dots i_r},$$

we shall have

$$\begin{aligned} \sum_s^{r+1} (-1)^s p_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+1}} q_{i_s, h_1, \dots, h_i} \\ = \sum_s^{i+1} (-1)^{u-1} N_u \sum_s^{r+1} (-1)^s p_{i_1 \dots i_{s-1} i_{s+1} \dots i_{r+1}} \frac{\partial \phi_{i_u}}{\partial x_{i_s}}. \end{aligned}$$

*Every function of order  $r - 1$  admits as divisor another function of the system of lower order* (see section II, article 3).

3. Let us consider specially the functions of the system of order zero; that is, the functions  $\phi_1, \phi_2, \dots, \phi_r, \dots$ . By means of the equations (I) we know that *there must be  $r$  of them,  $\phi_1, \phi_2, \dots, \phi_r$ , independent, of which all the others are functions*, and conversely, that *every function of  $\phi_1, \phi_2, \dots, \phi_r$ , will be an elementary function in the system, and will be of order zero*.

If we take two functions  $\Phi$  and  $F$  of order  $r - 1$ , they will be isogenous, and we shall have the relation

$$(2) \quad \frac{d\Phi}{dF} = \phi(\phi_1, \phi_2, \dots, \phi_r).$$

Further, if we take an arbitrary function  $\phi$  of order zero, that is a function of  $\phi_1, \phi_2, \dots, \phi_r$ , we shall have

$$(3) \quad \int_{S_r} \phi dF = 0,$$

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where  $S_r$  is the complete boundary of a space  $S_{r+1}$  within which  $\phi$  and  $F$  have no singularities. If we have

$$F \equiv (\phi_1, \phi_2, \dots \phi_r),$$

then (3) can be written in the form

$$\int_{S_r} \phi \frac{d(\phi_1, \phi_2, \dots \phi_r)}{d(\omega_1, \omega_2, \dots \omega_r)} d\omega_1, d\omega_2, \dots d\omega_r = 0,$$

$\omega_1, \dots \omega_r$  being the parameters of the hyperspace  $S_r$  (see section I, articles I, 2). If we take

$$\frac{d\phi_i}{d\omega_s} d\omega_s = d_s \phi_i,$$

we shall have 
$$\int_{S_r} \phi \begin{vmatrix} d_1 \phi_1 & d_2 \phi_1 & \dots & d_r \phi_1 \\ d_1 \phi_2 & d_2 \phi_2 & \dots & d_r \phi_2 \\ \vdots & \vdots & \ddots & \vdots \\ d_1 \phi_r & d_2 \phi_r & \dots & d_r \phi_r \end{vmatrix} = 0,$$

which is but a *generalization of Cauchy's theorem* (see the preceding section) put in a different form for the case of the elementary functions.

If  $S_r$  is not closed, but is bounded by two hyperspaces  $S_{r-1}^*$  and  $S_{r-1}$ , of which the first is fixed and the second variable, we shall have defined the expression

$$\Phi |[S_{r-1}]| = \int_{S_{r-1}^*}^{S_{r-1}} \phi \begin{vmatrix} d_1 \phi_1 & \dots & d_r \phi_1 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ d_1 \phi_r & \dots & d_r \phi_r \end{vmatrix}$$

*Third Lecture*

ON THE THEORY OF WAVES AND  
GREEN'S METHOD\*

SECTION I

LET a homogeneous liquid be subjected to certain forces and let it occupy a domain  $S$ . Let this domain be limited by a frontier  $\sigma$  which is composed partly of a set  $\omega'$  of rigid boundaries, and partly of a free surface  $\omega$ , where the pressure is  $P$ .

Let us suppose that the state of equilibrium is stable. We shall study the small oscillations of the fluid when it is displaced from the state of equilibrium.

The hydrodynamical equations of Lagrange are

$$\begin{aligned} \frac{d^2x}{dt^2} \cdot \frac{\partial x}{\partial x_0} + \frac{d^2y}{dt^2} \cdot \frac{\partial y}{\partial x_0} + \frac{d^2z}{dt^2} \cdot \frac{\partial z}{\partial x_0} &= \frac{\partial}{\partial x_0} \left( V - \frac{P}{\rho} \right) \\ \frac{d^2x}{dt^2} \cdot \frac{\partial x}{\partial y_0} + \frac{d^2y}{dt^2} \cdot \frac{\partial y}{\partial y_0} + \frac{d^2z}{dt^2} \cdot \frac{\partial z}{\partial y_0} &= \frac{\partial}{\partial y_0} \left( V - \frac{P}{\rho} \right) \left\{ \right. \quad (1) \\ \frac{d^2x}{dt^2} \cdot \frac{\partial x}{\partial z_0} + \frac{d^2y}{dt^2} \cdot \frac{\partial y}{\partial z_0} + \frac{d^2z}{dt^2} \cdot \frac{\partial z}{\partial z_0} &= \frac{\partial}{\partial z_0} \left( V - \frac{P}{\rho} \right) \end{aligned}$$

where  $x, y, z$ , denote the coördinates of points of the fluid at time  $t$ ,  $x_0, y_0, z_0$  the initial coördinates,  $V$  the potential function,  $P$  the pressure,  $\rho$  the density.

2. Let  $x_0, y_0, z_0$  be the coördinates which correspond to the state of stable equilibrium,  $\xi, \eta, \zeta$  the components of displacement of each particle with respect to its position of equilibrium.

Then  $x = x_0 + \xi, \quad y = y_0 + \eta, \quad z = z_0 + \zeta.$

\* Translated from the French by Professor Percy John Daniell, of the Rice Institute.

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If we consider the displacements as infinitesimals of the first order and if we neglect terms of order higher than the first, the equations (1) become

$$\begin{aligned}\frac{d^2\xi}{dt^2} &= \frac{\partial}{\partial x_0} \left( V - \frac{P}{\rho} \right) \\ \frac{d^2\eta}{dt^2} &= \frac{\partial}{\partial y_0} \left( V - \frac{P}{\rho} \right) \\ \frac{d^2\zeta}{dt^2} &= \frac{\partial}{\partial z_0} \left( V - \frac{P}{\rho} \right)\end{aligned}$$

For simplification the indices 0 are suppressed and  $x, y, z$  denote the coördinates of each particle in the position of equilibrium.

Then

$$\begin{aligned}\frac{d^2\xi}{dt^2} &= \frac{\partial}{\partial x} \left( V - \frac{P}{\rho} \right) \\ \frac{d^2\eta}{dt^2} &= \frac{\partial}{\partial y} \left( V - \frac{P}{\rho} \right) \\ \frac{d^2\zeta}{dt^2} &= \frac{\partial}{\partial z} \left( V - \frac{P}{\rho} \right)\end{aligned} \quad (2)$$

The condition of incompressibility can be written as

$$\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} + \frac{\partial \zeta}{\partial z} = 0. \quad (3)$$

On account of (2) we can put

$$\xi = \frac{\partial \Phi}{\partial x}, \quad \eta = \frac{\partial \Phi}{\partial y}, \quad \zeta = \frac{\partial \Phi}{\partial z},$$

$\Phi$  being the potential of displacement.

Then the equations (2) become

$$\frac{d^2\Phi}{dt^2} - V + \frac{P}{\rho} = c, \quad (4)$$

where  $c$  is constant with respect to  $x, y, z$ , but may vary with  $t$ .



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The equation (3) becomes

$$\Delta^2\Phi = 0.$$

At points of the liquid where it touches the rigid boundary

$$\xi \cos nx + \eta \cos ny + \zeta \cos nz = 0,$$

if  $n$  denotes the normal to the boundary.

This condition becomes  $\frac{\partial\Phi}{\partial n} = 0$ .

3. Let us return to the equation (4). If we put

$$V - \frac{P}{\rho} + c = H,$$

the equation (4) becomes  $\frac{d^2\Phi}{dt^2} = H$ . (4')

The free surface of the fluid has been denoted by  $\omega$ . Let us suppose that the potential function  $V$  and the pressure  $P$ , which correspond to each particle of fluid belonging to  $\omega$  are functions of the coördinates of the point occupied by the particle independently of the form of the liquid. If this hypothesis is not correct, since the displacements are infinitesimal, we can neglect the variations produced by the changes in form of the fluid so that we can always proceed as if the hypothesis were correct.

In the state of equilibrium  $H$  is constant on  $\omega$ . Therefore the equation of this surface will be

$$H = H_0 = \text{constant}.$$

Let us now calculate  $H$  when a point of the surface  $\omega$  is displaced when  $\xi, \eta, \zeta$  are the components of displacement.

If we neglect infinitesimals of a higher order than the first,

$$H = H_0 + \frac{\partial H}{\partial x}\xi + \frac{\partial H}{\partial y}\eta + \frac{\partial H}{\partial z}\zeta.$$

Then putting  $\lambda^2 = \left(\frac{\partial H}{\partial x}\right)^2 + \left(\frac{\partial H}{\partial y}\right)^2 + \left(\frac{\partial H}{\partial z}\right)^2$ ,

$$\frac{\partial H}{\partial x} = \lambda \cos nx, \quad \frac{\partial H}{\partial y} = \lambda \cos ny, \quad \frac{\partial H}{\partial z} = \lambda \cos nz, \quad (5)$$

when  $n$  is the normal to the surface  $\omega$ .

$$\begin{aligned} \text{Then } H &= H_0 + \lambda (\xi \cos nx + \eta \cos ny + \zeta \cos nz) \\ &= H_0 + \lambda \frac{\partial \Phi}{\partial n}; \end{aligned}$$

combining this with equation (4')

$$\frac{\partial^2 \Phi}{\partial t^2} = H_0 + \lambda \frac{\partial \Phi}{\partial n}$$

or 
$$\frac{\partial^2 \Phi}{\partial t^2} = \lambda \frac{\partial \Phi}{\partial n},$$

since  $\Phi$  is determinate except for a quantity which is constant with respect to the time.

Let us take the normal  $n$  as directed toward the interior of the fluid, and let us suppose that  $V - \frac{P}{\rho}$  increases on moving  $\omega$  and following the positive direction of  $n$ .

Then when  $n$  is positive,  $\frac{\partial H}{\partial n} > 0$ ,

or by virtue of the equations (5)

$$\frac{\partial H}{\partial n} = \frac{\partial H}{\partial x} \cos nx + \frac{\partial H}{\partial y} \cos ny + \frac{\partial H}{\partial z} \cos nz = \lambda,$$

it follows that  $\lambda > 0$ .

The problem of waves can be presented in the following manner.

4. To determine a function  $\Phi$  regular within the domain  $S$  which satisfies the equation

$$(A) \quad \Delta^2 \Phi = 0$$

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 within  $S$  and which in the part  $\omega'$  of the boundary satisfies the condition

$$(B) \quad \frac{\partial \Phi}{\partial n} = 0$$

and in the part  $\omega$  satisfies the condition

$$(C) \quad \frac{\partial^2 \Phi}{\partial t^2} = \lambda \frac{\partial \Phi}{\partial n},$$

where  $\lambda$  is a positive quantity independent of the time, and  $n$  is the normal to the boundary directed toward the interior of the domain  $S$ .

## SECTION 2

1. We can make a comparison between the problem we are about to consider and that of the vibrations of elastic media, and other problems of mathematical physics. The problem of the vibrations of elastic media is based upon the equation

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \Delta^2 u. \quad (6)$$

The problem of the propagation of heat in the case of varying temperature leads to the equation

$$\frac{\partial V}{\partial t} = a \Delta^2 V. \quad (7)$$

The problems of potential and of stationary temperatures in isotropic bodies depend upon the equation of Laplace

$$\Delta^2 W = 0. \quad (8)$$

These three equations are respectively of *hyperbolic*, *parabolic*, and *elliptic* types.

The question we have considered in section 1 belongs to the elliptic type on account of the equation (A) of section 1, which is the equation of Laplace; but it is the condition which must be satisfied on the surface  $\omega$  of the boundary

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which leads to the essential difference between this problem and the problems of potential and stationary temperatures. In fact, in the problems of potential the conditions at the boundary are reduced to that of giving the values of the unknown function or of its normal derivative; in those of stationary temperatures a linear relation between the unknown function and its normal derivative is known. But in the case of waves the condition at the boundary (equation (C) of section 1) introduces a new variable, the time, which makes the problem one of four variables. In respect to the number of variables the problem of waves is similar to the problems of vibrations and varying temperatures. It differs from them, however, because equations (6) and (7) have real characteristics. There are no real characteristics in the problem of the waves of liquids. We shall give a theorem in section 3 which will show the difference, from a physical standpoint, between waves in elastic media and waves in liquids.

2. There are two general methods in which the different problems we are investigating can be treated.

That of the separation of variables consists in separating the time from the space variables.

Let us put in the equation (6)

$$U = \sin mt \cdot u(x, y, z), \quad (9)$$

where  $m$  is a constant.

The equation becomes

$$m^2 u + \alpha^2 \Delta^2 u = 0, \quad (10)$$

where the time has disappeared. If, for example, on the boundary  $U = 0$ ,  $u$  must be taken  $= 0$  there. We are led to find values of  $m$  for which the previous equation has solutions which are not identically zero (special solutions). The general solution is obtained by forming an infinite series of

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solutions of the form (9) multiplied by arbitrary constants of such values that  $U$  and  $\frac{\partial U}{\partial t}$  for  $t = 0$  have the values of the given functions of  $x, y, z$ .

The question of determining the special solutions has been resolved by Poincaré; the theory of integral equations has been used and Mr. Hilbert, Mr. Schmidt, and others have founded the theory of series of special solutions.

Similarly an analogous process can be employed for equation (7) if we put  $V = e^{mt}v(x, y, z)$ ; that is to say, by separating the time from the variables  $x, y, z$ .

Equation (7) reduces then to

$$mv + a\Delta^2v = 0,$$

which is exactly analogous to equation (10).

3. The same method of the separation of the variables can be applied to the problem of waves in liquids.

If we put  $\Phi = \sin mt \phi(x, y, z)$  equation (A) of section 1 becomes

$$\Delta^2\phi = 0,$$

equation (B) is

$$\frac{\partial \phi}{\partial u} = 0,$$

and equation (C) must be replaced by

$$m^2\phi + \lambda \frac{\partial \phi}{\partial n} = 0.$$

Here again the values of  $m$  corresponding to solutions  $\phi$  which are not identically zero (special solutions) must be found.

By series of special solutions the general solution can be obtained. To calculate the values of  $m$  the method of Poincaré with those of integral equations can be used.

4. But we wish to set aside the process of the separation of variables and to pass on to the other general method. It

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is the method which is connected with the ideas which Green used for the first time for the equation of Laplace and which, little by little, has been also used for other types of equations. By this point of view Kirchhoff arrived at his celebrated formula which expresses the principle of Huyghens. He applied Green's method to equation (6).

Betti has also applied an analogous method to equation (7).

We wish to show that a general formula can be found in the case of waves of fluids of a type which presents some analogies to these formulæ. I have had occasion to mention this formula without giving any development from it in my lectures at Stockholm. We shall now develop it and demonstrate in detail some applications of it.

### SECTION 3

1. We shall begin by demonstrating in this paragraph some general theorems.

*First Theorem.* If  $\Phi$  is the function which satisfies the conditions (A), (B), (C) of section 1, it is determinate if the values  $\Phi_0, \left(\frac{\partial \Phi}{\partial t}\right)_0$  of  $\Phi$  and  $\left(\frac{\partial \Phi}{\partial t}\right)$  for  $t = 0$  on the surface  $\omega$  are known.

*Demonstration.* Let  $\Phi_1, \Phi_2$  be two functions which satisfy the conditions to which  $\Phi$  is subjected.

Their difference  $\Phi_3 = \Phi_1 - \Phi_2$  also satisfies the equations (A), (B), (C) and further we have

$$(\Phi_3)_0 = 0 \qquad \left(\frac{\partial \Phi_3}{\partial t}\right)_0$$

for  $t = 0$  on the surface  $\omega$ .

Let us now calculate

$$\Omega = \frac{1}{2} \frac{\partial}{\partial t} \int_{\omega} \frac{1}{\lambda} \left(\frac{\partial \Phi_3}{\partial t}\right)^2 d\omega.$$

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On account of equation (C) we shall have

$$\Omega = \int_{\omega} \frac{1}{\lambda} \left( \frac{\partial \Phi_3}{\partial t} \right) \left( \frac{\partial^2 \Phi_3}{\partial t^2} \right) d\omega = \int_{\omega} \left( \frac{\partial \Phi_3}{\partial t} \right) \left( \frac{\partial \Phi_3}{\partial n} \right) d\omega.$$

But on  $\omega'$   $\frac{\partial \Phi_3}{\partial n} = 0$  and therefore

$$\Omega = \int_{\sigma} \left( \frac{\partial \Phi_3}{\partial t} \right) \left( \frac{\partial \Phi_3}{\partial n} \right) d\sigma.$$

Applying a well-known transformation,

$$\begin{aligned} -\Omega = \int_S \left( \frac{\partial}{\partial x} \frac{\partial \Phi_3}{\partial t} \cdot \frac{\partial \Phi_3}{\partial x} + \frac{\partial}{\partial y} \frac{\partial \Phi_3}{\partial t} \cdot \frac{\partial \Phi_3}{\partial y} + \frac{\partial}{\partial z} \frac{\partial \Phi_3}{\partial t} \cdot \frac{\partial \Phi_3}{\partial z} \right) dS \\ + \int_S \frac{\partial \Phi_3}{\partial t} \Delta^2 \Phi_3 dS. \end{aligned}$$

The third term = 0; then

$$-\Omega = \frac{1}{2} \frac{\partial}{\partial t} \int_S \left\{ \left( \frac{\partial \Phi_3}{\partial x} \right)^2 + \left( \frac{\partial \Phi_3}{\partial y} \right)^2 + \left( \frac{\partial \Phi_3}{\partial z} \right)^2 \right\} dS$$

and it follows that

$$\frac{1}{2} \frac{\partial}{\partial t} \left[ \int_{\omega} \frac{1}{\lambda} \left( \frac{\partial \Phi_3}{\partial t} \right)^2 d\omega + \int_S \left\{ \left( \frac{\partial \Phi_3}{\partial x} \right)^2 + \left( \frac{\partial \Phi_3}{\partial y} \right)^2 + \left( \frac{\partial \Phi_3}{\partial z} \right)^2 \right\} dS \right] = 0.$$

Integrating with respect to the time,

$$\int_{\omega} \frac{1}{\lambda} \left( \frac{\partial \Phi_3}{\partial t} \right)^2 d\omega + \int_S \left\{ \left( \frac{\partial \Phi_3}{\partial x} \right)^2 + \left( \frac{\partial \Phi_3}{\partial y} \right)^2 + \left( \frac{\partial \Phi_3}{\partial z} \right)^2 \right\} dS = c, \quad (11)$$

where  $c$  is constant with respect to the time.

Then if  $(\Phi_3)_0 = 0$  for  $t = 0$  on  $\omega$ , since  $\frac{\partial \Phi_3}{\partial n} = 0$  on  $\omega'$   $(\Phi_3)_0$  must be zero in the domain  $S$ . Consequently, the second integral in the formula (11) will be 0 for  $t = 0$ . In the same way, since  $\left( \frac{\partial \Phi_3}{\partial t} \right)_0 = 0$ , the first integral will be 0 for  $t = 0$ . It follows that  $c = 0$ , and the conclusion can be drawn that  $\Phi_3$  will be 0 for every value of  $t$  and therefore  $\Phi_1 = \Phi_2$ .

Q. E. D.

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2. *Second Theorem.* If at a certain instant the molecules belonging to a part of the domain  $S$  are not displaced from the position of equilibrium, any molecule of the fluid is not displaced from the position of equilibrium.

*Demonstration.* If  $\xi, \eta, \zeta$  are 0 in any part of  $S$ ,  $\Phi$  will be constant in this part, and since it is an harmonic function regular in  $S$ , it will be everywhere constant. Consequently  $\xi, \eta, \zeta$  will be 0 at all points of  $S$ . Q. E. D.

*Third Theorem.* If at a certain instant the molecules belonging to a part of the domain  $S$  are not displaced from the position of equilibrium and have no velocity, the fluid will remain always in the position of equilibrium.

*Demonstration.* If  $\xi, \eta, \zeta$  and  $\frac{d\xi}{dt}, \frac{d\eta}{dt}, \frac{d\zeta}{dt}$  are 0 in one part of the domain  $S$  at a certain instant,  $\Phi$  and  $\frac{d\Phi}{dt}$  will be constant in this part and therefore they will be constant in the whole domain  $S$  at the same instant. By virtue of the first theorem they will be constant in  $S$  at every instant and consequently the liquid will have no motion. Q. E. D.

3. These propositions show us the essential difference which exists between waves in liquids and waves in elastic media. In elastic media the motion is propagated with a certain velocity from one part to another; in liquids the motion reaches the whole mass contemporaneously, at least when the fluid does not remain in a constant state of equilibrium. In the case of liquids there is no propagation of motion and consequently one cannot speak of the velocity of propagation.

### SECTION 4

I. Let  $\Phi$  and  $\Psi$  be two functions which satisfy the conditions (A), (B), (C) of section I.



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By virtue of Green's theorem

$$\int_{\sigma} \left( \phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n} \right) d\sigma = 0$$

on account of (B)  $\int_{\omega} \left( \phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n} \right) d\omega = 0.$

Using (C) this becomes

$$\int_{\omega} \left( \Phi \left( \frac{\partial^2 \psi}{\partial t^2} - \Psi \frac{\partial^2 \phi}{\partial t^2} \right) \frac{1}{\lambda} \right) d\omega = 0. \quad (12)$$

Let us now suppose that

$$\Psi = \frac{1}{r} + \chi,$$

where  $r$  denotes the distance between a point  $A$  ( $x_0, y_0, z_0$ ) interior to the domain  $S$  and a point  $(x, y, z)$  and where  $\chi$  is a regular function. Then the preceding formulæ are no longer valid for they presuppose that  $\psi$  is regular in the domain  $S$ . In this case formula (12) must be replaced by

$$4 \pi \Phi_A + \int_{\omega} \left( \Phi \frac{\partial^2 \psi}{\partial t^2} - \psi \frac{\partial^2 \Phi}{\partial t^2} \right) \frac{1}{\lambda} d\omega = 0, \quad (12')$$

where  $\Phi_A$  denotes the value of  $\Phi$  at the point  $A$ .

Then 
$$4 \pi \phi_A = - \frac{\partial}{\partial t} \int_{\omega} \left( \phi \frac{\partial \psi}{\partial t} - \psi \frac{\partial \phi}{\partial t} \right) \frac{1}{\lambda} d\omega.$$

Integrating between the limits 0 and  $t_1$ , we obtain

$$\begin{aligned} 4 \pi \int_0^{t_1} \phi_A dt = & - \int_{\omega} \left\{ \phi_1 \left( \frac{\partial \psi}{\partial t} \right)_1 - \psi_1 \left( \frac{\partial \phi}{\partial t} \right)_1 \right\} \frac{1}{\lambda} d\omega \\ & + \int_{\omega} \left\{ \phi_0 \left( \frac{\partial \psi}{\partial t} \right)_0 - \psi_0 \left( \frac{\partial \phi}{\partial t} \right)_0 \right\} \frac{1}{\lambda} d\omega \end{aligned}$$

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where  $\phi_1, \psi_1 \left( \frac{\partial \phi}{\partial t} \right)_1 \left( \frac{\partial \psi}{\partial t} \right)_1$  denote the functions  $\phi, \psi$  and the derivatives  $\frac{\partial \phi}{\partial t} \frac{\partial \psi}{\partial t}$  for  $t=t_1$ , while  $\phi_0, \psi_0 \left( \frac{\partial \phi}{\partial t} \right)_0 \left( \frac{\partial \psi}{\partial t} \right)_0$  denote the same quantities for  $t=t_0$ . Let us now suppose that  $\psi_1$  and  $\left( \frac{d\psi}{dt} \right)_1$  are 0 on  $\omega$ .

Then

$$(D) \quad \Phi(x_0, y_0, z_0, t_1) = \frac{1}{4\pi} \frac{d}{dt_1} \int_{\omega} \left\{ \phi_0 \left( \frac{\partial \psi}{\partial t} \right)_0 - \psi_0 \left( \frac{\partial \phi}{\partial t} \right)_0 \right\} \frac{1}{\lambda} d\omega.$$

The above formula gives us a knowledge of  $\Phi$  at every point in  $S$  and for every value of  $t$  when the values of  $\phi_0 \left( \frac{\partial \phi}{\partial t} \right)_0$  are known on  $\omega$ . (Compare with the first theorem of section 3.)

It is necessary to calculate further the function  $\Psi$  and consequently  $\chi$ . This function plays, in this case, *a part which can be compared with that played by Green's function*.

It must be remarked that  $\psi_0$  and  $\left( \frac{d\psi}{dt} \right)_0$  should depend on  $t_1$  since  $\psi_1$  and  $\left( \frac{d\psi}{dt} \right)_1$  should be 0. The variable  $t_1$  appears then in the second member of the equation (D) because it is contained in  $\psi_0$  and  $\left( \frac{d\psi}{dt} \right)_0$ .

### SECTION 5

In this paragraph we shall give some applications of the fundamental formula (D) of the preceding paragraph. Let us suppose that  $S$  is a sphere of radius  $R$  and that  $\omega$  is the surface of the sphere in such a way that there are no rigid boundaries.

Let us put

$$\psi = a_0 + \frac{(t_1 - t)^2}{2!} a_2 + \frac{(t_1 - t)^4}{4!} a_4 + \dots,$$

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$a_0, a_2, a_4 \dots$  being coefficients independent of  $t_1$  and  $t$ . We shall have

$$\psi_1 = a_0, \quad \left(\frac{d\psi}{dt}\right)_1 = 0.$$

But

$$\psi = \frac{1}{r} + \chi.$$

$$\therefore a_0 = \frac{1}{r_A} + (\chi)_1,$$

and since  $a_0$  should be 0 on  $\omega$  and  $\chi$  should be a regular and harmonic function if we use the method of images we obtain

$$(\chi)_1 = -\frac{R}{l} \frac{1}{r_{A'}},$$

where  $A'$  denotes the image point of  $A$  with respect to the sphere,  $r_{A'}$  is the distance of the point  $A'$  from the point  $(x, y, z)$ ,  $l$  is the distance from the center of the sphere to the point  $A$ .

Then

$$a_0 = \frac{1}{r_A} - \frac{R}{l} \frac{1}{r_{A'}}.$$

Let  $\rho$  be the radius vector, the pole being at the center of the sphere; then

$$\frac{\partial \psi}{\partial n} = -\frac{\partial \psi}{\partial \rho} = -\frac{\partial a_0}{\partial \rho} - \frac{(t_1 - t)^2}{2!} \frac{\partial a_2}{\partial \rho} - \frac{(t_1 - t)^4}{4!} \frac{\partial a_4}{\partial \rho} \dots$$

$$\frac{\partial^2 \psi}{\partial t^2} = a_2 + \frac{(t_1 - t)^2}{2!} a_4 + \dots$$

Consequently on the surface  $\omega$ , i.e. for  $\rho = R$

$$-\lambda \frac{\partial a_0}{\partial \rho} = a_2, \quad -\lambda \frac{\partial a_2}{\partial \rho} = a_4, \quad -\lambda \frac{\partial a_4}{\partial \rho} = a_6, \dots$$

Since  $a_0$  is known, the regular harmonic functions  $a_2, a_4, a_6 \dots$  must be determinate when their values on the boundary of the sphere are known.

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Let us begin by transforming the expression for  $a_0$ . Let us denote by  $\gamma$  the angle between the lines joining the center of the sphere to the points  $A$  and  $(x, y, z)$ .

$$\text{Then } a_0 = \frac{1}{(l^2 + \rho^2 - 2 l \rho \cos \gamma)^{\frac{1}{2}}} - \frac{R}{l} \frac{1}{\left( \frac{R^4}{l^2} + \rho^2 - 2 \frac{R^2}{l} \rho \cos \gamma \right)^{\frac{1}{2}}}$$

$$\text{or } \frac{\rho}{R} \frac{\partial a_0}{\partial \rho} = \frac{\rho}{R} \frac{\partial}{\partial \rho} \left[ \frac{1}{(l^2 + \rho^2 - 2 l \rho \cos \gamma)^{\frac{1}{2}}} \right] \\ - \frac{\rho}{R} \frac{\partial}{\partial \rho} \frac{R}{l} \frac{1}{\left( \frac{R^4}{l^2} + \rho^2 - 2 \frac{R^2}{l} \rho \cos \gamma \right)^{\frac{1}{2}}}$$

is a harmonic function which is equal to  $\frac{\partial a_0}{\partial \rho}$  on the surface of the sphere; but it is not regular in the interior of the sphere. In fact, the first term of the second member becomes infinite for  $\rho = l$ ,  $\gamma = 0$ . Then to calculate  $a_2$  we cannot take the previous expression and multiply it by  $-\lambda$  for  $a_2$  must be regular in the interior of the sphere. But the following artifice may be used to calculate  $a_2$ .

Let us transform the first term of the second member by a transformation of reciprocal radii with respect to the sphere and let us multiply by  $\frac{R}{\rho}$ . The expression remains harmonic, possesses the same values on the boundary of the sphere, but becomes regular in the interior. To make the transformation of reciprocal radii it is sufficient to replace  $\rho$  by  $\frac{R^2}{\rho}$ . Thus the first term of the previous expression becomes

$$- R^2 \frac{R^2 - l \rho \cos \gamma}{(l^2 \rho^2 - R^4 - 2 l R^2 \rho \cos \gamma)^{\frac{1}{2}}}.$$

The second term equals

$$\frac{\rho l (l \rho - R^2 \cos \gamma)}{(R^4 + l^2 \rho^2 - 2 l R^2 \rho \cos \gamma)^{\frac{1}{2}}}$$

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It is found then that

$$a_2 = -\lambda \frac{l^2 \rho^2 - R^4}{(R^4 + l^2 \rho^2 - 2 l \rho R^2 \cos \gamma)^{\frac{3}{2}}}.$$

In calculating  $a_4, a_6 \dots$  there are no more difficulties and

$$a_4 = \frac{R \partial \rho}{\lambda^2} \left[ \frac{R^4 - l^2 \rho^2}{(R^4 + l^2 \rho^2 - 2 l \rho R^2 \cos \gamma)^{\frac{3}{2}}} \right].$$

In general,

$$a_{2n} = (-1)^{n-1} \frac{\lambda^n}{R^{n-1}} \frac{\partial^{n-1}}{\partial (\log \rho)^{n-1}} \left[ \frac{R^4 - l^2 \rho^2}{(R^4 + l^2 \rho^2 - 2 l \rho R^2 \cos \gamma)^{\frac{3}{2}}} \right].$$

Consequently,

$$\begin{aligned} \Psi &= a_0 + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\lambda^n}{R^{n-1}} \frac{\partial^{n-1}}{\partial (\log \rho)^{n-1}} \\ &\quad \left[ \frac{R^4 - l^2 \rho^2}{(R^4 + l^2 \rho^2 - 2 l \rho R^2 \cos \gamma)^{\frac{3}{2}}} \right] \frac{(t_1 - t)^{2n}}{2 n !}. \\ \frac{\partial \Psi}{\partial t} &= - \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\lambda^n}{R^{n-1}} \frac{\partial^{n-1}}{\partial (\log \rho)^{n-1}} \\ &\quad \left[ \frac{R^4 - l^2 \rho^2}{(R^4 + l^2 \rho^2 - 2 l \rho R^2 \cos \gamma)^{\frac{3}{2}}} \right] \frac{(t_1 - t)^{2n-1}}{(2 n - 1) !}. \end{aligned}$$

In order to calculate the formula (D) of section 4 it is necessary to evaluate  $\psi_0$  and  $\left(\frac{d\psi}{dt}\right)_0$ , that is to say, to put  $t = 0$  in the previous series. Further it is the values at the surface of the sphere which have to be found. Finally, this expression must be derived with respect to  $t_1$ .

Let us then adopt polar coördinates and put

$$\begin{aligned} x &= \rho \sin \theta \cos \phi, & y &= \rho \sin \theta \sin \phi, & z &= \rho \cos \theta, \\ x_0 &= l \sin \theta_0 \cos \phi_0, & y_0 &= l \sin \theta_0 \sin \phi_0, & z_0 &= l \cos \theta_0. \end{aligned}$$

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Then  $\cos \gamma = \cos \phi \cos \phi_0 + \sin \phi \sin \phi_0 \cos (\theta - \theta_0)$ .

Let us write

$$\Theta(l, \theta_0, \phi_0, \theta, \phi, t) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\lambda^{n-1}}{R^{n-1}} \frac{\partial^{n-1}}{\partial (\log l)^{n-1}} \left[ \frac{R^2 - l^2}{R^2 + l^2 - 2 R l \cos \gamma} \right] \frac{l^{2n-1}}{(2n-1)!}$$

Formula (D) can be written

$$(D_a) \Phi(l, \theta_0, \phi_0, t) = \frac{R}{\gamma \pi} \int_{\omega} \phi'_0(\theta, \phi) \Theta(l, \theta_0, \phi_0, \theta, \phi, t) \sin \theta d\theta d\phi \\ + \frac{R}{\gamma \pi} \frac{d}{dt} \int_{\omega} \phi_0(\theta, \phi) \Theta(l, \theta_0, \phi_0, \theta, \phi, t) \sin \theta d\theta d\phi,$$

where for simplification we have written

$$\Phi_0(\theta, \phi) = \phi_0(R, \theta, \phi, t), t = 0$$

$$\Phi'_0(\theta, \phi) = \left\{ \frac{d}{dt} \phi_0(R, \theta, \phi, t) \right\}, t = 0.$$

The formula we have been seeking to find is the general formula in the case of the sphere.

If, instead of a sphere, the liquid occupies a hemisphere and the diametral plane constitutes the rigid boundary so that the curved surface is free, the method of images will provide the solution in a similar manner. The same holds in the case where the liquid occupies a section of a sphere between two rigid diametral planes the angle between which equals  $\frac{\pi}{n}$ , where  $n$  is an integer.

VITO VOLTERRA.

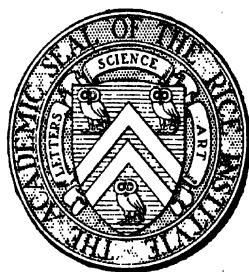


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## FOREWORD

THE following five public lectures were delivered during the winter of 1914-15 at The Rice Institute. The writer's immediate purpose was to stimulate further interest in the reading of Russian literature; his further aim, to share with his audience, in a very informal manner, some of the ethical ideas dominating Russian fiction. This small collection aspires neither to bibliographic nor to biographic completeness; it does not begin at the beginning, nor is it finished; it does not deal with the Great War, nor with Russia's recent wonderful transfiguration and more wonderful prospect. What has been attempted here is not a critical analysis either of Russia's novel or of Russian life, but rather a broad survey of the Russian novel as itself a criticism of life. To that end the writer has limited himself to Russia's four greatest masters, and has used their own words wherever possible, hoping in this way to deepen the reader's interest in their works as well as in the problems which they raise.

Among the English translations which have been utilized by the writer, and which he desires to recommend to those interested in further reading, are the following, easily accessible: Constance Garnett's versions of Turgenev, Dostoyevsky, and Tolstoy; Isabel F. Hapgood's translations of Turgenev and Tolstoy; Louise and Aylmer Maude's and N. H. Dole's translations of Tolstoy. Several of the novels have been published in Everyman's Library, and Gogol's "Dead Souls" by the F. A. Stokes Company. The reader will also find Aylmer Maude's "Life of Tolstoy" and Count Ilya Tolstoy's "Reminiscences of Tolstoy" of great interest and value, and, for a general discussion of the whole subject, Prince Kropotkin's "Russian Literature."



# THE PROBLEM OF LIFE IN THE RUSSIAN NOVEL

## LECTURE I

### THE RUSSIAN SOIL AND NIKOLAI GOGOL

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RUSSIA'S greatest glory is her literature. One may wonder how a people so ignorant as the Russian people are proverbially supposed to be could produce such a literature. Yet the mass of Englishmen are certainly better educated to-day than they were during the age which produced Shakespeare; nor was Shakespeare himself an erudite scholar. Russian literature is great, not because Russia is learned, but because her literature is the tragic utterance of one hundred million people aching to be born into real humanity.

To the Russian, literature is a serious business. The Czar's bureaucracy blocked all the avenues which the social consciousness of man usually follows in actualizing its ideals. Economic and religious reform, political and educational reorganization, public assembly, free speech,—all these have been closed to the Russian. A society in which the individual is denied these obvious ways of self-expression will be a society in which the mass will be mute, stolid, bovine,—even as the Russian *mujik*.

But what of the tortured spirit, the genius, the prophet,—what of him who must unburden his soul or perish? In Russia until yesterday he could not make a speech in parliament, because there was no parliament; he could not address a mass-meeting, since mass-meetings were not allowed; it

were useless to write a newspaper article or a fiery pamphlet, as these would never leave the printer. In any of these cases the futile attempt itself might send him to prison or to Siberia. One avenue alone has been open. The Russian soul has uttered itself in song and story. And thus in an odd sense it may be said that the bureaucracy, which has ground under its heel the Russian millions and has kept them brutalized, has, in leaving only the literary road open, virtually compelled Russian genius to follow that road, and has thus made Russian literature great. This peculiar nature of Russian literature explains in a measure its distinguishing features: its profound seriousness, its penetration and realism, and above all, its social character. The drama and the novel are not the recreations of Russia's esthetic leisure hours, nor poetry an idle pastime: in them we can see Russia's spirit groping toward the light of freedom and culture. The Russian writer, great or near-great, feels himself the spokesman of one hundred millions, and this social sense, dominating his work, lends it gravity, earnestness, and dignity, makes mere wit, clever pretense, and shallow optimism inadmissible, and compels sincerity. This social sense may lead to crude realism, but to sham and frivolity, never. The literature of no other modern nation can be characterized so truly as a criticism of life. And life is vast and various in the Russian land, a land of contrasts, of bright lights and black shadows, of beauty and of squalor, of glory as great as her shame, a land in which can be heard, not only the middle octave of mediocrity, but also the highest notes of human attainment and the lowest depths of human despair.

Hence the significance of Russian literature to the student of Russia, for the history of Russian letters is the history of the Russian people. The characters which move through the pages of Russian novels are living sons and daughters of

million-voiced Russia. The message of Russia's literary masterpieces is a message of gloom, of Slavic disenchantment and of what the Pole Sienkiewicz called Slavic unproductivity,—the message of the pitifully inadequate Russia that *is*. But the compelling truth and the matchless artistry of that message give us a vision of the glorious Russia yet unborn. And the absence of frivolity, the sincere, genuine realism of Russian literature, while making it truly national, at the same time save it from provinciality. The heroes of Turgenev and Tolstoy are true Russians, of course, but their struggles are human struggles and their appeal is indeed universal. Unless one digs deep enough, one cannot reach genuine Russian nature; and when one has dug deep enough to reach genuine Russian nature, behold! he is face to face with human nature.

In speaking thus of Russian literature, it is not of course intended that all Russian writers are sincere, serious realists. Russia is plentifully supplied with all sorts and conditions of writers and typewriters. Besides, Russia takes up the literary fads of France and Germany as readily as America apes Parisian millinery. But when we think of genuine Russian literature, we think of the real masters, of those whom Russians themselves regard as their prophets, and who alone have a claim to our attention,—those few men whose life work is a consistent attempt to portray truly Russia's men and women, to analyze the ethical problems of Russian life, to face honestly the question of Russia's national destiny. To four of these masters—the four greatest—I would invite your attention: Gogol, Turgenev, Dostoyevsky, Tolstoy.

Nikolai Gogol made realism a tradition in Russian literature. He enjoys the double distinction of being not only the first great analyst of the Russian soul, but also Russia's greatest humorist. It is something to have written the first



master-comedy in a language, which, after a century, is still the master-comedy, and also with a humorous novel to have inaugurated a movement in literature which must of necessity lead to heroic pessimism so long as Russia is what she is. Gogol did both, and the greatness of the man is but poorly appreciated outside his native land.

Nature endowed him well for his great work. He came from what is known as Ukraina, or Little Russia. It is now a province in Southern Russia, but once it was a free land of blossoms and battles, when the ancestors of the saddle-born Cossacks, who to-day form the stormy heart of the Czar's army, were roving knights of adventure, unvanquished spirits roaming the vast stretches of the Russian steppes. Gogol was born in the very heart of it. A son of the soil, himself of Cossack descent, lulled to sleep from infancy with the luring tales of the long-vanished days, he longed for his native steppe from the clerk's desk which he occupied in St. Petersburg. In poetic homesickness he wrote and at the age of twenty startled Russia with his beautiful sketches of Little Russian and Cossack life. Is one to believe that geniuses come in showers? The year of his birth, 1809, gave to the world Tennyson, Poe, Darwin, and Abraham Lincoln. To Russia it surely gave a master, free not only from the mock classicism which Pushkin and Lermontov had cast away, but free also from the morbid, self-engrossed manner which Byron had made a fad in Europe and, for a time at least, a literary fashion in Russia.

Gogol had been collecting materials for a nine-volume history of the Middle Ages; he actually wrote and published a prose epic of the Dnieper Cossacks, a comedy of Russia's corrupt officialdom, and a novel in which Russian life is revealed as the spirit of the Middle Ages is revealed in Dante and knight-errantry in "Don Quixote."

Gogol's humor is Homeric; there is indubitable Homeric influence in his work, and there is no Russian book which possesses such a Homeric sweep of elemental nature as "Taras Bulba," his story of the Dnieper Cossacks. A prose epic is "Taras Bulba," an epic of Cossack heroes, brave rather than beautiful, more hardy than cultured, intensely patriotic and martially Christian, but neither broad-visioned nor wise; an epic of braggarts who make good their boasts, of half-brutal, half-divine horsemen of the Russian plains, resistless, irrepressible, outwitting Tartar and Pole in battle, yet veritable children in the primitive naïveté of their minds. In old Taras Bulba Gogol has pictured a masterly Cossack, a truly gigantic figure, one half Ulysses, one half Sir Toby.

These restless riders of the endless steppes were a constant check to Mongol invasion. They saved Western Europe from the Tartar, and helped to save Northern Europe from the Turk. This was no regular standing army of soldiers who knew only how to fight, and knew naught else. "There was no craft the Cossack did not know; he could make wine, build a cart, grind powder, do a farrier's or a gunsmith's work, and, last but not least, riot and drink and feast as only a Russian can; it all came natural to him."

But these were, after all, avocations, and the least danger of invasion was enough to let the Cossack show his real business in life. "A captain had but to enter the squares and market-places of a Cossack settlement or village, stand up on a cart, and cry out: 'Hark ye, ye beer-swillers and brewers! Enough of this ale-brewing, yea, and wallowing on stoves, yea, feeding the flies with your greasy bodies! Come to, and win knightly fame and honor! And ye plowmen and mowers, sheep-tenders and women-lovers! Enough of following the plow, yea, shoving your greasy boots in the earth, yea, dangling after women and wasting knightly

strength! . . . ' And these words were ever as sparks falling on dry wood. The plowman broke his plow, the brewers and beer-venders left their vats and smashed their barrels, the craftsman and trader sent their craft and shop to the devil, broke all the pots in the house, and sprang into the saddle. . . . When the campaign was over the warrior returned to his meadows and pastures by the Dnieper river fords,—went a-fishing, bought and sold, brewed beer, and was, in short, an independent Cossack."

The story opens with the arrival of Bulba's two sons from the Academy of Kiev, where their father has compelled them to stay and finish their classical education, in order that, having acquired learning, they can afford to despise it as Cossacks should. Instead of welcoming his sons with a kindly word, Bulba compels the elder to quarrel with him, and after a warm fist fight has assured him that his first-born is no milksop he embraces and kisses him. "Let no man mock at you, son! Drub everybody as you have drubbed me!" And he orders the feast of welcome. "Put everything upon the table! We want no cakes, nor gingers, nor poppy-pasties nor other fangles! Drag us a whole sheep or a goat, yea, hundred-year-old honey, yea, plenty of brandy, not faked with raisins and rubbish, but clear, sparkling brandy that can pinch and sparkle like mad!"

The meal is barely half over when old Bulba announces that the very next morning he will take his sons to Sietch-beyond-the-Rapids, the great stamping-ground and corral of the Dnieper Cossacks. He is eager to exhibit his sons, and the thought of once more meeting his old fighting comrades maddens him with joy. A sudden contempt for his established home takes hold of him; the nomad in his soul is in the saddle. "We will away in the morning. Why tarry here? What enemy is there to fight hereabouts? What

do we want with this hut? What do we want with it all? What are all these pots for?" And he begins to knock down the mugs and flasks and fling them about the room.

During all this scene, one person is a mute spectator—Bulba's wife. Gogol has painted her in all the crude tragedy of her life, sitting on the bench and sadly gazing at the children with whom she is doomed soon to part. We can see her mutely tending her warrior-husband as he orders his sons to be ready to start the first thing in the morning; we watch with her all through the night, as she bends over her boys' heads sleeping under the clear southern sky, combing their tangled locks and moistening them with her tears.

A man's epic is "Taras Bulba," for the Cossack's life was a man's life. "Her lot was a hard one, as was every woman's in those distant times. She had been loved but for an instant, in the first heat of youth and passion; then her stern charmer had cast her aside for his sword, his comrades, and his carousals. She would see him for three days in two years, and then probably not hear of him for years again. Aye, and when she did, and they lived together, what a life was hers! She suffered insults and even blows while she dreamed of caresses bestowed in the fullness of love. Hers was a strange existence among the crowd of wifeless knights on whom dissolute Sietch had thrown its stern mantle. Her joyless youth faded before her; her fair fresh cheeks and bosom lost their bloom for lack of tenderness, and became covered with premature wrinkles. All love, all sentiment, all that is tender and passionate in woman was turned into maternal instinct. She hovered over her children like some solitary lapwing of the steppes, full of pain and passion and tears."

The picture of this unloved old mother flinging herself on the younger of her sons early the next morning, grasping

his stirrup, clinging to his saddle, fighting madly with two burly Cossacks until she is twice torn away from her offspring and the horses gallop away, is, alike in its simplicity and its intensity, a very gem of elemental passion. The retrospect of it makes the scene doubly poignant, for the Cossack mother never again sees her boys.

But they are a-horse and away across the boundless steppes. I wish I might quote all of the remarkable description which Gogol gives of this vast Cossack-breeding soil. A brief passage must suffice: "The sun had long since come out in the once dull sky, and was bathing the steppe in its cheerful light. The farther the steppe reached, the more beautiful it became. . . . Nature has nothing fairer than these steppes with their surface like a green-gold ocean strewn with a million flowers. Posies, lilac, blue, and green, shimmered in the tall, slim grass; yellow gorse and white clover danced upon the surface. An ear of corn, brought God knows whence, had taken root, and partridges pecked here and there among its thick stalks. The songs of a thousand birds filled the air, and hawks, suspended therein with outstretched wings, gazed on the earth below them. A flock of wild geese, wheeling cloudwards, sent their piercing cry from some distant lake. A lapwing rose with measured stroke from the grass and bathed delightfully in the air's blue waves, now lost in the heights until one black spot alone was visible, now turning on the wing and soaring sunwards. . . . The devil take you, steppes, how beautiful you are!"

Brilliantly full of life the steppe is in the daytime; alluring in twilit even; at midnight it is mystic, infinite. "Ever and anon the night sky was lighted by the distant glare of dry rushes burning on the meads and river banks, and dark flights of swans, hastening northward, shimmered with a

pinkish hue till it seemed as though red kerchiefs were flapping against the sky."

At last they arrive at their destination, the stamping-ground Sietch. Bulba looks himself over, twists his long moustachios, and assumes an air of martial dignity. A martial place is Sietch-beyond-the-Rapids: it has everything that the soldier wants, and nothing else; wealth or poverty, parentage—respectable or otherwise—count for naught here. The initiation ceremony is simple: "Good day! Do you believe in Christ?" the commander would ask of a newcomer. "Do you believe in the Holy Trinity? Do you go to church? You do? Well, then, cross yourself. Very good! Now join whatever regiment you like."

It takes small provocation to rouse this many-hued assembly of daredevils into a campaign. They start for Poland to avenge the insults heaped upon the Orthodox Christ by Catholic unbelievers and unchristian Jews. And if individually they are free-lances, collectively they are a well-organized army. They lead stern lives, and stern is their morality while the campaign lasts. Their picturesque spirits demand vivid manifestation of the austerity of law. A thief is tied ignobly to a post; a club is placed beside him, with which every passer-by must hit him as long as he remains alive. For a murderer a deep hole is dug and he is put in it alive, a coffin with his victim's corpse is lowered over the offender, and living and dead are then covered together.

It is a bloody story that follows, a Homeric recital of battle and iron courage and fierce revenge. Of Bulba's two sons, the younger falls victim to the charms of a Polish daughter and proves traitor to the Cossack cause. Old Taras kills him with his own hands. And when the battle goes against the Cossacks and his brave elder son is cap-

tured, taken away to far-distant Poland, imprisoned, and tortured to death; when his long-tested fortitude at last gives way and the young soul cries out in agony: "Father, where art thou? Do you not hear all this?" a cry rings through the Polish crowd: "Aye, I hear it!" For it is Bulba himself, daring the whole might of Poland in order to see his son once more before avenging his death.

And most terribly does he avenge it. True enough, he is burned at the stake in the end. But what boots it? Cossackdom lives and grows in might after him. Gogol writes: "The land of the Russ has an army founded upon religion, than which no foundation is more powerful. Hard and stern it is as the rock in the midst of a strong ocean. It rears its unbreakable walls from the sea's deep bed and gazes long and sternly at the waves which break over it. Woe betide the ship that strikes it, for her rigging shall float in broken pieces and her sides be ground to powder, whilst her drowning crew's despairing cry fills the air. . . . Think ye there is aught in the world can frighten a Cossack? What force or flame can overcome Russia's strength?"

Taras Bulba is a barbaric story. There is no point in trying to disguise what indeed Gogol himself freely brings out: the blood-thirstiness, the coarse, crude spirituality, the wifeless, roughshod manner, the bigotry and superstition, the brutal religiosity and racial savagery of these Cossack knights. Taras is a hero in the sense in which a magnificent bulldog or a bellicose ram is a hero when fighting with beasts of low degree. Estimate him as generously as you will, he remains a diamond in the rough—very rough—and, judged by any standard save the martial, doubt might arise as to whether he is a diamond at all. Gogol has portrayed a semi-savage life in which the very austerities of existence produce virtues as fierce as vices. Taras Bulba and his comrades

are demigods in battle, but we seek in vain for any expression of Christlike sweetness and charity in Cossack Sietch. These fierce riders of the steppes style themselves defenders of the Christian faith, but their true ideal of sublimity approximates Attila the Scourge of God more nearly than it does Jesus the Saviour of Man.

Gogol had painted with giant strokes, with a titanic brush, and the Homeric canvas caught the imagination of Russia. But the artist in Gogol could not have overlooked a fatal circumstance. A Taras Bulba could be the hero of a barbaric, martial Russia; where was the heroism of modern, westernized, peaceful Russia to be sought? On religious zeal and savage valor a martial autocracy had surely rested supreme in Russia, and still does rest. But what were to be Russia's bulwarks in her climb upward to genuine modern culture? Gogol was fast turning a realist. "Taras Bulba" had been an occasionally realistic treatment of an essentially romantic theme. But life in the Russian capital, close contact with the somber Russian actualities led Gogol to realistic themes. From the epic songs of the past he turned to the analysis of a prosaic present, which in Russia had retained the ancient brutality after it had lost the ancient glamour, and was pettily, unheroically grasping and cruel. Gradually we see tears mixing with the laughter of Gogol, bitterness and pathos behind the humor, until in the comedy "Revizor" an unspeakably farcical situation serves only to point out a state of social and political corruption, the tragic realization of which sobered laughing Russia and made her shudder guiltily.

The theme of "Revizor," or, as it is usually translated into English, "The Inspector-General," is perfectly simple. The situation is announced in the first twenty words of the play. News has reached the governor of a provincial town



that an inspector-general, sent secretly from St. Petersburg to investigate local administration and report to the central government, is about to arrive incognito in their midst. A friend of the governor has apprised him of the fact and warned him to put his administrative house hurriedly in order. And he needs the warning, as the governor himself informs his various colleagues.

The scene is decidedly different from the barbaric steppes of "Taras Bulba." We are introduced to "a Russian inland town, from which you may gallop for three years before you reach a foreign country." A pack of clumsy officials mismanage it most pitifully; every corner and crevice of it shrinks from the slightest inspection. The hospital is organized on the simple basis that if a man dies, he dies, and if he gets well—why, then, he gets well. The patients smoke strong tobacco, their nightcaps remind one of the mob in "Julius Cæsar," and they are as dirty as blacksmiths. The justice of the peace is a sportsman with a fondness for greyhound puppies, and his assistant reeks of spirits as if he has just come out of a distillery. The court attendants breed geese in the antechamber. The governor tells the postmaster kindly to open every letter and see that no complaint about the administration is sent out by some of the misgoverned citizens. The warning is superfluous; the postmaster informs him that as a matter of course he opens every letter that passes through the post-office!

Behold the setting! Only one thing is needed: an inspector. Two of the local male gossips rush into the room and announce that they have seen the inspector in the hotel. As a matter of fact, the supposed inspector is a penniless young spendthrift on his way home, who has lost all his money gambling and is on the verge of starvation, the landlord allowing him no further credit. The comical situations

which follow the mistaking of this penniless scapegrace for an inspector-general may well be imagined. The governor asks the young man to his house: in an excruciatingly ludicrous scene the young gambler thinks he is about to be arrested for evading his board bill, while the governor interprets his ill humor as a mark of official displeasure. One after another, the local officials take their turn at paying their respects, and from each the "inspector-general" borrows a few hundred rubles, the amount of the bribe varying with the trepidation of the guilty official. He makes silly love to the governor's wife, and in order to extricate himself from an embarrassing situation asks the governor for the hand of his daughter. Everything is working up to a climax, when finally the young rascal's servant, a lout possessing more real intelligence than the entire bureaucratic staff, persuades his master to tempt fate no longer and escape as quickly as he can. Just before leaving, however, he narrates all his experiences in a letter to a friend in St. Petersburg. In due course the postmaster opens this letter, and while it is being read to the consternated officials a policeman enters and announces that the real inspector-general, sent by imperial command, has arrived at the hotel and requests the governor's immediate attendance. Curtain.

"The Inspector-General" is, of course, Russia's greatest comedy; it would occupy a high place in any literature, so fully does it meet the requirements of dramatic art. Shamed Russia laughed, or else felt affronted and declared that there was not a single honest character in the play. Which is perfectly true, Gogol retorted, but there is honest ridicule throughout. Gogol did not intend that his audience should merely laugh. In later years he wrote: "In 'Revizor' I tried to collect in one heap all that was bad in Russia, as I then understood it; I wished to turn it into ridicule. The real

impression produced was that of fear. Through the laughter the spectator feels my bitterness and sorrow." Russia could not laugh gaily at this tragi-comic exhibition of her spiritual nakedness. At the end of the play the governor, chagrined at his ridiculous position, suddenly turns on the laughing audience and shouts words that Gogol was flinging at all Russia: "What are you laughing at? You are laughing at yourselves!"

All the corrupt officialdom of Russia conspired to keep the play off the boards. But it had caught Czar Nikolai's fancy. He laughed imperially at its performance. Imperial sanction had been accorded that penetrating analysis of Russian life, which later was to turn Czar and government against literary realism—in vain. For while the comic exhibition of provincial corruption could count on the approval of imperial St. Petersburg, the new note which Gogol struck in "Revizor" was to be repeated in other less comic, more relentless revelations of Russian life, which made Russia cease laughing, which brought it to a sudden realization of its unspeakable misery, which potentially and actually incited to revolt, for they called forth the dangerous question which is already the title of two Russian books: "What Is To Be Done?"

In "Revizor" Gogol established realism as a tradition of the Russian drama. He performed a similar service for Russia's novel by writing the first capital work of Russian fiction, "Dead Souls," the first part of which, written in 1838, was published in 1846.

"Dead Souls" is the Russian "Divine Comedy." Dante's travel-notes through the world beyond were, as a matter of fact, a mirror of medieval life—a mirror with a thousand faces. "Dead Souls," similarly, is a succession of prose cantos in which the thousand and one sides of Russian life

in town and country are subjected to the most searching scrutiny. Again we may compare it with "Don Quixote." Cervantes' masterpiece showed the pompous folly of cavalier Spain and made knight-errant heroics' ridiculous. Gogol's novel turned the pitiless light of unqualified realism on Russia; revealed Russia so veraciously that after "Dead Souls" had been published Russian novelists could continue to write only in the spirit of "Dead Souls."

A complete outline of the plot of Gogol's masterpiece is impossible to give here, just as it is impossible to summarize in a few words the entire substance of the "Divine Comedy"; and yet a paragraph is sufficient to state what "Dead Souls" is about. In Gogol's day the Russian peasantry were still serfs of the rich landowners. They could be bought and sold like any other property. The government collected taxes, not from them, but on them. If a "soul" (serf) died, the landowner had to pay taxes on him for several years, just as if he were alive, until the taking of the next census should alter the statistics.

Now this is the animus of the novel. A nobleman, Pavel Ivanovitch Tchitchikov, who is in bad financial straits, concocts a bold scheme for restoring his credit. The plan is as simple as it is audacious. He would go from landowner to landowner and either obtain free of charge or buy at a nominal price the deed to those of their serfs who had died since the last census, thus ridding them of so many taxable burdens. When he had acquired in this way the legal ownership of a large number of "souls" whose non-existence was not likely to be discovered until the next census, he would "transport" them to a tract of land somewhere on the southern plains, where the Russian government was offering homesteads to colonizers. He would thus have the legal ownership of a supposedly large estate. Using these actu-

ally dead, but legally alive, souls as security, he planned to borrow a large sum of money from the Council of Guardians' Bank, and thus restore his shattered credit. The plan of the story is thus seen at a glance, and Gogol's idea in his long account of Tchitchikov's journeys was obvious: in chronicling the travels and adventures of this trafficker in dead souls, Gogol undertook the creation of a gallery of Russian portraits. As he himself expressed it, "I wished to show, at least from one point of view, all Russia."

And what a portrayal it is! As Tchitchikov's inebriate coachman Selifan drives his rickety *britchka* from estate to estate, all the thousand-charactered immensity of Russia reveals itself before us in a living panorama. The hero himself, too greedy of immediate opulence to follow the tedious path of slow-coming prosperity and too unstable in his moral sense to relish the path of virtue, is a very chameleon of obsequious adaptability and opportunism, alternating with a certain haughty touch-me-not dignity. A nobleman of obscure origin, he has shaped his entire life in accordance with his father's precept: "Friends and comrades will cheat you, but money will never betray you, no matter in what straits you may be."

Having inherited from that profound ancestor, in addition to the aforesaid wisdom, four badly worn waistcoats, two ancient surtouts lined with lambskin, and an insignificant sum of money, Tchitchikov enters the department of justice, determined to win in the race of official preferment over his colleagues, the faces of some of whom are described by Gogol as "looking exactly like badly baked bread." By intrigue, subterfuge, contemptible stooping and hypocrisy, Tchitchikov twice gets within sight of venal affluence, once by the road of a public building commission, another time by custom-house bribery. On both occasions,

however, he is caught in his own meshes and, while he avoids imprisonment, he is plucked of all his ill-gotten finery and is compelled to begin anew. He begins always in a different manner, it is true, but it is ever the same motive which incites him. From the losses of his inglorious past Tchitchikov learns caution and cunning, but cupidity he seems unable to unlearn. Yet there is something attractive and, one is ashamed to add, something almost lovable about the man, something making one wish that, even by trickery if necessary, he could somehow succeed in amassing the wealth which he demands as a prerequisite for settling down and realizing his lifelong dream of becoming a respected paterfamilias and a virtuous, useful citizen in the last chapter.

Gogol's genius has achieved a masterpiece of character delineation in the portrayal of Tchitchikov. It is not a stock-villain or picaresque intriguer that we have before us. Moving in a social medium which in turn amuses, repels, and disgusts the spectator, Tchitchikov is at least respectable and externally refined: the author himself is obviously impressed by the cleanliness of his hero's linen and his liberal use of perfumed soap and eau de Cologne. Occasionally his eyes seem to catch a gleam of the pure light of moral decency and one almost dares to hope that the man in his soul will master the vulture; but a new prey comes in sight and the old hunt continues. As in the case of the besotted miser Pliushkin, there flashes "a ray of light which expresses, not feeling, but the pale reflection of a feeling: an apparition similar to the sudden appearance of a drowning man, which appearance elicits a joyous shout from the crowd assembled on the shore. But he is not seen again, and the calm surface of the unresponsive fluid seems still more terrible and more desolate than before."

We find Tchitchikov in prison, almost on bended knees in

his repentant decision to accept the proffered help of the millionaire benefactor Murasov and turn a new leaf; but the kind gentleman has barely left the room when the rascally offer of a shyster to get him out of jail by foul means rouses the prisoner's hopes to the pitch of enthusiasm, and all his tears of repentance are wiped dry.

The society in which Tchitchikov plies his traffic in dead souls does not retain our moral approbation which he forfeits: more frequently the contrast is one between vice and vice than between vice and virtue. In many respects he is morally the equal of his victims and judges, and in point of personal qualities he is undoubtedly their superior. Gogol's portraiture of this society is at once humorous and tragic: irresistibly mirth-provoking as his manner is on the surface, there lurks beneath the laughter an ocean of infinite sorrow for the nation whose nakedness of soul he ridicules. The tragedy wells up through the humor, it is ever more in evidence, and the novel ends in increasing spiritual gloom. For sheer comedy, of course, one doubts if Russian genius has ever produced the peer of "Dead Souls," especially the early part of the novel. In the portraits of Russia's landowners with whom Tchitchikov deals, the ridiculous and the contemptible in human nature are chastised in a manner which manifests the infinite variety of Gogol's art.

The brainless sentimentalist Manilov, of agreeable features, but "rather too much permeated with sugar," dreams his life away in maudlin projects of the most utopian philanthropy, but he never gets beyond the fourteenth page of the book he is about to finish, and in his house there is forever something lacking. His heart would doubtless be in the right place, were it only in the vicinity of some backbone and not altogether removed from intelligence. As it is, he plans to make his estate a paradise of comfort for his serfs; yet when

Tchitchikov offers to buy his dead souls, neither he nor his overseer knows who is alive and who is dead on the estate. Tchitchikov's affable manner so melts his soft heart that he makes him a present of the dead peasants, and bidding his guest farewell, seats himself in a chair and dreams all afternoon of how delightful it would be to dwell with his friend on the bank of some river.

The hero's progress is not invariably as easy, however. The next landowner, the widow Korobotchkina, is a grasping bargainer, in mortal fear of being cheated. She has never sold dead souls before, she argues; she wants to wait awhile; "perhaps some merchants may come, and I can find out about the prices; . . . perhaps the dead souls may be needed some day on the estate!" Sophisticated, polished greed wrestles with greed in the rough throughout a magnificent chapter, and it is hard to make a moral choice between the two. Then there is Sobakevitch, too much concerned with his own interests to be at all inquisitive about the motives of the buyer of dead souls. He asks one hundred rubles apiece for the dead souls which Tchitchikov had offered to buy at the rate of eighty kopeks a name, and proceeds to read our hero a sublime catalogue of laudation in which the virtues of each dead serf are set forth: "Some other scoundrel will deceive you and sell you rubbish, and not real dead souls; but mine are as sound as nuts, picked articles; there is no better artisan than the healthy *mujik*. Just consider the matter: here's Mikhyev, the carriage-builder! Why, no better equipages are made than those he used to build. . . . And Maksim Telyatnikov, the cobbler; whatever he pricked with his awl became a boot at once. . . ." "But they are dead!" Tchitchikov protests and is finally compelled to pay three rubles a soul.

Pliushkin, the almost dehumanized miser, on the other



hand, welcomes Tchitchikov as a deliverer from so many taxable burdens and gives away not only his dead souls, but also all those who have absconded. But perhaps the most disgusting full-length portrait in the book is that of the suspicious tippler and gambler Nozdryov, who not only refuses to sell his deceased serfs, but, angered with Tchitchikov over a game of draughts, exposes his scheme at the very point when he is the object of the town's admiration. Dame Gossip makes a round of calls accompanied by Mistress Rumor, and "The Result is Our Hero's Flight," as Gogol entitles Chapter X.

He departs on fresh enterprises, and new scenes, new landowners appear on the stage, but the landscape is only a different copy of the same spiritual wilderness. Tentyotnikov is another Manilov—not as loving, perhaps, but certainly as useless: we know that he will never reach the end of his universal history of all Russia, but that his serfs will certainly make an end to that portion of Russia which he mismanages. Pyetukh is a prodigal son in the first chapter of festive gluttony, and Khlobuyov, whose mortgaged estate Tchitchikov buys, is a prodigal son in the last chapter, a wanton spendthrift in tatters. Nor is the bored Russian landowner absent, as witness the tedium-tortured Platonov. There is precisely one good man in the book, and only one possessing genuine efficiency. But the former is too old to inspire hope for the future, while the latter, Kostanzoghlo, is of foreign descent. And to match this portrayal of Russia's landowners, Russian officialdom is sketched in strokes, necessarily more cautious, but assuredly none the less telling. "It is impossible to convict you," the shyster jurisconsult tells Tchitchikov; "for before the trial is over, everybody in town will be implicated!"

Throughout the novel one seems to hear, as in an under-

tone, the groans of those millions of live souls whose dead brothers Tchitchikov is buying up from their masters. Yet "Dead Souls" is not a tract against serf-ownership or against anything else. In this utterly unconstrained chronicle, conceived in a sort of Rabelaisian humor, gigantic, embracing an entire race, yet minutely veracious to the least detail, a tragedy is revealed not in the telling but in the fact that such a story could be truthfully told of Holy Russia. Like the old cloak of the poor official in Gogol's story "Shinel," the Russian soul appears in "Dead Souls"—worn out, beyond repair; and, like the three-horse team, the bird-troika, Russia dashes onward, dashes resistlessly and yet without direction. The future of his native land overwhelmed the author: "Is it not thus, like the bold troika which cannot be overtaken, that thou art dashing along, O Russia, my country? . . . O Russia, whither art thou dashing? Reply! But she replies not! . . ." And so does Gogol despair of his country's pettiness, only to sink in awe before her inscrutable, immense inertia.

Russia read Gogol's "Dead Souls," and the story which the great novelist told was a story which every Russian could see and hear in his own midst: the characters, men and women, landed gentry and serf chattels, they were all about him, peopling every province. As the reader followed Tchitchikov on his rascally mission, the cumulative impression of his vast country brought laughter of kindly pity and laughter of contempt, brought smiles and sneers, but most of all it was spiritually depressing; it made men concerned, worried, anxious about the vast futility of their fatherland. And what Russian in all Russia could escape the question which Gogol was asking throughout the book: "Which of you, filled with Christian humility, will dive into the depths of his own soul, and not aloud, but in silence and solitude, in mo-

ments of isolated self-communion, will put to himself the weighty question: And is there not some taint of Tchitchikov in me also?"

Like "Don Quixote," "Dead Souls" is a satire on human vice, human pettiness and vanity; and, like "Don Quixote," it lacks artistic unity. Indeed, one should not say that the book lacks artistic unity; it does not attempt unity of any sort. Perhaps it is not a novel, strictly speaking; the telling of the story may have a beginning, but the story itself is without any artificial boundaries. Gogol never finished it, but what he did finish is not on that account radically affected. "Dead Souls" is a packet of leaves torn at random from the book of life; a collection of unforgettable etchings of human character, overwhelmingly true to nature.

Gogol possesses a Shakespearean, uncanny power of sketching a character full and complete with half a dozen strokes of the pen. One must read his works to appreciate this genius of his to the full. I shall quote only two examples, one of them from "Dead Souls." Tchitchikov's valet Petrushka "always carried about him a special atmosphere of his own, a peculiar smell which corresponded to some extent with that of a dwelling-room; so that it sufficed for him merely to install himself somewhere, to take off his cloak and belongings there, for people to think that the apartment had been inhabited for fully ten years." To match this passport of externals, behold a spiritual passport: "Ivan Ivanovitch was a man of the most accurate and systematic habits. When he had eaten a melon, he would wrap the seeds in a bit of paper and write on it: 'This melon was eaten on such and such a date'; and if there had been a friend at table he would add: 'in the presence of Mr. So-and-So.'" And in one moment you find yourselves completely acquainted with both gentlemen.

We cannot follow Gogol to the last days of his life: the mental depression which claimed him, as it has claimed so many Russian masters since his time; the fit of despondency in which he burned the manuscript of the second part of "Dead Souls"; the religious mysticism in which his last days flickered away. Of Russia's master-novelists, he was the first and the model of his successors. One sometimes wonders how realistic art could possibly surpass itself by surpassing Gogol; and yet, after Gogol, a writer came to Russia who combined Gogol's mastery of life-portrayal with a clearer philosophy of life and with a certain genius of artistic conception which made his novels not only true pictures of human character, but also artistic unities. These excellences were combined as they never had been combined before, in Russian or in any other fiction, in the art of Ivan Turgenev, the prince of novelists.

## LECTURE II

### THE ART OF IVAN TURGENEV

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INNOCENT Mark Twain learned that everything in Rome was the work of Michelangelo. The student of modern Russia finds that all things hark back to Peter the Great. The last two centuries of Russian history are but the continuation of Peter's work by his successors. Peter found Russia an Asiatic, Oriental power; the aim of his whole life was to make Russia European, to westernize Muscovy. To that end he spent years working as a common laborer in Europe, trying to learn Western methods; to that end he moved his capital from ancient Moscow to the modern city on the Baltic which he himself built as a window through which Russia could see and imitate Europe.

Peter was determined to cure Russia of her orientalism, and the remedies he administered were heroic. He began with the Russian exterior; his first step in making Russia European was to make the Russians look European. By imperial decree, he ordered the Russian *boyars* to alter their oriental flowing robes and to cut their beards—these two visible signs of their Asiatic kinship. On April 26, 1698, Peter the Great himself played European barber to the Russian nobility, and on that date, memorable in modern history, Peter's assistant in the work of modernization was his jester, Yakov Turgenev.

Exactly one hundred years after the death of Peter the Great, during the famous conspiracy of December 14, 1825, among those who were exiled to Siberia was one Nikolai

Turgenev, a critic of Russian ideals and an ardent champion of the abolition of serfdom.

These two members of the Turgenev family always come to my mind whenever I think of the greatest of all Turgenevs, the prince of novelists, as he has justly been called, Ivan Turgenev. His ancestor, the great Peter's jester, had done his share to ridicule Russia's orientalism and had held the mirror before the Russian *boyars* while Peter barbered their faces into some European shape. Ivan Turgenev spent a lifetime holding before the Russian people a mirror in which they could see all their shallowness and superficiality, all their ineffectual dreamings and fumings, all their aimless dilettantism, their veneered ignorance, their moral instability, their lack of consistent resolution, their prodigality of futile emotion, and their miserable poverty of will; and, through it all, their oriental inefficiency and their Tartar-like vulgarity and savagery, which made the unwashed lout peep out suddenly through the face of the Russian prince or princess "like a smell of cabbage wafted across the most delicate perfume."

And less than forty years after Ivan Turgenev's cousin, Nikolai, had been exiled to Siberia for holding liberal ideas and advocating the emancipation of the serfs, the novelist's portrayal of Russian peasant life (published under the title of "Memoirs of a Sportsman" in order to get by the censor) made all Russia thrill at the tragic spectacle, and moved Alexander II to become the Czar Emancipator and accord freedom to thirty million souls.

By a curious coincidence, at about the same time when Turgenev's "Memoirs of a Sportsman" caused the bloodless abolition of serfdom in Russia, "Uncle Tom's Cabin" was stirring passions and purposes in America which could end only in a bloody drama. But the coincidence is largely

chronological, for whereas Harriet Beecher Stowe's novel was melodramatic and an undisguised tract, Turgenev's stories were veritable gems of art, marvelous transcriptions of life itself, dispassionately objective, and therefore doubly poignant. Beecher Stowe is an advocate, a pleader; Turgenev is first and last an artist. In the novel "Smoke" it is related—by a confirmed gossip, to be sure—how, when Mrs. Beecher Stowe was in Paris, a Russian serf-owner ventured to seek an introduction to her. "What!" Mrs. Beecher Stowe cried. "He presumes to be introduced to the author of 'Uncle Tom'?" And she gave him a slap on the cheek. "Go away at once!" Now the worst gossip on this long-suffering earth could not have connected such a tale with the author of "Memoirs of a Sportsman."

Ivan Sergeyevitch Turgenev was born on October 28, 1818, at Orel, half-way between Moscow and Kiev. His father, Sergei Turgenev, an officer in a local regiment of cuirassiers, in marrying Varvara Petrovna Litvinova, did not make a bad match, financially speaking, but the marriage was hopeless otherwise. Both natures were high-strung, and the moral instability of the husband was exceeded perhaps only by the wife's rancor and spitefulness. He died when the future novelist was only seventeen years of age, but the mother lived to be seventy. Her son's liberal tendencies and his choice of a literary career wounded her arrogantly aristocratic spirit. She refused her son's request to see her before she died; one of the last deeds of her life was calculated to wreck him financially. Such was Turgenev's mother. The villainous wife of Lavretzky in "A House of Gentlefolk" bears the name of Varvara. Thus much of sweetness and light did the mother's name possess for the son.

Nevertheless he had excellent training. In his youth he

was tutored in foreign languages; but his Russian he learned from a serf on the family estate,—“a philanthropic and philosophic plebeian,”—whose enthusiastic radicalism doubtless influenced the spiritual attitude of the youth. Turgenev was ever a passionate hunter, and his close communion with nature is indicated by the intimate part which nature plays in his novels: clouds and rivers, storm and sunshine, and the forest in spring and in autumn enter into the life of his characters and play the part which they play in folk-songs and in poetry of the highest type.

Notice the nature-environment in which his action moves; it reflects in the finest, most delicate manner the inner, spiritual life of his men and women. Especially is this true of the great scenes in his novels. Take, for instance, the rain-storm in “On the Eve,” which brings Elena and Insarov together in the old abandoned oratory and draws from them both a brave confession of an exalted love. Compare it with the thunderstorm in “First Love,” where an untutored soul first awakes to the strange empire of love and feels himself baffled, intoxicated, and consumed by it. In this thunderstorm, which draws the unsophisticated Voldemar to the Princess Zinaida, the flashes of lightning were “quivering and twitching, like the wing of a dying bird.” Compare it again with the description of that other sudden, sinister tempest in the novel “Spring Freshets,” whose dull vibration echoes and peals in the deep fastness of the forest, whose lurid glare blinds Sanin to all that is fine and holy in life, draws him into the watchman’s wretched hut, and makes him passion-heated wax in the hands of the diabolically voluptuous temptress Marya,—Marya, who is lust incarnate and triumphant, possessing *le terrible don de la familiarité*, who makes wagers with her husband that she can seduce any man he may choose, and gives iron rings to all her victims. Tur-



genev is a poet, and just as his every character is a living being, so nature provides the overtones that swell the fundamental spiritual note of the action depicted and make his descriptions, not mere stage directions, as it were, but inevitable and indispensable elements of a unitary poetic effect.

In appreciating Turgenev's attitude alike toward his country and toward his own literary mission, it should not be forgotten that, in addition to being the greatest artist in the history of fiction, he was also, of all great novelists, the one blessed with the broadest and most genuine education and culture. Himself the guide of Russian thought and the unquestioned master of Russian speech, he made his home in Paris or Baden-Baden; the acknowledged lion of the literary lions of France and an Oxford Doctor of Civil Law, he was so perfectly at home with the ideals and the thought-currents of France, Germany, England, and even America that the shallowness and the thinly veneered vulgarity of his dilettante Russia outraged not only his honest patriotism and his faith in the Russian people, but outraged also his fine esthetic sensibility and his truly educated and cultured mind.

Turgenev found things Russian too disheartening to endure viewing them at close range day by day. His liberalism also, which had courageously expressed itself at the death of Gogol, whom he called "a great man," an offence for which Holy Russia caused his arrest and virtual banishment, made his life in his native land a source of constant danger and irritation. All these causes combined to make him during his later life a willing exile in western Europe. But his dealings with the everlasting Russian traveler and his own periodic visits to his native land allowed so keen an observer abundant opportunity to study Russian life, Rus-

sian humanity, the development of the Russian spirit. Like Gogol himself, Turgenev also found his vision of things Russian clarified by the distance which allowed him perspective and more genuine objectivity. And yet we have only to read his *Reminiscences* and *Letters* to learn how passionately he loved and longed for his native land, which he nevertheless knew too well to admire unreservedly. Yet he would love to live in Russia if he only could: "Russia is now passing through sad and gloomy times," he writes to Tolstoy; "but it is for this very reason that at this moment one feels the gnawing of conscience at living like a foreigner." We are reminded of the words of Lezhnyov in "Rudin": "Russia can get along without any one of us, but no one of us can get along without Russia."

Turgenev's novels are the record of Russia's spiritual growth during his life. Consider that he saw the abolition of serfdom, the rise of Slavophilism and Pan Slavism, and the beginnings of consistent, organized revolution in his native land, and you will see that million-voiced Russia provided him no end of material. In portraying this life, Turgenev showed a sort of novelistic clairvoyance; he seized concretely the dominant, essentially characteristic notes of Russian life,—he portrayed live human beings in which every Russian recognized something, perhaps all, of himself. And, as in the case of his character Bazarov, and indeed of the whole theme of "Fathers and Children," he was able sometimes to foresee the coming to life of a certain type of human character, a certain movement in Russia, to understand the conditions responsible for it, keenly to analyze its essentials, to anticipate its probable course and destiny, and even to baptize it before it was born,—as he certainly did baptize nihilism before there were any nihilists in Russia. He showed such prophetic insight in his portrayal of

the Russian revolutionist in his novel "Virgin Soil" that some of his enemies accused him of having been in touch with the revolutionists and others saw in him an agent of the secret police, while in reality Turgenev had been throughout only an observer of Russia who saw clearly what most Russians could perceive dimly, if at all. Even so does every great lyric poet publish our own unspoken thoughts and vaguely felt moods until, hearing in his lyrics the voice of our own heart, we feel as though some one has been prying into our own Holy of Holies.

And what is this Russian people which, depicted in terms of universally human portraits, unfolds itself before us in Turgenev's volumes? It is Russia million-voiced and Russia mutely enduring; and Turgenev's portraits, whether life-sized studies or the merest sketch-outlines, are true to nature not only in a physical sense: they possess spiritual verisimilitude. The human appeal which characterizes them all is genuine, born of the life with which the master has conceived them, and in no way the result of conscious or unconscious melodrama. If it can be said at all that Turgenev preaches, it is as life itself is a preachment which those who incline their souls may hear.

The message conveyed in "Memoirs of a Sportsman" is the more poignant and the impression doubly profound precisely because it is conveyed by means of pictures almost severe in their objectivity. From hamlet to hamlet, through forest and steppe and over country road, amid marshland and meadowland, in peasant-hut and feast-hall and dram-shop and counting-house, we follow the huntsman analyst, and dozens of living men and women meet our eyes, utter the burden of their souls, and pass on. Others come, and others, and one forgets to admire the artistry of the writer, so gripping is the living reality of his art.

The "Memoirs" abound in life-comprehending epigrams and master-strokes of description. "Russian maidens love eloquence." "The elder replied . . . languidly and awkwardly, as though he were buttoning his kaftan with half-frozen fingers." "Funtikov will serve us with fish worth a hundred rubles and prepared with tainted butter." The servility of the poor, shiftless Kalinitch earns him barely enough to keep body and soul together, while the peasant Khor moves to economic independence by the path of proud humility which makes him indifferent to formal freedom from serfdom. "Why should I buy my freedom?" he says. "As it is, I know my master." He is a very Socrates in his ironic self-depreciation, and yet perhaps of all Turgenev's peasants he is the most efficient and level-headed. And efficiency is indeed a mark of distinction in Turgenev's sportsman-land, because it is so rare.

A peculiar melancholy, a romantic melancholy at times, becomes ever more profound as we proceed from hut to hut, and the atmosphere of superstitious awe in the presence of a world of inscrutable, immense, malignant destiny is likewise an atmosphere of romance: the world is no mere machine for the animistic minds of the five lads in "Byezhin Meadow." The peasant's soul gropes in trepidation and spiritual squalor in a world whose natural beauties only heighten by contrast the desolate state of man. The hopeless misery of the poor is the more pathetic because it seems to leave the ordinary landed proprietor utterly unmoved and unaware of it. In these serf-lives, which to the reader are so intensely human, the masters of Russia's peasantry see no humanity whatever. Zvyerkov's wife is kindness itself and makes the life of her maids "a paradise visibly realized," but she simply refuses them permission to marry. When the servant Arina begs to be released from service,

so that she may follow the call of her heart, her prayer is to her mistress an evidence of rank ingratitude, and when, thwarted in the normal expression of her love, the poor maid becomes its prey, she is banished into misery.

A counting-house clerk is asked whether merchants give their servants larger wages than do landed proprietors. "God forbid!" he answers. "Why, a merchant would pitch you out of doors by the scruff of the neck if you were to ask wages from him. No; you must live in faith and in fear with a merchant." Life is one endless round of gloomy prospects,—“it is not cheerful to enter a peasant's hut by night.” The indifference toward the serf's woes is loftier in the master, more brutal in the counting-house clerk and overseer, but in either case it appears implacable, and it is implacable not because of premeditated malice and cruelty, but because of a certain lack of sympathy, perhaps lack of imagination: the peasant is born to endure, it is his lot; if he protests, his very rebelliousness proves him unnatural, worthless. A master owes it to his serf to punish him. "Whom he loveth, he chasteneth, you know that yourself," Mardary Apollonitch quotes in self-justification when censured for his cruelty. And the whipped serf agrees with his master. "I deserved it," he says sententiously. "We are not whipped for trifles, that's not the custom with us,—naw, naw!"

The same stolid submission to implacable authority and pitiless injustice is portrayed in two other, longer stories, "Mumu" and "The Inn." The laundress Tatyana is called by the majordomo: "The mistress has hunted a bridegroom for thee." "I obey, Gavril Andreitch. But who has been appointed as my bridegroom?" "Kapiton the shoemaker." "I obey, sir." "He is a reckless man—that's a fact. But the mistress pins her hopes on thee in that respect." "I obey,

·sir." In "The Inn," a house-serf, Akim, has by long endeavor managed to establish himself as an innkeeper. But his social standing does not allow him to hold legal title to his property. Naum, a cunning young laborer-merchant, insinuates himself into the graces of Akim's wife, induces her to give him all her husband's savings, and with that money buys Akim's inn from the landed proprietress and turns both innkeeper and wife out of their own house. Incendiary anger flames up for an hour in Akim's heart, but he submits to his fate, pardons every one, sets out on a pilgrimage to the holy places of Russia, and to his mistress, who has wronged him beyond all words, he keeps sending blessed bread from sundry sanctuaries.

At times, to be sure, the serf's rebelliousness rises to a pitch of genuine terror, as in "The Wolf," where the very despair and helplessness of a peasant, forced by his misery to steal timber and caught in the act, makes him awesome and hypnotizes the wolfish forester into releasing him, albeit the latter remains disgusted with his one manifestation of human pity. Nor must we forget Ivan Suhikh in "Old Portraits." By a mere subterfuge he is taken away from a master whom he loves to one whom he loathes utterly. After protests and threats, he apparently submits to his fate. But one fine day he splits his master's head open with an axe. "I killed him," he tells the police. "I told him I would do it, and I did it. Bind me."

But Turgenev's theme in "Memoirs of a Sportsman" is not merely the oppression of man by man. Already the reader has been warned against mistaking this work for a tract against serf-ownership. Turgenev portrays Russian life as he finds it, and while man-made misery is perhaps most clearly apparent in it, it is neither the most tragic nor the most profound. There is the tragedy of life for which

no one in particular is culpable, a tragedy of circumstance, a nature-born, fate-begotten tragedy, to witness which is an experience the more agonizing because it does not allow one the relief of growing indignant at the villain-oppressor. And more impressive too than the vast misery of Russia is the Russian's capacity of endurance. The poet Tiutchev has rightly put it in the couplet which heads Turgenev's sketch "Living Holy Relics":

"O native land of patient fortitude,  
Land of the Russian folk art thou!"

"Wonderful is the way in which the Russians die," Turgenev says. For sheer pathos and for the beauty of pathos, this sketch is easily the masterpiece in the collection. The love of plain earthly happiness and the love of the heavenly Christ blend most touchingly in the patient waiting of paralyzed Luker'ya for the angel of death. A dancing beauty in her youth, she is now resigned to her fate; she wishes no cure; she finds a strange happiness in the very absence of any hope, in her very resignation. "Don't touch me, Master; don't take me to the hospital. . . . Who can help another? Who can enter into this soul? Sometimes I lie here alone like this, and it seems as though there were not another person in all the world except myself. I alone am living. And I feel as though something were blessing me. Thoughts come to me—even wonderful thoughts." Alone in her hopeless illness, her mind does not beg for sympathy, it lavishes sympathy on others. From her miserable death-bed she begs the landed proprietor to have the quit-rents of the serfs reduced,—“for they are very poor. But I need nothing. I am content with everything.” The power of pain and suffering to regenerate a soul and bring it closer to God and Christ,—this typically Russian text upon which

almost every novel of Dostoyevsky is a sermon, is uttered here with unforgettable, concentrated intensity and pathos.

Perhaps the most significant way of entering into Turgenev's philosophy of life is by following its development through his six longer novels, which he himself advised us to read in the order in which he wrote them: "Dmitri Rudin"; "A Nobleman's Nest"; "On the Eve"; "Fathers and Children"; "Smoke"; "Virgin Soil"; the progressive revelation of nineteenth-century Russia, from the decay of the old nobility to the rebellious rise of the new democracy.

Dmitri Rudin, the hero of Turgenev's first great novel, has been called the typical embodiment of Russian character. Rudin is a man of lofty ideals, or rather he is loftily eloquent about ideals. Freedom, nobility of soul, moral courage, self-sacrifice are his daily topics. A most magnetic personality is Dmitri Rudin,—“that man not only knows how to move you, he lifts you up, he does not let you stand still, he stirs you to the depths and sets you on fire.” He is truly generous; one doubts if in his nature there is an iota of mean selfishness; a sincere and an ardent idealist, he himself lives in the world of his glorious ideas of human emancipation. But exactly of what stuff is this champion of independence really made? His eloquent advocacy of human rights and freedom and his contagious enthusiasm captivate Natalia, the daughter of Rudin's hostess. Rudin himself loves her devotedly. But his radicalism makes him lose favor in the eyes of Natalia's family. The time for words has passed; arrived is the time for action. What are the lovers to do? Natalia looks to her brave young model of independence to utter the decisive words.

“We must submit,” Rudin says.

Whereupon the young woman bursts out in words of spiritual disenchantment: “You spoke so often of self-sacri-



fice," she tells Rudin; "but do you know that, had you said to me now, this hour: 'I love you, but I cannot marry. I do not answer for the future; give me ~~your~~ hand and come with me!'—do you know that I would have come, that I would have confronted anything with you? But, alas, it is a long way from words to deeds!"

There is your Russian man as Turgenev saw him. His ideals are noble, but he is too inefficient spiritually, too weak-willed to translate them into action. His ideas are not motive forces; there is a deal of conversation, but it leads to no decision. He may be "upright, honorable, and simple," like Gagin in "Asya," a thoroughly lovable figure, or, like Ivan Afanasievitch in "Pyetushkov," he may be common, coarse, even unsightly, but in either case he is languid, without tenacity or inner ardor. A man who translates his thinking into concrete action may experience both fear and hope in anticipating the conclusions of his reasoning. But where will-energy is divorced from the rest of one's being, the froth of effusive emotion and the sparkle of clever wit and reckless thought-abandon are alike unavailing. If the mind feeds on mere ideas, then the more lofty the ideas, the more deadly is their narcotic influence: it lulls the soul of man to futile thought-spinning and inactivity; with all his aspiration, he remains superfluous, a stupid fifth wheel to the cart of actual life. So we read in "The Diary of a Superfluous Man": "I am falling into speculation, I think: that is a bad sign—am I not beginning to turn coward?" And again: "Emotional effusions are like licorice-root: when you take your first suck at it, it does n't seem bad, but it leaves a very bad taste in your mouth afterwards."

For such a human misfit one moment of genuine achievement would atone for an entire life-course of unfinished episodes. Rudin almost tastes the joy of atonement in the

struggles of '48; but Tchulkaturin, in "The Diary of a Superfluous Man," after living insignificantly to no purpose, is denied even the blessing of finding some meaning in death. He floats away with the last snows of early spring. As the messengers of death creep through his lungs, he has only one consolation: "In becoming annihilated, I shall cease to be superfluous."

Rudin fails because he is deficient in will-energy. A different man is Fyodor Lavretzky, the hero of Turgenev's next novel, "Dvoryanskoye Gnyesdo," variously translated in English as "A Nobleman's Nest," "A House of Gentlefolk," or "Liza." Here we have high ideals accompanied by strong will,—but here also life ends in futility: it is Lavretzky's past which makes a happy future impossible for him. Turgenev has lavished loving care on Lavretzky. The hero's father, an Anglomaniac, has sought to make of his son a hardy English gentleman; instead of producing an English gentleman, however, he turns out a man strong with men, but the easiest imaginable victim in the hands of a scheming woman like Varvara Pavlovna, whom he loves and marries before he knows what he is about. His dream of perfect bliss is destined to a rude shock when he discovers his wife's utter infidelity—he leaves her in Europe and, after training himself for his life's work, returns to Russia to till the soil in a civilized manner. Lavretzky lacks all the poetic flights of Rudin, but he is an efficient, sensible man who could bless Russia if only he were not such a rarity there.

One could scarcely imagine a woman more likely to be Lavretzky's ideal comrade in life than the heroine, Liza Kalitin. "In the most hidden nooks of the forest," Turgenev writes in "Yakov Pasyukov," "dreaming in primeval denseness, under fallen trees and thickets, grow fragrant

flowers." A fragrant flower, half opened, dreaming in primeval innocent goodness, is Liza. She is a rare creature. She "has no words of her own," but she has thoughts of her own, and she goes her own way; it is not her custom to ask others what she should do. From the first moment we have known her, we are certain that Panshin, the trivial, complacent, egotistic dilettante Panshin, has not the shadow of a chance of winning either her respect or her love. Yet she is young, innocent, fresh; love and passion are unspelled words to her. Turgenev has pictured greater, stronger women, but never so perfect a maiden as Liza. "Thoroughly imbued with a sense of duty, with the fear of wounding any one whatsoever, with a kind and gentle heart, she loved every one in general and no one in particular; God alone she loved with rapture, timidly, tenderly. Lavretzky was the first to break in upon her tranquil inner life." The inevitable melancholy of the unfortunate Lavretzky first evokes her pity; she tries to wake the unhappy husband to forgiveness for his undeserving wife, to purge his soul of bitterness. Gradually the woman's fresh, sympathetic charm and the man's grim tenderness deepen the friendship between them.

In "The Diary of a Superfluous Man," Turgenev portrays the miracle wrought by love in the soul of a dismal man: "I began to blossom out in spirit. Everything in me and round about me underwent such an instantaneous change! My whole life was illuminated by love—literally my whole life, down to the smallest detail—like a dark, deserted chamber into which a candle has been brought." Even such a miracle of transfiguration does Liza's friendship unwittingly produce in the life of Lavretzky. Neither of them has translated into words the actual character of the relation which binds them to each other. Then, by accident, La-

vretzky reads a report of his wife's death. Friendship declares itself to be love, for the man is now free to begin life once more.

But Lavretzky's wife is not dead. Varvara Pavlovna returns, and poor Liza enters a convent, while Lavretzky lives on, or rather, continues existing.

Love, happiness, fate: the dynamic, the goal, the lord of life; the one sets us afire, the other lures us on forever, the third lavishes on us unsought and unappreciated blessings or damns us undeservedly. Three powers in life, they are not for Turgenev three mystic entities; they are life itself. For what is love but the thirst for the thrill of genuine, intense living? In "The Region of Dead Calm," Veretyev tells the woman he loves: "Do you know why I drink? Look yonder at that swallow. . . . Do you see how boldly it manages its tiny body, and hurls it wherever it wishes? Now it has soared aloft, now it has darted downward. It has even piped with joy: do you hear? So that's why I drink, Masha, in order to feel those same sensations which that swallow experiences. . . . There is passion; . . . it produces the same effect. That is why I love you." Love is the longing freely to absorb another's life, to be absorbed into another. But that freedom is also a slavery—indeed, the quintessence of slavery. "In love there is no equality, no so-called free union of souls and other ideal things invented at their leisure by German professors. No; in love one person is the slave, the other is the sovereign, and not without cause do the poets prate of the chains imposed by love. Yes, love is a chain, and the heaviest of chains at that."

And just as love brings chains with it, so the longing for happiness brings disenchantment; the surest way not to attain it is to set out in pursuit of it. "Happiness is not to

be captured by battle. But we must not forget that not happiness but human dignity is the chief goal of life." So writes Turgenev in "Faust": "Life is a heavy toil. Renunciation, constant renunciation—that is its secret meaning, its solution; not the fulfilment of cherished ideas and dreams, no matter how lofty they may be, but the fulfilment of duty—that is what men must take heed to; not until he imposes upon himself chains, the iron chains of duty, can he attain to the end of his course without falling."

But herein precisely is the essence of Turgenev's melancholy: man, who understands this scale of values, whose noble soul admires the prospect of ascending that scale,—man finds himself thwarted in his upward striving by a Fate which dominates his life, and yet a Fate of his own making. "As clouds are first formed by the exhalations from the earth, rise up from its bosom, then separate themselves from it, and bear over it either blessings or ruin, just so around each one of us and from us ourselves is formed—how shall I express it?—is formed a sort of atmosphere which afterwards acts destructively or salutarily upon us ourselves. This I call Fate. In other words, and to put it simply: each person makes his own fate, and it makes each person."

This is a heroic philosophy, is it not? It should make one optimistic about the initial chance one has in his struggle with Destiny. But the same Alexyei who philosophizes so bravely in "A Correspondence" ends by lamenting his own fate. "And what a fate is mine!" he writes after he has sunk to sensual insignificance. "In my youth I was resolutely determined to conquer heaven for myself. . . . Later on, I fell to dreaming about the welfare of all mankind, the prosperity of my fatherland. Then that passed off: I thought only of how I might manage my domestic, my family life . . . and I tripped over an ant-hill—and flop! I

went headlong to the ground, and into the grave. What master-hands we Russians are at winding up in that fashion! . . ."

There is scarcely any more masterly example of the unutterable pathos of which Turgenev is capable than the closing paragraph of "A Nobleman's Nest." "'And the end?' perchance some dissatisfied reader will say. 'And what became of Lavretzky? Of Liza?' But what can one say about people who are still alive, but who have already departed from the earthly arena? Why revert to them? They say that Lavretzky paid a visit to that distant convent where Liza had hidden herself—and saw her. In going from one choir to another she passed close to him—passed with the even, hurriedly submissive gait of a nun—and did not cast a glance at him; only the lashes of the eye that was turned toward him trembled almost imperceptibly, and her haggard face was bowed a little lower than usual—and the fingers of her clasped hands, interlaced with her rosary, were pressed more tightly to one another. What did they both think, what did they both feel? Who knows? Who shall say? There are moments in life, there are feelings . . . we can only indicate them,—and pass by."

One has to read the great love-scenes and the scenes of great pathos in Turgenev to appreciate what a chaste, what a delicately tender spirit he was. But it is not with Turgenev's masterly portrayal of love that we are at present mainly concerned. Ask the question: What makes human life fail of greatness, what saps the energies of men, what makes the nobility of their thought ineffectual? In "Dmitri Rudin" we have the first demon of failure: weak-willed instability of spirit. In "A Nobleman's Nest" Lavretzky's life ends in unrealized futile aspirations because of the prejudicial influence of past folly, which claims one's mature

life as atonement in the stern course of conventional expiation. The weak will of man himself; the prejudicial influence of society; the third tragic demon, nature and circumstance, is portrayed in Turgenev's next novel, "On the Eve."

In many ways "On the Eve" is Turgenev's masterpiece, although the novel "Fathers and Children," which followed it, appears stronger and created a greater stir. Like Diogenes of old, Turgenev hunted over Russia with a lantern to find a *man*, a stern, strong personality of vigorous will, a man of consistent action rather than an eloquent dispenser of ideals. His failure to find that type in Russian life is itself a criticism of Russian life. To hold before the eyes of Russia this model of moral virility, Turgenev wrote "On the Eve." The title, seemingly puzzling at first, is very apt. "On the Eve" of what? "On the Eve" of true self-realization, true greatness for Russia: that is to say, in "On the Eve" Turgenev the artist depicts the men and women Russia must possess before the dawn of a new era can illumine the Russian twilight.

Elena, the heroine, is Turgenev's strongest, noblest woman. She is not a paragon of beauty. Indeed, Turgenev has made her the more compelling by the very blemishes in her, which only heighten the effect of the charms that she does possess. She yearns for a living ideal that would compel her unqualified devotion. "The life that surrounded her seemed at its best trifling and unbearable. 'How live without love? To love no one!' she thought, and her heart was filled with a strange and indefinable desire. . . . She would be oppressed with a vague longing for something, she knew not what, something that none before her had ever wished for, something that none in all Russia had ever imagined."

But she is also proud, stern, implacable in her demands: she is no sentimental enthusiast. One feels that this girl of

nineteen would be a very priestess of love, and brave danger, and death itself, could she find an ideal, a cause worth her devotion. She could love so perfectly the man who could perfectly meet her demands, only because she could love absolutely no other. She may be of the sex which men please themselves to call weaker, but she is not lacking in strength, either of will or of emotion. "Any exhibition of weakness irritated her, stupidity made her angry, a lie she could never be brought to pardon, nothing could move her when once she had formed a resolution. . . ." Such a rare nature, in which lofty aspirations combine with demands as lofty, in which longing emotions are grounded in indomitable will, a nature tender, but not weakly tender, susceptible to the least flutter of emotion, yet firm as a rock in its refusal to rest satisfied with unrealized ideals,—such a woman Turgenev has painted in Elena Nikolaevna.

But who is to answer the call of this rare soul? Two men love Elena: Shubin, the irrepressible artist, a sculptor, is a very butterfly of flitting, unstable emotions. Not one note of sternness sobers the song of his life. Undoubtedly he has talent, yet he takes seriously neither his art nor himself. Simpering damsels like Zoe he could perhaps make happy, or flower-girls like Annette, but Elena he frankly confesses himself unable to understand.

The other suitor, Bersenyevev, is a scholar, a bookish university valedictorian, calm, intellectual, nothing if not serious, with a high conception of his academic future, eager to write a ponderous history of something or other. He is tender-hearted, too; generous, as only Russians can be generous,—witness his self-effacing, honorable loyalty to his friend who becomes his rival, so masterfully portrayed in the course of the novel. But Bersenyevev's life is the theoretic life; one doubts if it is in him to cast everything aside and



burn his life out to light any torch whatever. And it is that sort of man alone that Elena Nikolaevna can possibly love. Though she likes his honest devotion to his academic task, we feel that Bersenyev lacks at once the romance and the strength of spirit that would be necessary to win Elena.

"I love Elena; Elena loves you!" the artist Shubin informs Bersenyev. "What a lovely night, quiet, fresh, and full of shadows! . . . Believe me, there will never be such another glorious night as this in all your life!"

But the future professor goes home, lights a candle, puts on his dressing-gown, and taking from his bookcase Raumer's "History of the Hohenstaufen," begins reading from the page where he had left off the night before.

Shubin the artist is too effervescent a spirit for Elena; Bersenyev is a mind too calm. And thus Elena's youth, like water beneath a frozen stream, flows on silently and quickly, in outward inactivity, but in inward uneasiness and strife.

It is one of the points of this story which made it a criticism of Russian character that the nation which produced such a woman as Elena could not supply a corresponding man. Turgenev finds his man outside of Russia, as if to hold before Russian men at once a picture of their own insufficiency and a model of possible regeneration. Dmitri Insarov, the hero, is a Bulgarian patriot. Himself orphaned by a Turkish massacre, his whole life is dominated by one concrete idea. Goodness, wisdom, courage, self-sacrifice, nobility, love, hate, life and death—all these possess for him one clear, concrete meaning: the liberation of his country from the Turks. Shubin sketches him well in speaking to his philosophic friend: "Talents, none at all; poetry, deuced little; capacity for hard work, enormous; memory, almost as enormous; mind, neither discursive nor deep, but healthy and quick; speech, dry, energetic, and even eloquent when

he gets upon his—between ourselves we may say—stupid Bulgaria. . . . But further, you will never be on such a footing with him as to say *thou*, and no one ever was. I, of course, an artist, can be no favorite of his; and I am proud that I cannot. Cold-hearted, cold-hearted, and capable of grinding us all to powder! He is attached to his country; not like our empty-heads who merely flatter the common people; but he tries to unite the whole nation in one common work. Thus his task is more easy and more intelligible: to drive out the Turks, that is all; that is the work to which he dedicates himself. But, thank God, all these fine qualities do not take with women. There is nothing attractive, no charm about him; he is not like either of us." Which last remark shows Shubin a better judge of Insarov, whom he has just met, than of women like Elena.

From the very first we know that she will love Insarov. "He not only talks," she writes in her diary; "he acts, and will act."

"Do you love your country dearly?" she asks him.

"As yet I cannot say," he answers. "Only when a man has died for his country can you truly say that he loved it."

"Why is he not a Russian?" she thinks. "No, he never could be that."

Action, not mere words; decisions, not mere sighs and longings: such are the motive forces in this man. He does not scatter his energies. Turgenev saw Russia flapping its million wings enthusiastically, but not knowing whither to fly; dabbling in everything, accomplishing nothing. But listen to his Bulgar hero: "Just think," he tells the Russian woman, "the veriest peasant, the lowest beggar in Bulgaria, not a whit less than I myself, awaits one and the same thing; we have all but one and the same thing in view—and think what strength, what assurance, this unity of aim must give us!"

Again we read in Elena's famous diary: "I think that the reason why Dmitri is so tranquil is because he has devoted his entire self to one work, to the realization of one dream. What can disturb him? He who gives himself up entirely, entirely, entirely, is superior to all contingencies. It is not *I* wish, but *it* wishes."

Can we trace the growth of the heroic love which follows? Determined not to allow his passion for Elena to interfere with his patriotic mission, Insarov would stifle all emotion and leave her. But she proves herself his equal. For the sake of him and his ideal she abandons her family, her native land, and starts with him to share his perilous life in Bulgaria. An attack of quick consumption kills Insarov on the way; Elena goes to Bulgaria alone to continue his work, and is lost among the oppressed compatriots of her husband.

A noble tragedy is "On the Eve," but a tragedy just the same. The spiritual weakling Shubin lives and prospers on his idle fancies; Insarov, the grim hero, is stricken by hostile nature and dies. What avails his master-soul before that hacking cough? Life is a strange puzzle. "It will often happen that a man, with involuntary apprehension, asks himself: Can it be that I am already thirty—forty—fifty years old? How is it that life passes so quickly? How is it that death presses so quickly upon us? Death is like a fisherman who has caught a fish in his net, but leaves it for a while in the water; the fish still swims about and fancies itself to be free, but the net encircles it, and the fisherman seizes hold of it whenever the fancy takes him."

Profoundly disenchanted Turgenev was with Russia. And yet, even while he was writing "Rudin," "A Nobleman's Nest," "On the Eve," a new Russian type was coming into being, a new generation with new ideals which were

to set Russia afire, to give birth to Russian Insarovs and multiply the Elenas in Russia. A spirit of denial, of destructive radicalism was arising, as usual beginning with mere ideas, mental attitudes, but destined to pass into action with a definite goal, political or economic, and lead to all the noble bravery which lies buried in the frozen wastes of Siberia. This new spirit Turgenev analyzes in "Fathers and Children," the most famous Russian book, and the one containing Turgenev's strongest Russian hero, Evgeny Bazarov.

The contrast is between the respectable, orthodoxly radical, sentimental, formal, noble-hearted fathers and the blatantly radical, vigorous, defiant, negating children. A new generation has sprung up, and to it the old honors, principles, and duties mean nothing. The father of young Bazarov, an old regimental doctor, speaks in terms of profound respect of the noble-born Arcady's father and uncle. But the son has only contempt for their respectability and for the respectability of all other old fogies, while well-born Arcady himself looks up to the son of the poor regimental doctor.

Turgenev's hero utterly scorns all romanticism, all poetry, all conventional principles. Bred on the materialism of Büchner, he is a destructive genius, but a genius of unquestioned force. Along with the democracy of this modern son of the soil, there is a good deal of crude materialism and a certain cruel, even coarse irony.

Absolute independence, absolute freedom from all conventions, from all prejudices—this is Bazarov's aim. His aim is frankly destructive. The work of building up he leaves for future generations; his age needs men to tear down, to level this civilization of lies. For this task young Arcady, son of the aristocracy, proves too weak. The chil-

dren of the plebeians must do this work also, as they have done all other work in Russia. "You lack boldness, wickedness," Bazarov tells the youth when they finally part, "but in turn you are endowed with a youthful audacity and ardor. That does not suffice for the work which we others are pursuing. And then, you gentlemen cannot go beyond a certain indignation or a generous resignation, things which do not signify much. You think you are great men, you think yourselves at the pinnacle of human perfection when you have ceased to beat your servants; and we—we ask only to fight with one another and to beat. Our dust reddens your eyes, our mire soils you; you are truly not of our height; you admire yourselves complacently, you take pleasure in reproaching yourselves; all that bores us; we have other things to do than to admire and reproach ourselves; we must have other men broken on the wheel."

Perhaps the most pathetic thing in the book, from the point of view of literary art, is the trembling admiration with which Bazarov's old parents watch over their son, whom they at once revere and fear. The cold radicalism of the son is in tragic contrast to the loving orthodoxy of the mother and the heart-breaking effort which the father makes to keep up with his son's progressiveness, not to be a tedious old fossil in his own child's eyes. The scene where the son, leaving his parents after a three days' visit because their loving attention gives him no opportunity to work, and the courage with which the father receives the crushing news, is a scene which, for suppressed pathos, is scarcely to be matched in all fiction.

Bazarov does return to his parents; a wound which he receives while treating a peasant suffering from typhus infects him with the disease and claims his life. The nihilist, who in his life denied all, is himself denied by death.

"Fathers and Children" caused a national tempest in Russia. Both the fathers and the children thought themselves outraged in Turgenev's novel. Especially the new generation felt wounded, forever hurt by Turgenev, and it did not forgive him to his very death. Turgenev's was the unenviable lot of the sane mediator, the lot of Socrates, the lot of all who are too penetrating to go to either extreme and too honest to join either of the two parties of narrow thinkers which comprise the majority in all lands and particularly in Russia. In his "Poems in Prose" Turgenev pictures those two types. On the one hand the two toilers, anxious to get hold of a bit of rope with which the authorities have hanged the man who has tried to liberate them, for "they say that that brings the greatest good luck to a house." For these toilers, for those who exploited their toil, and for those whose orthodoxy regarded the old order as divinely ordained, "Fathers and Children" was doubtless too radical. For another class it was too conservative: for those whose rebellious souls found no virtue whatever in the established order, and even more for the type of man who always stirred Turgenev's sarcasm—the brainless dolt whose meager capital of intelligence exhausted itself in bold criticism. "You are behind the times," says the Fool in another "Poem in Prose." And society is impressed by the Fool's self-assured criticism. "There is a career for fools among cowards."

The failure of Russia to understand his works, her hatred for the man who revealed her true state, outraged Turgenev's sensibilities as artist and man. There is no hatred in Turgenev's next two novels, "Smoke" and "Virgin Soil," but there is less pity. Turgenev pictures the utter futility of the Russian society man and woman, frivolous, veneered sepulchers of corruption; the futility also of the unkempt,

disheveled, loud-voiced, word-mongering radicals; the utter inefficiency of those who would upset Russian society without having any well-thought-out ideals either of tearing down or of building up.

I wish I might dwell further on that immortal life-size portrait of the Russian society lioness—the portrait of Irina Pavlovna—in “Smoke.” But it was not the portrait of Irina Pavlovna that Russia did not like: it was the judgment of Russia, expressed in the words of Litvinov, the hero, expressed even more tragically by that derelict of a man, Potugin.

The Slavophiles and Panslavists, who wished to shut Russia from the rest of the world, who deluded themselves with complacent dreams about Russia’s supremacy, exasperated Turgenev. “Nothing to compare with Russia, indeed!” Potugin exclaims in “Smoke.” “Our bristles, for instance, are large and strong, because our pigs are poor; our hides are stout and thick, because our cows are thin; our tallow’s rich because it’s boiled down with half the flesh. . . . They talk to me about our inventive faculty! The inventive faculty of the Russians! Why, our worthy farmers complain bitterly and suffer loss because there’s no satisfactory machine for drying grain in existence, to save them from the necessity of putting their sheaves in ovens, as they did in the days of Rurik; these ovens are fearfully wasteful—just as our bast shoes and our Russian mats are—and they are constantly getting on fire. The farmers complain, but still there’s no sign of a drying machine. And why is there none? Because the German farmer doesn’t need them; he can thresh his wheat as it is, so he doesn’t bother to invent one, and we . . . are not capable of doing it! Not capable—that’s all about it! Try as we may!

From this day forward I declare whenever I come across one of these rough diamonds, these self-taught Russian geniuses, I shall say: 'Stop a minute, my worthy friend! Where's that drying machine? Let's have it!'

And the answer comes from the young painter Gagin in "Asya," whose studies show life and truth, but whose drawing is slovenly and inaccurate, and who has not finished a single picture. "I have not studied as I should have done, and that cursed Slavonic laxity is asserting itself. When one dreams of work, he soars like an eagle; it seems as though he could move the earth from its place; but in the execution he immediately grows slack and weary."

Why is this novel called "Smoke"? As Litvinov the hero turns his back on Baden-Baden and its Russian colony, "the wind blew facing the train; whitish clouds of steam, some singly, others mingled with other darker clouds of smoke, whirled in endless file past the window at which Litvinov was sitting. He began to watch this steam, this smoke. Sometimes the wind changed, the line bent to right or left, and suddenly the whole mass vanished, and at once reappeared at the opposite window; then again the huge tail was flung out, and again it veiled Litvinov's view of the vast plain of the Rhine. He gazed and gazed, and a strange reverie came over him. . . . 'Smoke, smoke,' he repeated several times; and suddenly it all seemed as smoke to him, everything, his own life, Russian life—everything human, and especially everything Russian. All smoke and steam, he thought; all seems forever changing, on all sides new forms, phantoms flying after phantoms, while in reality it is all the same and the same again; everything hurrying, flying towards something, and everything vanishing without a trace, attaining to nothing; another wind blows, and all is dashing



in the opposite direction, and there again the same untiring, restless, and useless gambols! 'Smoke, smoke, nothing but smoke!'

Smoke is not lacking in "Virgin Soil," but through the smoke we can see undoubted fire. Young Russia, as Turgenyev pictures it in this last of his great novels, does not show very much greater capacity for action than it does in "Smoke"; the builders of the radiant to-morrows still spend their time in "those nocturnal, interminable discussions, which in such proportions and in such a form can hardly be characteristic of any other race whatsoever." But there is a radical change: discussions are their own reward in "Smoke"; in "Virgin Soil," on the other hand, arguments, projects, speculations are very clearly wearisome to those taking part in them; very keen is the conscious thirst for action. Of this indifference to eloquent words and this insistent demand for action, two characters in the book are the embodiment; and of the two, the man Markelov plays a secondary rôle, even though he precipitates the dénouement. Markelov is headstrong, grim, "like John the Baptist when he had eaten the locusts,—the locusts alone without any honey"; he is not balanced enough for efficient leadership; but he is no Rudin; he is on more than speaking terms with his ideals. Genuine to the heart and aware of worse prospects than the scaffold, he can be cowed by no threats whatever. The peasants to whom he preaches revolt themselves betray him to the authorities; but his own failure makes him no pessimist about the cause as a whole. "I did not set about it rightly," he says, and takes his medicine with a firm jaw and without heroics.

But Marianna, the heroine, is the very personification of the new Russia longing to burn itself in the all-consuming fire of actual combat. When her lover Nezhdanov tells her

about his meeting with Markelov and Solomin, this young daughter of a dishonest official is impatient about all he has to say of their discussions; one question alone fires her whole being: When is the struggle to begin, and where, and how can she enter it without delay? She rejoices that her hands are growing red and hard; and she is in instant expectation that, if necessary, she will ascend the scaffold. "Do you believe in the cause, Marianna?" Nezhdanov asks her, and she draws herself up and raises her head: "Yes, Alexyei, I do believe in it. I believe in it with all the powers of my soul. And I will consecrate my whole life to that cause! To my very last breath!"

Nezhdanov himself is built of a different fiber. This student, whom men of action trust and who inspires valorous men, is himself the battle-ground of Rudins and Insaurovs. Ever longing to lose himself in self-forgetting struggle, he ever finds the fountains of action drying up in his soul. He writes verses and is ashamed of it; he would be proud of converting peasants to the cause, but their vodka nauseates him. "A romanticist of realism," the effervescent, timorous firebrand Pakhlin calls him. From the very start he is foredoomed to failure. He curses "this nervousness, this sensitiveness, this impressionability, fastidiousness" which are his very inheritance. "They say that one must learn the language of the common people, learn their habits and customs," he writes, wearing a stinking burgher kaftan. "Nonsense, nonsense, nonsense! One must believe in what one says, and talk as he pleases! But when I begin to speak it is like a guilty man, and I keep begging forgiveness." An insuperable barrier separates him from the masses whom he would awaken. And since he cannot live his life on the only terms he honors, he would not live at all. "I did not know how to *simplify* myself; the only thing that was left was to

erase myself altogether," he writes in his farewell letter to Marianna. He is not afraid of the prison walls. "But to be incarcerated for the sake of a cause in which one does not believe is—entirely unfitting." So he blows out his brains.

Marianna marries Solomin. Of all Turgenev's characters, Solomin is the least typically Russian, if we can judge of what is typically Russian from Turgenev's gallery of portraits. But the Russian workmen to whom Nezhdanov is an alien obey Solomin like a master, respect him as their superior, yet treat him as an equal. "He is one of us," they declare proudly. He is the very reverse of a firebrand, and he even quenches enthusiasm, but he inspires trust. Men wonder at themselves that he compels their respect and admiration; but they respect and admire him notwithstanding. At first glance he strikes Marianna as indefinite, impersonal. "But the more she scrutinized him, the more she listened to his remarks, the stronger did her feeling of confidence—precisely that, of confidence—in him become. That calm, not exactly awkward, but ponderous man not only could not lie, dissimulate: one could rely upon him as upon a stone wall. He would not betray; he would understand and uphold."

Solomin talks little, not because he has no thoughts, but perhaps because he has no doubts about his thoughts. "I always know what I am talking about," he simply informs the peppery, contemptible upholder of reaction, Kalomyetzev. He has grown up from the soil, and he knows that soil full well; he has no roseate illusions about the common people like Marianna, and he is thoroughly acquainted with the Russian nobleman's inadequacy to cope with the problems of Russia. This is a man who works hard and well, and without eloquence; he cannot be stampeded either by

cowardly panic or by firebrand enthusiasm. He is thorough; he is genuine; he is likewise cautious.

He establishes a school and a hospital at the factory which he superintends, but he refuses to allow the distribution of revolutionary pamphlets among the men under his charge. He is not anxious to evade death, not he; but he thinks too much of his life to sacrifice it recklessly; he is too thrifty for that. He expects no quick solution of Russia's problems, nor does he believe in quick solutions. He believes in daily simple service. He can distinguish between the great and the merely grand: Marianna sighs for heroic sacrifice, but he tells her: "In my opinion, to comb the hair of a scabby little child is a sacrifice—and a great sacrifice, of which not many are capable. You will wash pots like a dirty-faced scullery-maid, and pluck fowls. . . . And then, who knows but you will save the fatherland?"

"I should like to justify your expectations," she says; "and then—I should be ready to die."

"No, live—live! . . . You are already, all of you Russian women, more capable and more lofty than we men!" And therein speaks Turgenev himself, whose disenchantment with the weak longings of Russian manhood found consolation and hope in the idealistic strength and energy of Russia's women. Almost all his genuine heroes are heroines. In picturing Solomin's recognition of this truth, Turgenev pays a rare tribute to the intelligence of the factory superintendent. It is an essential part of Turgenev's philosophy of life, this failure to find spiritual strength and energy of soul in Russia's manhood. Homeric physical strength is in Kharlov, the "Lear of the Steppes"; Gerasim, in "Mumu," has brutal might, but the motive agencies in the life of the spirit Turgenev finds in womankind. His women are saving angels and evil geniuses alike, their energy of

will and life-ardor yield them the shears of destiny: they are Liza, Elena, Marianna, Varvara, Irina, Marya; but among Turgenev's men heroic virtue is as rare as aggressive vice. Kalomyeitzev, in "Virgin Soil," is despicable; Naum, in "The Inn," is crafty and mean; Vassily Ivanovitch, in "Three Portraits," is perhaps the most accomplished villain one could desire; but these are exceptions, just as Solomin and Bazarov are exceptions. A virgin soil is Russia, and "virgin soil should be broken up, not with the primitive plough that skims along the surface, but with the modern plough which cuts deep."

When Sienkiewicz, in "Without Dogma," speaks of Slavic unproductivity, when it is written that Hamlet is Russia, the words are not wholly without meaning. Turgenev himself was something of a Hamlet. Asked once to give a definition of perfect happiness, he answered: "Laziness without remorse," thus proving himself a true Russian. But if this greatest artist of Russia was himself no Brand, he was too honest a genius to play the Peer Gynt. Always he criticized Russia, yet always he loved her, always he believed in Russia. Note his description of the old couple Fomushka and Fimushka; it is a message of hope in the native strength of Russia's virgin soil. "There are pools on the steppes of that sort: although they have no outlet, they never become covered with scum because they have springs at the bottom. And my old folks have springs—there at the bottom of their hearts—pure, exceedingly pure springs."

Georg Brandes speaks of the harmony between Turgenev's own estimate of a character and his actual delineation of it. This harmony is perhaps in no case so exquisitely exemplified as in "Asya." From her first appearance to her last, the impression which Asya produces has a certain lyrical unity; the author does not change his mind about her

character, nor, on the other hand, does he impose a character upon her. And Asya's irresolute lover shows the same true relation of the artist to his creation. He languishes for happiness. It is offered him in the self-revealing love of Asya. But happiness comes too suddenly; he who languishes for it finds himself disconcerted by its arrival. Like the story-teller in "Andrei Kolosov," like Fustov in "A Hapless Girl," he too postpones its attainment until the morrow. But "the word to-morrow was invented for irresolute people." "To-morrow I shall be happy! There is no to-morrow for happiness; neither has it any yesterday, and it reckes not of the future; it has the present, and not even a day at that, but a moment."

Like some of the sons and daughters of his fancy, Turgenev himself was a lifelong pilgrim on the road to the happiness with which true attainment blesses man. Nor are his characters alone subject to disenchantment. The lyrical intermezzo "It Is Enough," which he wrote after "On the Eve" and before "Fathers and Children," is a poignant self-revelation of a knight on the battle-field of ideals, who in the midst of the combat asks himself whether it is worth while. The satanic energy of man, which blasts effort, vitiates achievement, and poisons the joy of lofty endeavor, is not pain or misery or death, or even failure. It is a dread of pettiness. "The terrible thing is that there is nothing terrible, that the very substance of life itself is petty, uninteresting, and insipid to beggary." One spends his life-blood in battling for noble ideals, in wrestling with the powers of evil, or one burns his being in devotion to beauty, one holds aloft the torch of artistic truth—*cui bono?* What is the use of enlightening mankind? "Why demonstrate to gnats that they are gnats?" "Man is the child of Nature; but she is the universal mother and has no preferences. . . . She cre-

ates by destroying, and it is a matter of perfect indifference to her what she creates, what she destroys, if only life be not extirpated, if only death does not lose its rights. And therefore she as calmly covers with mould the divine visage of Phidias' Jupiter as she does a plain pebble, and delivers over to be devoured by the contemned moth the most precious lines of Sophocles. . . ." So writes the poet of "A City of Dreadful Night":

"The sense that every struggle brings defeat  
Because Fate holds no prize to crown success;  
That all the oracles are dumb or cheat  
Because they have no secret to express;  
That none can pierce the vast black veil uncertain  
Because there is no light beyond the curtain;  
That all is vanity and nothingness."

"How are we, poor men, poor artists, to come to an agreement with this deaf and dumb force, blind from its birth, which does not even triumph in its victories, but marches, ever marches on ahead, devouring all things?"

Has Turgenev answered this question which most writers do not even ask? He produced several masterpieces after the writing of "It Is Enough." What gave him the inspiration to persist in a task which his reason found so hopeless, so futile? Perhaps life also is futile, but we who treasure the hope that it is not futile—can we dash that hope to bits by putting an end to our lives? Man lives on and hopes. And who knows? Perhaps this determination to live on and hope is bringing the hope itself to realization.

The blind stupidity of nature and of man oppressed the creative soul of Turgenev, yet he kept on to the end—and at the very last he points out the guardian angel of his ideal pilgrimage, the rock of his support and the source of his

inspiration: the indomitable might and beauty of the Russian Word, the Word which for his artist soul was God. This is the last "Poem in Prose," the last page of Turgenev's works. It is entitled "The Russian Language":

"In days of doubt, in days of painful meditation concerning the destinies of my fatherland, thou alone art my prop and my support, O great, mighty, just, and free Russian language! Were it not for thee, how could one fail to fall into despair at the sight of all that goes on at home! But it is impossible to believe that such a language was not bestowed upon a great people!"



### LECTURE III

#### FROM DARKNESS INTO LIGHT: FYODOR DOSTOYEVSKY

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IN becoming Europeanized, Muscovy had of course become more nearly civilized; but it had also become more corrupt. The unthinking faith of Holy Russia had given way in the upper classes to a highly sophisticated skepticism. The old Russia had been, and the *mujik* still was, ignorant, superstitious, yet genuine. The nobles and educated classes had a veneer of learning, but the varnish was thin.

Now those among the leaders of Russia who believed that her destiny was in Europe, not in Asia, those who were themselves imbued with the Western spirit, were not blind to the fact that Russia aped instead of emulating Western culture; but they did not on that account regard the European ideals themselves as unworthy of emulation. The trouble with Russia, they said, was that she simply aped Europe, instead of getting her inspiration from the West and seriously attacking her own problems in a modern European manner. Such a champion of world-culture was Turgenev, —indeed, Russia has never had a better prophet of world-culture than the author of "Smoke." Near the close of this novel he puts in the mouth of the seared Potugin a bit of counsel to his countrymen: "Every time it is your lot to undertake any piece of work ask yourself: Are you serving the cause of civilization in the true and strict sense of the word; are you promoting one of the ideals of civilization; have your labors that educating, Europeanizing character

which alone is beneficial and profitable in our day among us? If it is so, go boldly forward; you are on the right path, and your work is a blessing!"

In opposition to the shallow imitators of European manners, in opposition also to the advanced advocates of thorough European culture, there have always been the champions of Holy Russia; men who believe that Russia should remain Russian, spiritually self-complete; men willing to build a cultural Chinese wall around Muscovy. Leaving aside the early opponents of Peter the Great's occidental policy, we find in the nineteenth century the orthodox clergy of Russia opposing the Franco-German influences which made infidels and agnostics of the educated classes; we find apostles of Russian Russia, Slavophiles, Panslavists, voicing their message in literature, urging Russia's self-sufficiency, the superiority of Russian ideals, Russian art, Russian literature and music, Russian morality and religion, laboring for the social and political self-assertion of self-dependent, Slavic Russia.

Self-conscious Panslavism began with Aksakov's rhapsodical praise of the ancient Russian, Slavic virtues, his protest against the tendency to look up to Europe, and his championship of Russia's isolation. Holy Russia has nothing to learn from Europe, the Slavophile said; on the contrary, Europe must and will make pilgrimages to Russia. Above all, this was the Slavophile program: Purge Russia of all European influences. Needless to say, the opponents of this cultural chauvinism resented the idea that they were not good Russians simply because they wanted Russia to share in the ideal treasures of Europe and believed that Russia's cultural star was in the West. Turgenev's firm opposition to this narrow nationalism cost him the friendship, or rather won him the violent hostility, of the Slavo-

phils, of their political leader, Katkov, and of him who may be called their literary apostle, Fyodor Dostoyevsky.

Turgenev's portrayal of shallow, frivolous, culture-needing Russia outraged the uncritical Russianism of Dostoyevsky. Especially was he offended by the novel "Smoke." He hated Germany and despised France. At a meeting in dissolute Baden-Baden, Turgenev told him: "You want to save Russia? There is only one universal and irrefutable way, that of civilization. All attempts to create an independent Russian culture are but folly and pigheadedness." Dostoyevsky thereupon recommended to Turgenev, who was then living abroad, to order a telescope from Paris so that he could study the Russian people conveniently from afar.

The theme of Dostoyevsky's novel "The Possessed," or "The Demons," is the same as that of Turgenev's "Fathers and Children"; and its significance as a means of comparing the art of the two novelists is equaled by its importance in understanding the totally negative attitude which Dostoyevsky took toward the revolutionary propaganda, toward that movement in literature, art, intellectual and social ideals which Turgenev had christened nihilism. Turgenev's book is an honest endeavor to analyze the new type, clearly to delineate it in all its admirable and unlovely strength and weakness, and to contrast it with the old. Dostoyevsky's portrayal is a confessed attack on the new-born demons of destruction. In the new movement of revolt, Dostoyevsky sees the manifestation of cynicism, of embittered, distorted estimates of life; the manifestation of the spirit which kills; a danger, not a promise, for Russia. Human failures, men lacking spiritual orientation, disgruntled men, and emotionally seared women,—these comprise the rank and file of "The Possessed"; although, in justice to Dostoyevsky's art, his portrayal of the pathetically heroic idealist Erkel should

not be forgotten. Dostoyevsky exhibits almost maliciously the credulous curiosity of these folk, eager to be initiated into the red mysteries of revolution.

And the one to initiate them is Pyotr Stepanovitch Verhovensky, a sinister figure, conscienceless, implacable, utterly unscrupulous in his choice of methods. He is completely emancipated from the conventional standards of veracity, honor, and decency; he is willing to sacrifice the happiness or the good name of any man or woman if it serves his purpose. Like an evil genius, he fastens his tentacles on all whom he can use; he binds them to himself in unfaltering loyalty by staining their hands in conspiracies and crimes. This champion of freedom and equality would make supine slaves of all under his charge; he makes them execute his orders in blind submission: such is his idea of organization. He even deceives them about the strength of the movement and its prospects of success; he allows, he encourages them to follow phantoms of delusion. He seems worse than a demagogue; there is occasionally a veritable satanic gleam about the man. He loathes the prospect of reform. He wants things to become worse; the more rotten they become, the more radical will be the ultimate change. "One or two generations of vice are essential now; monstrous, abject vice by which a man is transformed into a loathsome, cruel, egotistic reptile. That's what we need." "We will proclaim destruction!" says this brutal Bazarov of Dostoyevsky. Turgenev's hero has excluded, along with other sentimentalities, filial love; but Bazarov is not maliciously insulting, designedly brutal to his father. Pyotr Stepanovitch is both; the record of his conversations with Stepan Trofimovitch does not increase our affection for the son. In his portrait of the elder Verhovensky, Dostoyevsky caricatured the advocates of "civilization," of Western ideals; the portrait is more prop-

erly a cartoon which Dostoyevsky might have offered to the author of "Smoke." The father is maudlin, theatrical, pathetic, ridiculous, but the son is vicious and morally repellent.

And yet his integrity cannot be doubted: Pyotr Stepanovitch may be a Jesuit in his methods, but his devotion to "the Cause" has the fervor of a Loyola. It is because universal destruction has become for him a mania, an obsession, because it has filled his life entire, that his being contains no room for pity or conscience. And it is this demoniac spirit of modernism, of nihilism, of revolutionism which Dostoyevsky denounces in his novel: a demoniac spirit because it has never felt the glow of sympathy and love, because it is a spirit of destruction, of hatred and denial, because it sets one man against another, because it is the negation of the spirit of Russia's Christ. Verhovensky and his followers must hate; they demand a world which they can hate. Dostoyevsky puts his challenge to "The Possessed" in the mouth of that tragic victim of nihilistic enthusiasm and nihilistic persecution, Shatov: "They'd be the first to be terribly unhappy if Russia could be suddenly reformed, even to suit their own ideas, and become extraordinarily prosperous and happy. They'd have no one to hate then, no one to curse, nothing to find fault with. There is nothing in it but an immense animal hatred for Russia which has eaten into their organism."

And over against this anti-Russian alien spirit, against this foreign-bred heresy of denial and destruction, Dostoyevsky preached the saving power of implicit orthodoxy, the saving grace of pity and charity and love. Not European culture, but faith and piety and Russian simplicity and devotion are saving and will save the world. I quote some

extracts from Dostoyevsky's letters recently published in English: "What has European culture done for Europe? Wherein does she surpass Russia? In Germany everybody can read and write, but everybody is terribly unintelligent, obtuse, stubborn, and devoid of high ideals. . . . In Western Europe the people have lost Christ, and Western Europe is tottering to its fall. . . . Russia is to reveal to the world her own Russian Christ, whom as yet the people know not, and who is rooted in our orthodox faith. . . . Veneration and love of the Russian people's God and its faith—this is fellowship with the people, and only from the people is anything worth while to be expected."

Here you have the quintessence of Pan-Russianism and literary populism—two of the fundamental characteristics of Dostoyevsky's art. Of all the Russian writers, Dostoyevsky is the most professedly, defiantly Russian; hence the difficulty of the Western mind to understand his message, which difficulty has led on the one hand to exaggerated praise of his strange, gloomy genius, and on the other hand to undeserved criticism of it. There is certainly no occidental veneer about Dostoyevsky. He himself could not live outside of Russia; he felt in Europe "like a fish out of water, like a slice cut from the loaf." "If you only knew," he writes in his Letters, "what a deep-drawn revulsion, almost approaching hatred, I have conceived for Western Europe!" Turgenyev's genius is also Russian, but Turgenyev has more clearly a universal appeal because his vision embraces all humanity. Dostoyevsky's novels are insistently Russian; Russian in their spirituality and their pessimism; even their very immensity and prolixity are Russian; and of the great triad—Turgenyev, Tolstoy, Dostoyevsky—the last is the least understood by those outside of Russia. To appreciate

the optimistic gloom which pervades the art of this great Russian genius, some slight acquaintance with his agonized life is indispensable.

Fyodor Dostoyevsky was born in 1821 in a hospital for the Moscow poor. His father, the descendant of what had once been a noble family, was a military surgeon in retirement, and supported his family by serving in the hospital. Their life was simple and their circumstances by no means over-comfortable, but the father had some of the old family pride left in him and the children were educated at home as sons of the nobility. The young Fyodor was not allowed to come in contact with the life of the city, or in any way to associate with the children of the streets. The father spared no effort to retain in his children all the lofty manner of the aristocracy. It was a proud poverty.

The mother died when Fyodor was sixteen, and the following year the father removed to St. Petersburg with his two boys to enter them in the military engineering school. Owing to a curious circumstance, at the entrance examinations the older brother, Michail, who was strong and robust, was rejected as sickly and unfit, while our future writer, who was distinctly weak and delicate, was approved and accepted. The brother Michail went to another school, and Fyodor, who had never before been away from the jealous care of his parents, was now left alone in the confused immensity of the Russian capital. By nature of a melancholy disposition, Dostoyevsky felt like a lost soul. He devoted himself to his military lessons, however, and indeed gave a good account of himself, graduating third in his class of thirty.

But a secret thirst consumed this engineer—a passion for literature which he could not suppress. From his early youth he had been fed on poetry and novels. Pushkin and

Gogol he read, of course, but Fenimore Cooper and Walter Scott were also early favorites. His life in St. Petersburg roused in him a great desire to write. For the first time this son of a decayed nobility came in direct contact with the squalid life of the masses, and he burned to put on paper that which oppressed his young spirit. Like Schiller with his "Robbers," Dostoyevsky wrote his first work in odd moments when he could avoid the surveillance of the military school authorities. In his efforts to evade the eyes of the proctors he was the more successful because his distaste for jolly company and his leaning toward solitude were familiar to his fellows. Soon he had written his first story, "Poor Folk."

About that time—1844—Dostoyevsky's father died. The son promptly resigned his commission in the army, for which he had no liking, and decided to stake all on his success as a writer. His only tangible capital was the manuscript of his story. For a while he dared not submit it to the critics. "I wrote that story with passion, almost with tears," he says. "Had it failed, I should have hanged myself." One of his friends, Grigorovitch, who had had only one thing published, but who knew Nekrasov, one of the great authors and editors of that day, encouraged him to submit his story to Nekrasov for publication. Dostoyevsky spent the next evening with some friends, reading Gogol's "Dead Souls," and returned home about four o'clock in the morning. It was one of the white nights of St. Petersburg. Unable to go to sleep, Dostoyevsky stood by his window musing. Suddenly the bell rang and two men, his friend Grigorovitch and the great poet and editor Nekrasov, rushed into his room, too deeply moved to speak. They had been reading his "Poor Folk" all night, until finally in a burst of enthusiasm they had come to his room to wake him up



and tell him about it; for his story, they said, was more important than sleep. To sensible Anglo-Saxondom this may sound like romantic moonshine, but it is characteristically Russian.

The next day Nekrasov took the manuscript to the famous critic Byelinsky. "A new Gogol has appeared," shouted Nekrasov, entering with the manuscript. "With you Gogols spring up like mushrooms," Byelinský<sup>\*</sup> remarked severely; but he took the manuscript. When Nekrasov returned that same evening, Byelinsky met him with perfect enthusiasm. "Bring him, bring him as soon as you can."

"Poor Folk" is written in the form of a correspondence between Makar Dyevushkin, a shabby copyist-clerk, and the poor, sickly seamstress Barbara Dobroselova. They lodge in adjoining tenements. The shy clerk, who is neither young nor handsome enough to court the seamstress openly, nor sufficiently old and dignified to risk calling on her without arousing comment, is reluctant to accept her invitation to visit her, and remains satisfied with letter-writing. By this literary device Dostoyevsky allows the two characters to reveal themselves and their environment. The impression produced is one of humble pathos rather than of tragic suffering. The petty joys and sorrows of Makar, his concern over his handwriting,— "I write a neat and pleasant hand, but my writing lacks style,"—his admiration for the dime-novelist Rateziaev, whom the more intelligent Barbara naturally despises; his pathetic prodigality in buying her bonbons and bouquets, which she as pathetically condemns while admiring; his own charity toward the poor Goshkov, whose coat is "worse even than my own,"—all these reveal a character pitiable, even lovable in its kindly insignificance. Makar is sentimental, but somehow, with all his shabbiness, sartorial and intellectual, he defies scorn, defies even con-

descension. Surely the portrayal of such a soul is a genuine achievement for a twenty-year-old.

When the wealthy Bwikov, who in time past had wronged Barbara, offers to marry her in orde., with one and the same act, to repair the wrong he had done her and to thwart an heir whom he dislikes, she accepts him and proceeds to make elaborate preparations for her trousseau. The news stuns Makar, but he submits to his fate; indeed, he is almost happy at her good fortune and runs her errands from shop to shop, looking at embroideries, laces, silks, and jewelry. She leaves St. Petersburg for Bwikov's country estate, and Makar is left, hoping the floods will stop her carriage and send her back to the city. There is deep sorrow in his soul, but rancor there is none whatever. Barbara has left him to marry the wealthy Bwikov: he plans to buy her a new cloak with his next salary.

This is the story, but what one carries with him is not so much the story itself as its spiritual atmosphere of pathos, compelling pity. Readers of Turgenev will remember Bazarov's father in "Fathers and Children," and his tragic effort to win the love of a son he worships. In Barbara's story of her youth, Dostoyevsky draws a like portrait of a drunkard's adoration for his learned son. Twice a week the old Pokrovski dares to visit his son, always hesitant, almost in trepidation lest his visit displease the stern student. "The old man would make up his mind to enter, and quietly and cautiously open the door. Next he would protrude his head through the chink, and if he saw that his son was not angry but threw him a nod, he would glide noiselessly into the room, take off his scarf, and hang up his hat (the latter perennially in a bad state of repair, full of holes, and with a smashed brim), the whole being done without a sound of any kind. Next the old man would seat himself warily in

a chair and, never removing his eyes from his son's face, follow his every movement as though seeking to gauge Petinka's state of mind. If the son was not in good spirits, the father would make a note of the fact and at once get up, saying that he had 'only called for a minute or two,' that, 'having been out for a long walk, and happening at the moment to be passing,' he had 'looked in for a moment's rest.' Then silently and humbly the old man would resume his hat and depart with a forced smile on his face—the better to bear the disappointment which was seething in his breast, the better to help him not to show it to his son."

"Poor Folk" is clearly the soul-revealing maiden effort of a man who possessed more than any other modern novelist the capacity for infinite pity. There is no indignation in "Poor Folk," there is no abnormal psychology, no panacea. It is as clear an echo from the depths of humble life as is to be heard in all Dostoyevsky. The young writer has chosen his field: let others write of kings and principalities; his song will be the piteous wail of suffering humanity. "One would have but to see what is passing within those great, black, grimy houses of the capital and to penetrate within their walls for one at once to realize what good reason there is for self-depreciation and heart-searching. . . . Let us look at what is passing within those houses. In some dingy corner, perhaps, in some damp kennel which is supposed to be a room, an artisan has just awakened from sleep. . . ." Here is promise of Dostoyevsky's later works.

Needless to say, "Poor Folk" was a brilliant success. The magazines were now open to the author and he published much. He could not duplicate his first success; perhaps he wrote too fast; and if he did, perhaps it was from necessity, for while there was no lack of fame, the poor writer found fame alone an insufficient diet. He was wretchedly paid;

he lacked literally the necessities of life. Besides, he was the sort of man who is ordinarily called shiftless—unless he is a genius, in which case he is styled an idealist. Without any faculty for the practical affairs of life, Dostoyevsky was the easiest person in the world to cheat or defraud and repeatedly lost in foolish ventures what little money he had.

A reckless dreamer, he became interested in the communistic and extreme socialistic views of some Russian students and others in the city. Under the leadership of one Petrashevsky, they studied the communistic theories of Fourier, Proudhon, and Saint-Simon, and planned a radical reconstruction of society. But one April day, Dostoyevsky and thirty others were arrested by the police and imprisoned in the terrible fortress of St. Peter and St. Paul. After ten months of suffering, all of them were condemned to death. At the last moment the sentence of death was changed to eight years' penal servitude in Siberia and many years' subsequent exile. For five years at Omsk he was forced to share the company of thieves and cutthroats, to mingle with the veriest dregs of Russian society,—“murderers by imprudence and murderers by profession, simple thieves, masters in the art of finding money in the pockets of passers-by, or of wiping off no matter what from the table.” One doubts if heaven and hell ever came into such intimate contact on the face of this earth. In his “House of the Dead” Dostoyevsky narrates the story of his own exile under the transparent disguise of a noble, Goriantchikov, sentenced to ten years' hard labor for killing his wife through jealousy. “Such crimes are looked upon as misfortunes which must be treated with pity.”

Unutterable is the squalor of body and soul which Dostoyevsky depicts in this society of human beings buried alive. “In one single room we herded together, more than thirty

men. It was no wonder that we were shut up early. Four hours at least passed before every one was asleep, and until then there was a tumult and uproar of laughter, oaths, rattling of chains, a poisonous vapor of thick smoke; a confusion of shaved heads, stigmatized foreheads, and ragged clothes, disgusting, filthy."

Of all Dostoyevsky's books, "The House of the Dead" is perhaps the least sentimental: there is about this chronicle of prison life in Siberia a certain relentless objectivity, a certain calm massing of details which produces an impression of cumulative force and compels gripping terror, awe. Gorianchikov's companions are no banished heroes. "Those who were not already corrupt when they arrived at the convict establishment became perverted very soon. Brought together in spite of themselves, they were perfect strangers to one another. "The devil wore out three pairs of sandals before he got us together," they would say. Intrigues, calumnies, scandal of all kinds, envy and hatred reigned above everything else. "It would seem that during so many years I ought to have been able to notice some indication, even the most fugitive, of some regret, some moral suffering. I positively saw nothing of the kind."

What is a man of heart and cultivated mind, a man of delicate conscience, to do if destiny has thrown him for ten years' sojourn in this society? What he feels kills him more certainly than the material punishment, for he is daily his own judge. And alongside of him are men who have no conception of their crimes, men who even boast of their atrocious deeds, and creatures who enjoy themselves. Dostoyevsky has sketched this latter type in a way to make us ashamed of our humanity. There is the man who has been worked to death to enrich his master, whose whole life has been one sorry round of hungry stupidity. Here in

prison his work-hours are shorter, his food is better, and "the society one meets in the convict prison—is that to be counted for nothing? The convicts are clever, wide-awake people who are up to everything. The new arrival can scarcely conceal the admiration he feels for his companions in labor. He has seen nothing like it before, and he will consider himself in the best company possible."

But keener than the misery of squalid labor, deeper than the disgust, physical and moral, with one's environment, more deadening than all to Dostoyevsky was "the poignant and terrible suffering of never being alone even for one minute during ten years. Working under escort in the barracks together with two hundred 'companions'; never alone, never!"

The years pass one after another. "Man is a pliable animal," he says; "that would be perhaps the best definition that could be given of him." The years roll on, and "the winter so long, long prayed for is come, come at last." But while the prisoner has believed himself the same day by day, when the hour of departure comes there comes also the realization of the irremediable change which has affected his whole being. "One day I saw a prisoner, who had undergone his punishment, take leave of his comrades. He had had twenty years' hard labor. More than one convict remembered seeing him arrive, quite young, careless, thinking neither of his crime nor of his punishment. He was now an old man with gray hairs, with a sad and morose countenance. He walked in silence through our six barracks. When he entered each of them he prayed before the holy image, made a deep bow to his former companions, and begged them not to keep a bad recollection of him."

After his years of Siberian exile, Dostoyevsky, a noble, was put in the army as a private, and suffered indignities

for another five years before he regained his freedom. He returned to Russia prematurely old, a physical wreck; his nervous system shattered; an invalid, subject to epileptic fits due to a barbarous punishment inflicted on him during his exile. One should expect to find him a sworn enemy of Holy Russia. Not so Dostoyevsky. For this is the marvel: he went to Siberia a pessimist weary of life; he returned an optimist with the firmest belief in the inherent goodness of human nature, an idealist who believed that even his own terrible sentence had been a real good because it had opened his eyes to the wisdom and goodness of life, and who wrote the Czar a public letter of thanks for having exiled him; an optimist, not in spite of his sufferings, but because of them. What could be the meaning of this strange Russian paradox? Where does Dostoyevsky find his reasons for looking at life through such bright glasses?

Dostoyevsky thanks Siberia for saving his soul by revealing to him the essential goodness of human nature. But what goodness did he find in "The House of the Dead"? In this graveyard of souls buried alive are there any gleams of humanity? The sympathetic eye alone may occasionally discover the half-stifled but not dead capacity to respond to pity and Christian love. We find there Ali, the romantic child of Daghestan, a yataghan-wielder on general principles of fraternal obedience, and a dreamy, nostalgic soul. Goriantchikov teaches him Russian, using the New Testament, the only book not forbidden in the prison. When they reach the Sermon on the Mount the face of the highway robber lights up. "'Forgive those that hate you!' Ah, how divinely he speaks!"

Petrov is a man who "will assassinate any one for twenty-five kopeks simply to get himself a pint of vodka. On any

other occasion he will disdain hundreds and thousands of rubles." He steals Goriantchikov's Bible, which he sells for drink. And yet in the bath scene—which one wishes to quote entire in all its Dantesque gloom, so worthy of the admiration which it evoked in Turgenev—in the bath scene Petrov certainly shows himself human. Isaiah Fomitch's ambition to saturate himself with steam and outdo all others in heat endurance; the Christmas theatrical performance with its happy actors and its convict audience, grateful to the non-commissioned officers for their condescension in attending their show; and the Old Believer grandfather praying for the Orthodox Christians—these are all flashes of glimmering humanity and possible goodness which may atone for all the sad trickery displayed in the hospital—atone even for the unspeakable depths of moral degradation revealed in the episode, "The Husband of Akoulka."

A wounded, half-dead steppe eagle, brought to the prison camp by a convict, sullenly rejects any food that is offered him, waiting for death, refusing to be reconciled. He is ignored for two months, but at last he evokes something in the souls of this society of cutthroats.

"Let him die, but let him not die in prison," said the prisoners.

"He is not like us; he's a bird, and we're human beings."

"They threw him from the rampart on to the steppe. It was just at the end of the autumn, a gray, cold day. The wind whistled on the bare steppe and went groaning through the yellow, dried-up grass. The eagle made off directly, flapping his wounded wing as if in a hurry to quit us and get himself a shelter from our piercing eyes. The convicts watched him intently as he went along with his head just above the grass.



" 'Do you see him, hey?' said one very pensively.

" 'He doesn't look round,' said another; 'he hasn't looked behind once.'

" 'Did you happen to fancy he'd come back to thank us?' said a third.

" 'Sure enough, he's free; he feels it. It's *freedom*.'

" 'Yes, freedom.'

" 'You won't see him any more, pals.' "

And just as under the light of a genuine lofty emotion gleams of real humanity issue from the caverns of murderous crime, so even the mere promise of a generous love is sufficient to work miracles in the embittered and besotted depths of a lost soul. In "The Gentle Maiden," a grim money-lender, crushed by the contempt of others, an alien among his fellow-men, is exalted and transfigured by love. That he cannot adapt himself to his new state, that he ends in unwillingly causing the death of the woman he loves, is, after all, irrelevant. The fundamental fact remains: as he stands meditating beside the dead body of his gently proud wife, we know that to the end of his days he will never be happy again, but we know also that never again can he be the frozen soul he was before her misery had roused his pity and saved him from spiritual torpor.

The entire theme of that agonizing book, "The Idiot,"—agonizing in its unwieldy prolixity, agonizing in its insistence on morbid analysis of character,—what is its fundamental theme but precisely this: the depicting of a man who, lacking all except the fountain of Christian charity, compels men to yield to him, to obey, even to worship him in spite of his stupidity and repulsiveness? In the portrait of Prince Myshkin, Dostoyevsky seems consciously to have set himself to point out "the one thing needful." Myshkin is an epileptic; his inability to maintain his balance socially invites

the scorn of those with whom he comes in contact. He is the topic of pleasantries; intelligent, *comme il faut* ladies and gentlemen find his presence irritating and distasteful; he is freely insulted; he is even slapped in the face. But his very simplicity, the generous nature of his stupidity disarms hatred: one scorns him for not resenting an insult, but in time the outraged epileptic hero, who weeps instead of fighting duels, obtains his apologies freely. "Oh, how ashamed you will be of what you have done!" he exclaims in a breaking voice after Tanya, mad with rage, has given him a violent slap in the face. And he proves a good prophet. The idiot's docility, his simple-hearted sympathy breaks through walls of opposition which resist all the able arguments and fine rhetoric of cleverer men. He conquers the hearts of those whose intellects despise his own. "Where there is love, there is no need of wisdom."

Aglaia, who loves him in spite of herself, sums it up in genuine Dostoyevsky fashion: "I consider you the most honest and truthful of men, more honest and truthful than any one; and if they do say that your mind—that is, that you're sometimes afflicted in your mind, it's unjust. I made up my mind about that, and disputed with others, because, though you really are mentally affected (you won't be angry at that, of course; I'm speaking from a higher point of view), yet the mind that matters is better in you than in any of them. It's something, in fact, that they have never dreamed of. For there are two sorts of mind: one that matters and one that doesn't matter."

Nor is it only in lofty natures like Aglaia's that Myshkin produces this profound impression. This idiot who lacks the manner of the cultivated parlor sage, or anything imperious or impressive, this man who has epileptic fits, is capable of evoking in the miserable, shameless, venal Na-

stasya Filippovna a passion which maddens and destroys her, but a passion also which lifts her to a higher spiritual plane: she loves him too deeply to allow him to link his life to such a one as herself. So does generosity beget generosity. Love is the sovereign conqueror, says Dostoyevsky; no barrier of evil can withstand its gentle pressure. Pity and love open freely the most hopelessly locked hearts, open **them** to welcome and to be welcomed into the hearts of others. "In scattering the seed, scattering your 'charity,' your kind deeds, you are giving away in one form or another part of your personality, and taking into yourself part of another; you are in mutual communion with one another."

To understand adequately Dostoyevsky's philosophy of life, however, one must read his masterpiece, "Crime and Punishment." This book, published in 1866, immediately took all Russia, and later all Europe, by storm. No novel of Dostoyevsky's has achieved such a success. And, on the whole, there can be no doubt that it manifests his genius at its best. While his last unfinished novel, "The Brothers Karamazov," is a more colossal book, in its delineation of human character a work more typical of his manner, and in its scrutiny of the deeper reaches of human iniquity a more harrowing and perhaps a stronger book than "Crime and Punishment," it contains likewise some of the most exasperating examples of the unwieldiness, the prolixity, and the morbid psychiatry which poison so much of Dostoyevsky's art. Siberian tortures made Dostoyevsky an epileptic; the poor man had frequent fits all the rest of his life, and he took a morbid interest in his malady. He described all his symptoms to doctors, and naturally had an inquiring sympathy for all manner of nervous sufferers and lunatics. Holy Russia had thrown him in strange company, and the autobiographical leaning which is apparent

throughout his works led him to introduce into his novels the products of that unholy trinity of Russian life: the prison, the insane asylum, and the house of prostitution.

In "The Brothers Karamazov" we find ourselves in the darkest midnight of spiritual confusion and moral depravity. The plot of this eight-hundred-page novel turns on the murderous hatred between Fyodor Karamazov and his son Mitya, fanned by the passion, bestial in the father, tragic in the son, for the same Grushenka, a passion resulting in the death of the father and the consequent trial and conviction of the son. This is our theme. It is significant, too, that the main characters in this tragedy of human iniquity are a gallery of bodily, mental, and moral invalids; not one of them is healthy and normal. It is an immense book; it is more than a novel, it reminds one of "Les Misérables"; philosophical and religious digressions, while detracting from its artistic perfection, heighten its value as a human document, as in the life and thought of Elder Zossima and the conception of "The Grand Inquisitor," to mention only two. Through hundreds of pages Dostoyevsky pursues the souls of his characters, racking, probing, dissecting, an uncanny seer in a depressing world.

In point of morbid character analysis, "The Brothers Karamazov" is easily Dostoyevsky's masterwork—indeed, the masterwork of Russian literature. The portrait of Alyosha is an example of the ideal life as Dostoyevsky saw it, an example for which we shall hunt in vain through "Crime and Punishment"; while in the characters of the other three Karamazovs Dostoyevsky has sounded lower depths of human depravity than are disclosed in "Crime and Punishment." Yet one doubts if this straining of the extremes of human nature has yielded a book either of more compelling artistry or more profound philosophy.

Needless to say, the Russian masters do not lean to polite evasions in describing the frailties of man. Personally I should resent an expurgated Shakespeare as I should resent an expurgated Bible; it is not prudishness, therefore, which motivates my brevity in dealing with "The Brothers Karamazov." Turgenev, the purest of all the genuine masters of the novel, by no means turned his back on the seamy side of life. I doubt if in all Dostoyevsky, with his murderers and libertines and drunkards and lunatics and epileptics, there is to be found a character embodying the quintessentially diabolical in human nature to match Turgenev's Marya in "Spring Freshets," yet it would be quite possible to read Turgenev's "Spring Freshets" aloud from beginning to end before the most conventional audience. For Turgenev never crosses the barrier which separates the tragically evil from the repulsive.

In saying this I am not necessarily condemning Dostoyevsky. Of all the Russian masters, he is by all odds the strangest. He is an undoubted master, but one must cultivate a taste for his art. Now one of the objects of these lectures—perhaps the main object—is to stir interest in Russian literature, and I fear that if one begins the study of Dostoyevsky with "The Brothers Karamazov," one may go no further; while if one begins with "Crime and Punishment," one may perhaps be able to read even "The Brothers Karamazov." Aside from this, from the point of view of sheer literary art, in its construction, technical balance and unity, and spiritual artistry, "Crime and Punishment" is Dostoyevsky's best novel.

After what has just been said, it should be quite clear that if Dostoyevsky can look at life through bright glasses, he certainly does not limit his vision to the bright spots in life. Shame, degradation, squalid misery, and moral stench com-

prise his materials; and yet, in a manner half ghastly, half sublime, he sees and insists that we see the image of God in all this apparently diabolical creation. The hillside is not dew-pearled, as in Browning's "Pippa Passes." No larks are on the wing in Dostoyevsky, and yet the final refrain is the same:

"God's in his heaven,  
All's right with the world."

The hero of "Crime and Punishment" is a Russian student by the name of Raskolnikov. Melancholy, sensitive, kind and charitable to a fault, he lives in a dirty St. Petersburg hovel, in a room as big and as dismal as a coffin. His mother, a widow, living with his sister Dunia on a pension of a few rubles a year, still manages to scrape together some money for her son, whom they both believe to be making a success at the university. As a matter of fact, Raskolnikov has been obliged to leave the university for lack of funds, and, unable to find work and almost starved, has become hopelessly despondent. His whole outlook upon life is gloomy; he hates existence, and his despair is the deeper because he feels within himself gifts, intellectual and moral, which could benefit the world not a little if only a chance were given him to exercise them. This young man with fine sensibilities and unusual—indeed, keen—intelligence, is compelled because of his misery to associate with men who disgust him, to mix with the dregs of society, to soil his being with the dirt and soot and stench of the dismal city. To raise money on which to live he pawns, first, a gold ring which his sister has given him, and, later, his dead father's watch, to an old woman who thrives on poor students, a woman who squeezes out the last penny from penniless men.

And behold how the first seed of crime is sown in this

despondent, melancholy soul; how this sensitive, kind, generous student begins to harbor the dread idea. Just as Raskolnikov is drinking the tea he has bought with the old woman's money, he hears two other men discussing her. One of them says: "Here is a silly, flint-hearted, evil-minded, sulky old woman, necessary to no one—on the contrary, pernicious to all—who does not know herself why she lives. . . . A dozen families might be saved from hunger, want, ruin, crime, and misery—and all with her money! Kill her, I say; take it from her and dedicate it to the service of humanity and to the general good. What is your opinion? Shall not one little crime be effaced and atoned for by a thousand good deeds? For one useless life, a thousand lives saved from decay and death? One death, and a hundred beings restored to existence? There is calculation for you. What in proportion is the life of this miserable old hag? No more than the life of a flea, a beetle; nay, not even that, for she is pernicious. . . . She preys on other lives. . . ."

These words haunt the poor student: the old woman is nothing more than a vermin; she ought to be done away with. But kill her? How? He has no money with which to buy a weapon; he has only a hatchet at his disposal. Yet—kill an old woman with a hatchet? The idea is esthetically no less than morally repulsive; still, he cannot on that account get rid of it. He is feverish, dizzy with hunger, but his own misery cannot make him decide to commit the murder.

One already anticipates that Dostoyevsky is to have a murderer for his hero. But that is not enough: his heroine is a prostitute. Which is no shocking matter, after all, considering that the Christ whom Dostoyevsky worshiped did not scorn publicans and fallen women. The penniless student comes in contact with an old drunkard's family in

which the eldest daughter has been forced to become a street-walker for the sake of saving her own father's and her stepmother's children from starvation. Here is a girl who has sacrificed herself, soiled her pure being for the sake of a drunken sot father and a consumptive stepmother's children. Raskolnikov's moral nature feels no revulsion. He pities the girl, but scorn her he cannot. A time comes when he kneels before her: "I do not bow to you personally," he says, "but to suffering humanity in your person."

At his room he receives a pathetic letter from his mother, apprising him that his own beloved sister Dunia has consented to marry an undesirable man in order to obtain money with which to help him—Raskolnikov—through the university. The news compels his decision. He would do anything rather than let his sister wreck her life's happiness for the sake of money. If Dunia marries without love, she would be no different from the miserable drunkard's daughter, even though she remain respectable. Why should he allow his sister to kill her virgin soul for his sake? He would rather destroy the wicked, useless money-lender. Killing an old hag, he reasons, would be far less of a crime than condemning a young girl to an entire life of wretchedness.

By the merest chance he learns that at a certain hour the old money-lender will be alone in her flat—a most unusual thing for the suspicious woman. Raskolnikov decides not to miss his one opportunity. He goes to her rooms and kills her. Unfortunately, however, the money-lender's sister, Elizabeth, a good, pious woman, enters the fatal room just as the assassin is about to leave it, and the one premeditated and self-justified murder is followed by another murder, wholly unpremeditated, which shocks the sensitive student as much as the former had satisfied him. I will not urge you



through the sorry details of the tragedy. The fact is that this gifted, generous student has become overnight an atrocious murderer, the author of a crime which shocks even St. Petersburg and causes many arrests.

Raskolnikov manages to escape inquiry; he has himself, and only himself, to fear. But what a burden it is! With his unspeakable genius for horror, Dostoyevsky has portrayed the torments of this man who imagines himself suspected, the visions of agony, the ghastly dreams and hallucinations that torture this soul on the brink of spiritual disintegration, the delirium which gives him no peace, and the crafty cunning of his criminally insane brain, fighting a losing battle with itself, yet defying the whole world. His crime has brought him only suffering; it weighs upon him; it becomes an obsession, a fixed idea, which pursues him every minute of his life. Through hundreds of pages we follow the horrors of this man whom an unkind destiny and an elastic conscience have hurled into the depths of crime.

Finally Dostoyevsky leads us to an intensely dramatic situation. The student-murderer, who has committed a crime to save his mother and sister from servitude and dishonor, finds a true fellow-sufferer in the miserable daughter of the sentimental drunkard. He feels that her anguish must be similar to his, for in both cases a noble soul has been dragged into the mire. To Sonia, the street-walker, the student-assassin goes and confesses the crime, the secret which he has jealously concealed from his closest friend and from his family.

It is a literal marvel of genius, that description where the street-walker Sonia reads to the student-murderer Raskolnikov the chapter from the Bible about the resurrection of Lazarus. Lazarus, dead in the grave four days, was brought to life. There is hope, then, for the student-assas-

sin. But he must confess his crime; he must undergo the suffering it involves. That alone—only the truth—will purge his heart and soul and give him back peace in life. At first Raskolnikov would rather die than confess. "What crime?" he tells his sister, when she also has heard about it. "Is it a crime to have killed some vile and noisome vermin . . . a vampire living on the life of the poor? Murders of that kind ought to *make up* for many a crime!"

But his own life-agony compels him to do what his distorted logic refuses. Is it really repentance, is it fear, is it change of mind and faint-heartedness which lead the murderer to confess his crime? Or is it not perhaps the intolerable state in which he finds himself, the horror of the fixed idea which gives him no peace? He has confessed, and is sentenced to long years of exile and hard labor in Siberia. Sonia accompanies him there, and becomes a sister of mercy and grace to the wretched prisoners in that sepulcher of the living.

For a long time Raskolnikov is in despair. His life has no aim. How is it that in that gloomy, diseased spirit there finally gleams again the light of hope and happiness and new-born life? How is it that the new history commences: the story of the gradual rebirth of a man, of his slow, progressive regeneration and change from one world to another? What power accomplishes this miracle? Dostoyevsky answers: the power of suffering to purge the human heart, and the wondrous magic of charity and love.

These are the great doctrines, the main ideas which underlie all of Dostoyevsky's writings, which inspire his work, which make him an optimist in spite of the fact that few men have delved more deeply into the darkness, the slime, and the misery of life. In the first place, Dostoyevsky declares,—and that is what makes him essentially an

optimist,—in every life, no matter how evil and disgusting, no matter how hopeless to the superficial spectator, there is a germ of goodness, and therefore the possibility of regeneration, of purity and virtue. Go through the whole list of characters in his novels; you can scarcely find a single one that does not at times show the gleam of kindness, of virtue and goodness and light. Aye, even of that old beast Fyodor Karamazov Dostoyevsky writes: "People, even the wicked, are much more naïve and simple-hearted than we suppose." Man is not essentially bad, and even when he sinks in the mire of vice there is still hope. There is hope, but only through suffering. Voluntary suffering, willing expiation—this is the only cure for a shattered and diseased soul. And this is the second great idea in Dostoyevsky's mind, which led him, on his return from Siberia, instead of complaining of his sufferings, publicly to thank the Czar for sending him there! The third idea is that the power of sympathy, of charity and love can work miracles in any human heart, no matter how evil and degraded it may appear. "What is hell?" Father Zossima asks in "The Brothers Karamazov," and answers: "The suffering of being unable to love." It was suffering and sympathy and love which changed the student-murderer's outlook upon life until at the end he could exclaim: "What are now all the torments of the past?" All—even his sin and sentence and exile—appeared to him . . . as if they had not occurred, or were swept away. . . . Life, full, real, earnest life, was coming and had driven away his thoughts. Seven years—only seven years! They seemed now as seven days. . . . A new life is not given for nothing: it has to be paid for dearly, and only acquired by much patience and suffering, and great future efforts. . . ."

All his life Dostoyevsky taught these ideas, and this

Christian gospel of love and atonement is the dominant note in his novels. He was a Russian mystic with the Russian immense capacity for suffering and enduring, and with a Russian's defiant orthodoxy. The least profound study of Dostoyevsky's art, from the purely literary point of view, reveals his deficiencies. So intent is he on uttering his gospel in his novels that he neglects the artistic perfection of his work. He is prolix, he digresses to exasperation, he is careless alike in matters of style and in plot-construction; the most heart-breaking realist, he is also capable of maudlin sentimentality and reckless romanticism, and even of the cheapest sort of melodrama. Those who worship at his altars remind us that the prophet was poor, that he had to turn out as much work as possible to buy bread for his own and his brother's families, and was therefore not always able to revise and polish what he wrote. The fact remains, however, that his novels do show sadly the need of the blue pencil.

If his art is more profound than Gogol's, it is unrelieved by that kindly gleam of humor which makes "Dead Souls" immortal. He is more colossal than Turgenev, but not on that account a greater giant. He lacks alike Turgenev's faultless art and the penetrating, thousand-eyed vision of Tolstoy. All his life he measured himself with these two Titans, and his failure to come up to their level must have embittered his last years. That he *did not* come up to their level only a blind devotee of Dostoyevsky can deny. One has only to compare "The Possessed" with "Fathers and Children" and "Virgin Soil," or "The Idiot" and "The Brothers Karamazov" with "Anna Karenin," or "Poor Folk" with "Memoirs of a Sportsman," or "The Gambler" with "Smoke," to realize the gap which separates Dostoyevsky from his two contemporaries, a wide gap in spite of Dosto-

evsky's uncanny genius for sounding the depths of the human soul.

Dostoyevsky lacked the knightly nobility of spirit. He had to fight with hunger, a fight which does not always show the warrior at his best. The recent publication of his letters has produced in the fastidious *Spectator* "a disagreeable surprise." They reveal him, we read, as "a rather meanly egotistical nature, disagreeable, complaining, fault-finding, apparently without a trace of nobility. His perpetual topic is money. In fact, altogether, the book is of a sort to make one pray that the correspondence of Shakespeare is not lurking in some Jacobean cupboard, ready to spring upon a dismayed and disillusioned world." Against this clever but ungenerous estimate of a martyred spirit, behold Prince Kropotkin's no less candid but more illuminating, because more sympathetic, appreciation: "One pardons Dostoyevsky everything because when he speaks of the forgotten children of our town civilization he becomes truly great through his wide, infinite love of mankind—of man even in his worst manifestations. Through his love of those drunkards, beggars, petty thieves, whom we usually pass by without even bestowing on them a pitying glance; through his power of discovering what is human and often great in the lowest-sunken being; through the love which he inspires in us even for the least interesting types of mankind, even for those who will never make an effort to get out of the low and miserable position into which life has thrown them—through this faculty, Dostoyevsky has certainly won a unique position among the writers of modern times; and he will be read, not for the artistic finish of his writings, but for the good thoughts that are scattered through them, for their real reproduction of slum life in the great cities, and for the

infinite sympathy which a being like Sonia can inspire in the reader."

In the darkness and poverty which surrounded him, in the misery which pursued him to the grave, Dostoyevsky could see the gleam of a new life, a greater, purer life for Russia, a life whose foundation was human brotherhood, whose law was the law of charity and love. And when, in 1881, at the age of sixty, he finally collapsed under the unendurable strain of existence, all Russia felt the loss of a man she needed. It was a nation-wide sorrow. Forty thousand Russians followed Dostoyevsky's body to the grave—a procession of grief which extended for miles. And in Yasnaya Polyana, Count Tolstoy, who had not been personally acquainted with the dead novelist, but who nevertheless had been touched by his art and his ideas, experienced a genuine sense of bereavement. "I never saw the man," Tolstoy wrote, "and never had any direct relations with him, yet suddenly when he died I understood that he was the nearest and dearest and most necessary of men to me. Everything that he did was of the kind that the more he did it, the better I felt it was for men. All at once I read that he is dead, and a prop has fallen from me."

In the small company of nineteenth-century immortals, next to Russia's master-artist, Turgenev, next to the bravest apostle of an exacting gospel, Lyof Tolstoy, the world must reckon this long-suffering spokesman of Russia's millions, who endured much and yet saw in the dark shadows of life the light of human goodness; and the world must overlook the errors of the weak man and honor Fyodor Dostoyevsky.

## LECTURE IV

### COUNT TOLSTOY THE NOVELIST

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A TRANSLATOR of Turgenev, writing some thirty-five or forty years ago, mentioned several Russian authors of that day, among them Tolstoy, whom he described as "a writer of military stories." He suggested that perhaps some selections from Tolstoy's works might be translated into English, but that his novels in their entirety would not appeal to the English-speaking world. Yet Tolstoy's works have been translated into English, not once, but several times over, and for the last twenty-five years have made a deeper impression on mankind than the works of any of his contemporaries, Ibsen scarcely excepted. His estate, Yasnaya Polyana, was the Mecca of literary pilgrims. Tolstoy himself, "the grand *mujik*," as he has been called, stood somehow over and above our strenuous civilization—a prophet, a seer, a judge, whom mankind venerated even while criticizing. Dostoyevsky's novels barely kept the wolf from his door; his death did not impress the English-speaking world sufficiently to evoke even a notice in the English press. But the demand for Count Tolstoy's novels led a foreign publisher to offer him one million rubles for his copyrights; and when Tolstoy was dying at Astapovo no item of news was considered more important throughout the world. What can account for such astonishing influence?

Is it, perhaps, because human nature is enamoured of the unusual? The conventional, the mediocre produces little impression on us, nor do we feel drawn toward the perfectly

respectable; but the original personality is always sure to compel our attention, because of our normal demand for abnormal experiences. Mankind has repeatedly lost its heart to those who have defied and denounced it, who have dared boldly to deny its dominant conventions and principles of life. The sophisticated day of Athenian culture and Corinthian luxury, when the beauty-loving Hellenic soul sipped at superfine delights and weighed in delicate balance the slightest shades of bodily and mental pleasure, was just the right day for a Diogenes. The cynic of Sinope came to the Greek children of fashion and convention, and he spat at their artificial life. They vied with each other in gorgeous living and festivity; Diogenes fared on garlic and lived in the proverbial tub. They spent their life in a mad pursuit after social distinction and yielded admiration to success and power; Diogenes scorned all social honor as empty, and found his satisfaction in his own self-complete life. And yet few men in Greece commanded such respect as Diogenes. Even so the rich, sensuous Florentines turned to the scathing sermons of Savonarola. The formal, heartlessly intellectual eighteenth century was similarly shocked and conquered by Jean Jacques Rousseau's sentimental plea for a return to nature; and Rousseau's gospel was the more compelling precisely because it was the absolute negation of all conventional ideals. In all these cases the daring challenge of the prophet of the simple or austere or natural life had resistless power; for the world's ideal of life is like a pendulum and constantly oscillates between extremes.

This reflection may perhaps help us to appreciate the significance of Tolstoy's tremendous success as a writer and his profound influence as a prophet. At first sight the power and influence of this modern Diogenes are hard to understand. How is it possible for a man to command such uni-



versal attention when he negates the basic principles of our modern life? In an age of the most extreme universal struggle for self-advancement and conquest, Tolstoy preaches non-resistance; in an age whose great distinction is that of having worked the greatest wonders in the field of material achievement, he scorns material progress; in an age of the division of labor, he considers no man moral unless he produces for himself the necessities of life. Yet in spite of it all, he draws millions to himself. Not in spite of it all, some may say, but precisely because of it: Tolstoy appeals to us as he does just because he is so—one might almost say—perverse! Or again, perhaps because he is so genuinely in earnest, so human, and therefore such a puzzle.

When we come to examine Tolstoy's art, Tolstoy's gospel, and when we look into his life, we do feel puzzled. It is indeed not easy for us to understand his lifelong struggle with the problem of the meaning and worth of human existence. To us whose souls increase from year to year at so much per centum, and whose troubles and ideals are alike sordid, Tolstoy's spiritual trials and agonies seem abnormal, unreal,—so unreal that many of us even go so far as to doubt the sincerity of this Russian count who put on the peasant's blouse and went to the harvest fields to plow side by side with his one-time serfs. Tolstoy's writings are not the product of complacent leisure. Like Turgenev, like Dostoyevsky, Tolstoy also records in his works his own struggle with the problem of life. If his solution of life's problem strikes us as bold, we shall find the actual decisions he took in his own life much bolder,—and if they puzzle us, who shall say that Tolstoy is to blame? "When a book and a head strike against each other, and a hollow sound ensues, is the trouble always in the book?"

Lyof Nikolayevitch Tolstoy, like Turgenev, came from

the cream of society. Like Turgenev's ancestry, his also went back to the court of Peter the Great. Tolstoy's own father, Nikolai, had been prominent in Russian military history; his mother was born Princess Maria Volkonsky, and there are good reasons for believing that Tolstoy's own parents were in his mind when he told the romance of Nikolai Rostov and the Princess Maria Bolkonsky in "War and Peace." From the very start young Lyof was an uncommon boy. Keen mentally, but not over-diligent; suspicious, yet generous; morbid, with a certain inborn melancholy, and nevertheless a recklessly joyous and even mischievous youth, he must have been a boy of a thousand questions, the despair of nurses and tutors. Certainly he tells us as much in his autobiographic sketches, "Childhood," "Boyhood," and "Youth." Already, at the age of fifteen, he showed skeptical tendencies. The problems of life and death and the meaning of human existence troubled this unquiet spirit. He went to the University of Kazan, and, trying to do something unusual, entered the department of Oriental languages.

He was strong physically. When, in "Anna Karenin," he describes the physique of Konstantin Levin, we can well imagine the prototype. A vigorous body and a restless mind,—that was Tolstoy at the easy-going University of Kazan, the Mecca at that time of the gilded Russian youth with full purses and empty heads. Tolstoy found only tedium in the coarse jollity of his fellows. He did little work, but his soul was devoured with countless longings, taking the shape now of some ideal of perfect womanhood, now of some unrealized Utopia.

From the department of Oriental languages he changed to the department of law, but found it no less futile. He felt a sort of unreality in all academic scholarship. His pro-

fessors were fossils; they did not answer, they were not even interested in the life-and-death problems which engrossed him. They shut their eyes to him and his actual human questions, and they sing-songed their lectures on what happened a thousand years ago. And his fellow-students? What seekers after truth were these rakes who squandered their time and substance in indolence and revel and debauch? What spiritual light could be looked for from these card-fiends and midnight wanderers? The university disgusted Tolstoy, and he left it without bothering about his diploma.

The young count returned to his estate and for a while devoted his time to improving the miserable condition of his serfs. The aspirations of nineteen-year-old Prince Nekhludov, described in "A Russian Proprietor," are doubtless transcriptions from Tolstoy's own life. He had read with the greatest enthusiasm the twenty volumes of Jean Jacques Rousseau, wore a medallion with Rousseau's picture around his neck, and planned to lead a life close to nature. But, like Nekhludov, his altruistic dreams for his estate and serfs were unaccompanied by any knowledge of agricultural matters, and, as his enterprise failed, this sentimental pupil of Rousseau began imitating his master's manner of life instead of his teachings. Gipsy dancers and revelers, gamblers and roisterers became his daily company. He who had longed to become a saint turned a beast. Months passed in this way,—in riotous living which later necessitated the sale of the house in which he had been born to pay his gambling debts. The "Recollections of a Billiard-Marker" is a pithy record of the gambling Tolstoy's psychology: one can readily imagine Tolstoy anticipating for himself precisely such a contemptible end as Prince Nekhludov's in the "Recollections." But with the same objectivity with which Tolstoy paints his moral degradation and ignominy, does he deline-

ate also the deeper yearnings after nobility which stirred in his soul disgust for his riotous living.

Once more disenchanted, now with himself, he entered the army and went to the wild Caucasian country. Here in the untamed, unsophisticated life of nature he sought peace; and his Caucasian life and dreams are well reflected in his story "The Cossacks."

The hero, Dmitri Olyenin, is Tolstoy himself. Olyenin is a Moscow society young man who has squandered half of his patrimony, has never chosen any career or done anything. And yet he is not lacking in virtue so much as he is lacking in spiritual orientation. He feels in himself the fresh spontaneity of youth, and yet does not know in what channel of activity to invest it. His jolly life yields him no happiness; his long nights do not bring him contentment on the morrow. He craves contentment and happiness, but they elude him. "Hitherto the only object of his affection had been himself, and this was inevitable because he expected from himself nothing but what was good, and he had not as yet lost his illusions about himself." The love of others he has compelled, but his own heart is innocent of love's dominion. "Why have I never yet fallen in love?" he asks himself as his sleigh flies southward, away from Moscow's snows and sordid gaiety, flies southward toward the gleaming sun and winding Terek and smoking *auls* and the Cossacks and Tartars of the wild Caucasian mountainland.

And what a rare world is the world of these wild folk! Tolstoy's tale "A Prisoner of the Caucasus" possesses exquisite simplicity and directness, but there is more elemental intensity, more pervading atmosphere in "The Cossacks." Turgenev, in a burst of enthusiasm, called the story of Olyenin Russia's best novel. It reminds one of "Taras Bulba"; there is something of Gogol's savage magnificence

in the picture of old Uncle Yeroshka. This Terek land is a land in which money counts a good deal, but *jigit* bravery a good deal more. The Caucasian Cossack has no mean estimate of his own manner of life; "he regards himself as having attained the highest degree of culture," looks on the Cossack as alone worthy of the name of man, and affects to despise every one else." The former young lion of Moscow drawing-rooms finds himself in a land in which ragged and carelessly dressed, but richly armed bravos treat him, a prince and a wealthy serf-owner, with kind condescension, and where an untutored rustic belle, whose costume consists of precisely one garment, vouchsafes his elegance not one single glance.

"The Cossacks" is a novel with a Rousseauian theme. Tolstoy shows the transformation wrought in Olyenin's soul by the simple, elemental, nobly savage life of the Caucasian folk. Rousseau himself could not have drawn more magnificently the portrait of the man living close to nature: "Olyenin in appearance was an entirely different man. Instead of smoothly shaven cheeks, he wore a young mustache and a beard. Instead of the pale, unhealthy complexion of one whose nights are spent in dissipation, he showed a fresh and ruddy tan over his cheeks, forehead, and ears. Instead of a perfectly new black coat, he wore a dirty white *cherkeska* with wide lapels and carried a rifle. . . . His whole being breathed of health, happiness, and satisfaction."

But it is not enough for the Moscow-bred *junker* to don a Cossack costume in order to find happiness and meaning in life. The inner life of the Cossack is one of animal simplicity. When the intrepid Lukashka kills a Chechenetz, his conscience does not bother him in the slightest: they may kill him, he may kill them—it is part of life. Killing an enemy, stealing a horse, rioting, feasting and drinking, and

having luck, and excelling in all—these comprise Lukashka's life, these make him famous. Maryanna the virgin frankly admires this dissolute galliard. And Uncle Yeroshka's philosophy of life is easily expounded: "Every one has his own customs. But, in my opinion, it's all one. God made everything for man's enjoyment. There is no sin in anything."

The Cossack lives close to nature. Nature rouses in him no spiritual problems, because he has not yet attained the problem level. But to Olyenin life itself is a problem. He has come to the Caucasus to find meaning, genuine happiness in life. If nature is to exalt his life, it must do for him more than make it contentedly animal. Uncle Yeroshka may spend the night perched up on a tree-branch watching for wild boars, and his consciousness is blankly identical with that of the beast he hunts. But when Olyenin, with seven pheasants hanging from his belt, stretches himself on the ground in the damp, dark lair of the stag he is pursuing, he cannot become one with his environment. He cannot help philosophizing: "Here I, Dmitri Olyenin, an entity distinct from all others, am lying all alone, God knows where, in the very place where lives a stag, an old stag, a handsome fellow, and in a place, likewise, where no human being has ever been before, or thought of being. . . . Around me, flying among the leaves which seem to them like vast islands, the gnats are hovering in the air and buzzing; one, two, three, four, a hundred, a thousand, a million gnats, and each one of them is buzzing something for some special reason around me, and each one of them is a Dmitri Olyenin, an entity distinct from all the others as much as I am."

But how is a Dmitri Olyenin to conduct himself in the midst of this world of gnats and stags and Lukashkas and Maryannas in order to find true happiness? "How must I

live so as to be happy? . . . Man is endowed with a craving for happiness: therefore it must be legitimate. . . . Circumstances may make it impossible to satisfy this craving. . . . What cravings can always be satisfied independently of external conditions? Love, self-denial." Rousseau had sent the stags and gnats to raise the problem; Christ and Schopenhauer offered its solution.

Olyenin tries the altruistic path; he would sacrifice himself for others. But those whose contentment he would attain do not understand his motives. When he presents Lukashka with his horse, the Cossack brave suspects that he would bribe him for some purpose of his own. Olyenin, who respects and loves the simple *stannitza* folk, is alien to their affections; but they call the jovial rake Byeletzky "the little grandfather." Olyenin looks wistfully upon Maryanna's virgin simplicity, but Maryanna first becomes aware of his existence when, a bit tipsy at a party, he embraces her.

Olyenin's life is not so easily ordered as the life of a Lukashka. Lukashka's love for Maryanna is described in very simple language; it is a common enough emotion. But Olyenin's passion for the pink-shirt-garbed daughter of the shrewish Dame Ulitka rouses a riot of perplexities. When Lukashka is wounded in a scouting expedition, all the tenderness which Maryanna has ever felt for the child of Moscow vanishes in her wild outburst of passion for her savage gallant. To us Olyenin may be an exalted soul; to her he is contemptible, insignificant alongside of Lukashka. The love-story is ended; and—need we say it?—ended is also un-mixed Rousseauism.

Tolstoy had sought peace in the Caucasus. But wild, unmeaning peace began to tire him. The success of his writings, which in the early fifties began to appear in Nekrasov's *Sovremennik* ("The Contemporary") and caused his name

to be uttered with that of Turgenev, roused in him new ambitions. Elemental nature had not satisfied this spirit; the Crimean War roused his martial instincts, and so we find him fighting the Turks, first in Silistria, Bulgaria, and then at the siege of Sevastopol. His experiences during this terrible ordeal, Tolstoy has recorded in his three sketches, "Sevastopol in December," "in May," "in August."

War has been painted before and since in more majestic and in more terrible colors; the externals of martial life have been more tellingly delineated even by Tolstoy himself in "War and Peace"; but we may doubt if any one has succeeded in conveying the inner human atmosphere of warfare more hauntingly than Tolstoy has conveyed it in these Sevastopol sketches. He shows slight interest in the mechanics of war, it is war's spirit he would portray,—the spirit of war as it manifests itself in the manner and life of captain and sergeant and common soldier. Tolstoy possesses the uncanny power of picturing the psychology of a simple, innocent, lovable young man who comes to Sevastopol to serve his God and Czar, save his country, and win an order of merit, and who finds that to do this he must murder people and risk being murdered himself; the initial physical fear and moral recoil, the intoxication of ever-present danger, the hardening of soul by the intimate visitant Death, and the genuine and assumed stoicism which, in the life of the experienced officer and private, is so frequently accompanied by defiant daredevil gambling and joviality. Striking is the picture of the two Koseltzov brothers. The younger's eyes dim with tears because he imagines his brother is displeased with him, but on the morrow he must kill human beings callously, brutally, and gain the order of Anna or the order of Vladimir for his pains.

And, while we follow the reckless officer, the brutal offi-



cer, and the corrupt officer, and the young idealist who comes to Sevastopol to die for God and the Czar, we see and hear in the background the thousands who count only as so many privates, pawns on the chessboard of war. They live from day to day, they die stolidly without any philosophizing, jesting with death, thinking of a cross for bravery, perhaps, but much more thinking of their discharge. " 'You may say what you think, but when we've peace, we're sure to have an imperial review at Warsaw, and then, if we don't all get our discharge, we shall be put on the permanent reserve.' Just then a shrieking, glancing ball flew over the talkers' heads and struck a stone. 'Mind, or you'll get your discharge in full before to-night,' said one of the soldiers. They all laughed. And not only before night, but before two hours had passed, two of them had got their discharge in full and five more were wounded; but the rest went on joking just the same."

It is not war as glorious combat or noble self-sacrifice that Tolstoy portrays, but war in its terrible intensity, brutalizing, sordid war, war exalted in its inhumanity. He would tell the real inside truth about war: "The hero of my tale, whom I love with all the power of my soul, whom I have tried to portray in all his beauty, who has been, is, and will be beautiful, is—Truth."

Naturally the censor did not like this truth and blue-penciled the manuscript of the Sevastopol sketches in accordance with the demands of imperial Russia. But, even in their mutilated form, the stories, published in Nekrasov's *Sovremennik*, made Tolstoy the literary idol of Russia. Nekrasov wrote him: "Truth—in such a form as you have introduced it into our literature—is something completely new among us." Czar Nikolai even sent an order that special care be taken of the young writer and that he be re-

moved to a safe place, but Tolstoy insisted on sharing the dangers with his soldiers. Finally his commander ordered him to write an official account of the siege, and sent him with it as a messenger to St. Petersburg.

Here Tolstoy was received with ovations. Every favor St. Petersburg had to offer was at his disposal. Dinners, dances, fame, wine, women—all were his. But once more he turned away disenchanted. This effeminate, futile existence began to weary him. What was he supposed to be? A teacher of the people; a spiritual leader? So were all his fellow-writers. But what was he teaching Russia; what did he give in return for all this fame and money? He himself did not know. He turned to his fellow-writers, he tells us in his "Confession": "They disputed, quarreled, abused, deceived, and cheated one another. . . . Almost all of them were immoral men, most of them worthless and insignificant, and beneath the moral level of those with whom I associated during my former dissipated and military career, but conceited as only those can be who are wholly saints, or those who do not know what holiness is."

At that time the Czar Alexander II was already inaugurating the new era of liberalism, of reforms. The emancipation of the serfs was in the air; all Russia talked of the people's rights. But how were the Russian masses to be elevated? Tolstoy took his only trips to Europe—three of them—to study social and agricultural conditions, and in 1861 returned to his estate Yasnaya Polyana, freed his own serfs before the publication of the Emancipation Edict, and devoted his time to the improvement and education of his peasants. He established a school on his estate, teaching the peasant children on the theory that the child should have full freedom in his development; the teacher should not imprint his ideas and habits on the child, he

should assist the child only when assistance is demanded. This was a strange sort of education to advocate in a land like Russia. Holy Russia pestered Tolstoy until it made him ill; and while he went away to recuperate, his school was closed by orders from "on high," and thus ended Tolstoy's career as an educational reformer. On his way to Samara, where he was going to drink sour milk, he stopped over night at Moscow, and—backslid into a gambling party, with the result that he found himself with a debt of five hundred dollars. But the man who had beaten him, Katkov, was the editor of the *Russky Vyestnik*, and accepted as payment of Tolstoy's gambling debt the manuscript of "The Cossacks."

It was now the summer of 1862. Tolstoy was paying court to Sophia Behrs, and experienced the spiritual anguish which he has described in his "Confession," in the "Kreutzer Sonata," and in "Anna Karenin"; the anguish which the thoughts of his past debauches aroused in him when he thought of offering his life to a pure woman. But at least he would be honest, so just before he was married he asked his bride to read his diary, in which he had laid bare his whole life.

Sophia Behrs—or Sonya, as Tolstoy called her—made as helpful a wife as any Russian ever had. In the "Reminiscences of Tolstoy," one of the novelist's sons, Count Ilya Tolstoy, describes the daily life on his father's estate: "The chief personage in the house was my mother. She settled everything. She interviewed Nikolai the cook and ordered dinner; she sent us out for walks, made our shirts, was always nursing some baby at the breast; all day long she bustled about the house with hurried steps."

She bore Tolstoy thirteen children, five sons and three daughters of whom grew to maturity. But she was more

than a good mother and an efficient housekeeper; she was Tolstoy's literary assistant. And what a man to assist! Her son writes: "Leaning over the manuscript and trying to decipher my father's scrawl with her short-sighted eyes, she used to spend whole evenings at work, and often sat up late at night after everybody else had gone to bed. Sometimes, when anything was written quite illegibly, she would go to my father's study and ask him what it meant. But this was very rare, because my mother did not like to disturb him. When it happened, my father would take the manuscript in his hand and ask with some annoyance: 'What on earth is the difficulty?' and begin to read it out loud. When he came to the difficult place, he would mumble and hesitate, and sometimes had the greatest difficulty in making out, or rather in guessing, what he had written. He had a very bad handwriting and a terrible habit of inserting whole sentences between the lines, or in the corners of the page, or sometimes right across it. My mother often discovered gross grammatical errors, and pointed them out to my father and corrected them." Seven times over she copied the manuscript of "War and Peace."

These fifteen years, 1863-1878, were Tolstoy's most wonderful years as a creative artist. To these years we owe his two great novels, "War and Peace" and "Anna Karenin," two of the longest and greatest works of Russian fiction. Already in the fifties Russia looked up to Tolstoy for great things: Turgenev, for instance, wrote in 1854, "If heaven only grant Tolstoy life, I confidently expect he will surprise us all," and two years later Turgenev writes of him to the critic Druzhinin: "When this young wine has done fermenting, the result will be a liquor worthy of the gods." And now Tolstoy, for a few years at least, found inspiration in the life of his young and growing family. He writes in

his "Confession": "The new circumstances of a happy family life completely led me away from the search after the meaning of life as a whole. My life was concentrated at this time in the family,—my wife and children,—and consequently in the care for increasing the means of life. The effort to effect my own individual perfection, already replaced by the striving after general progress, was again changed into an effort to secure the particular happiness of my family."

He thought of portraying the spirit and the men of the uprising of December, 1825; but as he got deeper into his subject it grew; he went back to the conditions in Russia preceding the December uprising; his novel became vaster and vaster in scope until he realized that he was really writing an epic of Russia during the Napoleonic wars, an epic of Russian war and peace.

"War and Peace" is properly called an epic. It is scarcely a novel in the strict sense of the term; and if we judge it by the canons of art which we apply, for instance, to the novels of Ivan Turgenev, we shall do it scant justice. Turgenev's novels are artistically unified, balanced delineations of life; Tolstoy's "War and Peace" is limitless in its scope, with climax after climax, like a rolling steppe, endless in its expanse. Like Thomas Hardy's Napoleonic drama in nineteen acts, "War and Peace" is a texture of a dozen dramas of human life, twined and intertwined into an epic. If Turgenev's art reminds us of the exquisiteness and spirituality of Raphael, Tolstoy paints with a giant brush, paints all the world at once, like Michelangelo.

"War and Peace" does not center on the career of one man, or even of one family. Out of the hundreds that move before us and the hundreds of thousands who fill the background,—emperors, kings and princes and courtiers, and common, honest men,—the members of five families of

the Russian nobility stand out, and around their destinies the action of the novel turns: the families of the Kuragins, the Bolkonskys, the Rostovs, the Bezukhois, and the Drubetzkois. I must confess at the very start the utter hopelessness of any attempt I may make to summarize the plot of "War and Peace." Are we to take it from the military angle? But Tolstoy's action embraces several entire campaigns, and records not only the ascendancy and decline of Napoleon and his generals, but the military careers of Austrian and Prussian generals, more learned but not wiser than Russia's "Old Man Kutuzov." The battles of Schoengrabben and Austerlitz and Borodino, the Peace of Tilsit, the sacking and burning of Smolensk and Moscow, and Napoleon's tragic retreat from Russia determine the course of events in this novel.

Or are we to approach it from the romantic angle? And what a gallery of romances! There is Count Pierre's animal passion for Princess Ellen Kuragin,—Ellen with the marble-like shoulders, enameled by the gaze of a thousand eyes that had feasted on them. Her father, Prince Vassily, had failed to rob the young man of the enormous fortune to which he is about to fall heir, but what the father's cunning had failed to obtain, his voluptuous daughter easily grasps. And an infernal family life follows. There is the purely calculating, passionless romance of the clever climber Prince Boris Drubetzko for the meadows and forests of Julia Kuragin; again there is the love of Count Nikolai Rostov for his devoted Sonya, for the Czar Alexander, and finally for Princess Maria Bolkonsky, the richest heiress in Russia, whom he saves from the French invasion beyond Smolensk, becoming knight beyond compare in her eyes. When his father's estate shows him a bankrupt, Rostov refuses to ask for her hand and she practically has to do the proposing for him.

And there is Natasha, wonderful, inexhaustible Natasha

Rostov, Tolstoy's dearest woman, who sings her life through the entire scale of love in all its varieties,—loving Boris adolescently, just after she had left off loving her nurse; loving her dancing-master esthetically and Captain Denisov pitifully; devotedly worshiping Prince Andrei Bolkonsky, loving him with fear and trembling as one loves a god, and ready to throw her life in passionate abandon for Prince Anatol Kuragin, whom she loves as one loves a devil; poisoning herself and almost dying of a broken heart over the deceit of a shameless libertine, and all but withering away as she tries to nurse back to life the hero of Austerlitz and of Borodino; and, after all, meeting her real destiny in Count Pierre Bezukhoi and finding perfect and lasting happiness in worrying over her many babies. This love story, or rather this story of many loves, is unutterably beautiful, for truly Tolstoy has achieved a triumph in his portrayal of Natasha Rostov, who has made herself the heroine of a novel which has a dozen heroes.

But while the "grand passion" is revealed in all its shades in this Russian epic, we must leave the consideration of it for the study of the deeper ideas in "War and Peace,"—ideas which make this book more than a novel, just as Hugo's "Misérables" is more than a novel—a great human document. Of course it is needless to say that "War and Peace" is a work of art, not a sermon; Tolstoy wrote it before he turned to sermons. And yet "War and Peace" has a message,—it is a protest against what Tolstoy regards as a mistaken interpretation of history; against the view, namely, which we generally associate with Carlyle: the ordering of mankind by heroes and geniuses. In opposition to the hero and genius, Tolstoy points to the mass, to the millions, to the people. Tolstoy's "War and Peace," in its description of the great Napoleonic battles, is a constant proclamation

of this thesis. Napoleon, who had won every battle, lost the battle of Borodino. Why? "Many historians assert that the battle of Borodino was not won by the French because Napoleon had a cold in the head; that if it had not been for this cold, his arrangements before and during the battle would have displayed still more genius, and Russia would have been conquered and the face of the world would have been changed. . . . If it had depended on Napoleon's will to fight or not to fight the battle of Borodino, on his will to make or not to make such and such dispositions of his forces, then evidently the cold in his head, which had such influence on the manifestations of his will, may have been the cause of the salvation of Russia; and the valet who, on September 5, forgot to provide Napoleon with waterproof boots was the saviour of Russia."

But this is entirely wrong, as wrong as it is to believe that there is such a thing as a science of war, or that one man can really prearrange, not in details, but even in its most general outlines, the course that a battle will take. "No one can possibly know," Prince Andrei says, "what will be the position of our army and that of the enemy a day from now, and no one can know what is the force of this or that division. Sometimes, when there is no coward in the front to cry, 'We are cut off!' and to start the panic, and there is a jovial, audacious man there to shout, 'Hurrah!' a division of five thousand is worth thirty thousand, as was the case at Schoengraben; and sometimes fifty thousand will fly before eight, as happened at Austerlitz. Why did we lose the battle of Austerlitz?" Prince Andrei tells Pierre on the eve of the battle of Borodino: "Our loss was not much greater than that of the French, but we said to ourselves very early in the engagement that we should lose it, and we did lose it. And we said this because there was no reason



for being in a battle there, and we were anxious to get away from the battle-field as soon as possible. 'We have lost, so let us run,' and we did run. If we had not said this till evening, God knows what would have happened. But to-morrow we shall not say that. You say our position, the left flank, is weak, the right flank too much extended, . . . but that is all nonsense. It is not so at all. For what is before us to-morrow? A hundred millions of the most various possibilities, which will be decided instantaneously by the fact that either they or our men will start to run, this one or that one will be killed."

And in criticizing this view of war—and of human events in general—let us remember that Tolstoy fought at Sevastopol, fought not with the general staff, but in the trenches. The battle of Schoengraben was won for Russia by a captain who held his ground and disobeyed his orders,—and was almost court-martialed for his pains by the learned strategists who smelled the smoke from afar, while he was winning the battle for them in spite of their tactics.

Russia defeated Napoleon simply by keeping out of his way and letting him straggle along to his destruction. And therein lies the true greatness of Kutuzov, Russia's Grand Old Man, of whom this book is a eulogy. A plea for the masses, for the countless millions who are never taken into account, such is "War and Peace," and as such it already anticipates Tolstoy's later populism. Yet it is an immortal portrayal of Russia's aristocracy, too, in the days of the first Alexander. It is also an international novel, and in it Tolstoy has shown himself a master. Across his stage move French marshals, Austrian archdukes, German generals, and Russians of all varieties, and all true to life, typical bearers of their national characters.

Quite obvious it is, and quite interesting in view of the

present war, that Tolstoy is no worshipper of German wisdom. "The Frenchman," he says, "is self-confident because he considers himself individually, both as regards mind and body, irresistibly captivating to either men or women. The Englishman is self-confident through his absolute conviction that he is a citizen of the most fortunately constituted kingdom in the world, and because, as an Englishman, he knows always and in all circumstances what it is requisite for him to do, and also knows that all that he does as an Englishman is correct beyond cavil. The Italian is self-confident because he is excitable and easily forgets himself and others. The Russian is self-confident for the precise reason that he knows nothing and wishes to know nothing, because he believes that it is impossible to know anything. But the German is self-confident in a worse way than all the rest, above and beyond all the rest, because he imagines that he knows the truth,—the science which he has himself invented, but which for him is absolute truth!"

Already in "War and Peace" we see Tolstoy grappling with two great problems of a moral-religious character: the problem of the meaning of life and the problem of love and marriage. Count Pierre and Prince Andrei spend their whole lives in trying to find some meaning in life, some ideal which will illumine it and make it precious, worth while. And we notice that, while Pierre's marriage to Princess Ellen is the most piteous—indeed, the most wicked—mesalliance, Tolstoy would not let his hero love and marry the wonderful Natasha until after his wretched wife is dead. These two problems Tolstoy now takes up in portraying Konstantin Levin's spiritual struggle to find the meaning of human life and a worthy ideal, and the adulterous passion which works chaos in the lives of Alexey Vronsky and Anna Karenin. To this next novel, Tolstoy's masterpiece, we must now turn.

"Anna Karenin" is the greatest portrayal of adulterous love in the history of fiction. The heroine, Anna, is an honest soul whose virtue has made her famous in dissolute St. Petersburg; beautiful, deeply emotional, craving love, she has been married to Alexey Karenin, a high public official twice her age, who gives her a fine house and social prestige, but who is too much occupied with his career as a statesman and is of too cold a nature to give her what she desires above all,—plain, unqualified love. Alexey Karenin is the sort of man who, after saying a loving word to his wife, takes pains to preserve his dignity by laughing at himself lest she consider him sentimental. Another Alexey comes on the scene—Alexey Vronsky, dashing, brilliant, handsome, courteous, generous, passionate, a very knight of love, ready to throw his military career, his reputation—his all—to the four winds of heaven for her sake. And before very long Alexey Karenin finds that the wife he has been too busy to love is no longer his to love.

Adultery has never had so fine a chance to justify itself as it does in "Anna Karenin." At the beginning there is something volcanic—and something almost worthy, too—in this passion; it commands respect. What is it, then, that inevitably causes the tragic ruin of the dénouement? To be sure, "the greater number of the young women, who envied Anna and had long been weary of hearing her called 'virtuous,' rejoiced at the fulfilment of their predictions, and were waiting only for a decisive turn in public opinion to fall upon her with all the weight of their scorn. They were already making ready their handfuls of mud to fling at her when the right moment arrived." But Vronsky's social position is such that he can safely challenge society to insult him and his mistress. He is rich, he installs her in his home in all imaginable luxury. Anna's husband is ready to give her her freedom. Why, then, isn't she happy?

None of the hackneyed punishments would do for Tolstoy. It is not for men to punish those who have broken the law of God. "Vengeance is mine; I will repay," reads the motto of this novel. Anna is outwardly happy; she drinks the cup of love to the full,—and where could she find another lover like Vronsky! "But in dreams, when she had no control over her thoughts, her position presented itself to her in all its hideous nakedness. One dream haunted her almost every night. She dreamed that both were her husband at once, that both were lavishing caresses upon her. Alexey Alexandrovitch was weeping, kissing her hands and saying, 'How happy we are now!' And Alexey Vronsky was there, too; and he, too, was her husband. And she was marveling that it had once seemed impossible to her, was explaining to them, laughing, that this was ever so much simpler, and that now both of them were happy and contented. But this dream weighed on her like a nightmare, and she awoke from it in terror."

This is the punishment: Anna's own moral nature punishes her. Love has mastered her, and for the sake of love she has scorned all; but love has torn her spiritual nature in two. Love for Vronsky has led her to desert her husband, but the pure love for her son, which is the spark of God within her, keeps her from yielding completely to Vronsky, tears her soul, and makes her love a succession of the most intense happiness and the most unspeakable agony. The questioning, innocent eyes of her boy Seryozha haunt her: from the moment she has left her lover to visit her son, we know that she will never be happy. She hates her former husband for his forgiving spirit. "I have heard it said that women love men even for their vices," Anna exclaims to her brother; "but I hate him for his virtues. I can't live with him. Do you understand? The sight of him has a physical effect on me; it makes me beside myself. I can't—I can't

live with him. What am I to do? . . . Would you believe it that, knowing he's a good man, a splendid man,—that I'm not worth his little finger,—still, I hate him! I hate him for his generosity!"

But this sort of thing cannot last. Vronsky, who has been urging her to avail herself of her husband's offer, obtain her divorce, and marry him, at last begins to grow weary of the irregular relation. Or so it appears to Anna. She begins to be tortured by jealousy; she agrees to ask her husband to divorce her, but now he refuses. Anna is in despair; her groundless doubts of her lover, her growing irritability, her frenzied passion, which literally maddens her, complete the ruin which her adulterous passion had begun. Immediately after a quarrel with her, Vronsky has gone on a business visit. She writes him to come to her immediately. His delay in answering infuriates her; she will punish him, she says, in the only way she knows,—by killing herself. And the wheels of a railroad train under which she throws herself end the tragedy of her life.

Pestilential is illicit passion, Tolstoy tells us. But is human life truly blessed even by honest love alone? Can self-seeking love under any conditions supply men with the solution of the problem of life? Can it *by itself* make a spiritually sensitive man truly contented? This question Tolstoy attempts to answer in his story of the love and married life of Konstantin Levin and Kitty Shtcherbatzky, which forms the counterpart in this novel of the adulterous love of Alexey Vronsky and Anna Karenin.

The portrait of Konstantin Levin is autobiographic. There is, to be sure, something of Tolstoy's own character in the two main figures of "War and Peace," Prince Andrei and Count Pierre Bezukhoi; but Konstantin Levin utters in the pages of "Anna Karenin" the thoughts and dreams

which, a few years later, Tolstoy is to utter in his own name. Therefore it is essential to our understanding of this later Tolstoy—the Tolstoy from the year 1880 on—that we see what forces of spiritual unrest make this Tolstoyan hero unable to find perfect bliss in the love of a devoted, beautiful woman like Kitty Shtcherbatzky.

Levin is a wealthy landowner of an incurably serious turn of mind. He is not a saint; the book of his youth contains filthy pages of debauch; but his miserable memory of them is matched by an equally wretched consciousness of the futility of his outwardly blameless mature life. He is gloomy, not because he is vicious, but because he can see no meaning, no worthwhileness in a virtuous life. He is ambitious, and yet his every undertaking seems empty and futile. Needless to say, he scorns the silly fripperies of society; but even in the honest toil of his own farm he fails to find lasting satisfaction. Life is vanity to him. Life with Kitty Shtcherbatzky, he dreams, would be life worth living. Kitty respects him profoundly, but she loves Vronsky and refuses Levin. When Vronsky forsakes her for Anna Karenin, Kitty is almost killed by the humiliation, and even more by the realization of the sterling qualities of the man she has rejected. Kind fate intervenes, however, and in the course of time Levin and Kitty do marry.

The new life satisfies him for a while, even as married life contented Tolstoy himself; but the old questions and the old discontent return, due in part, though not entirely, to the inevitable frictions and quarrels of a married couple whose only fault is perhaps that they love each other too devotedly. But the death of his wretched brother and the birth of his child, two events which Tolstoy describes in an unforgettable manner, bring Levin face to face with the problems of life and death. What answer can he find to

those questions in the materialistic, so-called scientific beliefs which for him had replaced the simple Christian faith of his childhood? Like a man seeking for food in toy-shops and tool-shops, he reads over scientific and philosophical books, seeking an answer to the question: What makes life worth living? He abandons materialistic philosophy and reads and rereads thoroughly Plato, Spinoza, Kant, Schelling, Hegel, and Schopenhauer. But he can get no light, and his spirit is tortured. " 'Without knowing what I am and why I am here, life's impossible; and that I can't know, and so I can't live,' Levin said to himself. 'In infinite time, in infinite matter, in infinite space, is formed a bubble-organism, and that bubble lasts awhile and bursts, and that bubble is Me. . . .' And Levin, a happy father and husband, in perfect health, was several times so near suicide that he hid the rope that he might not be tempted to hang himself, and was afraid to go out with his gun for fear of shooting himself."

He takes good care of his peasants, he is honest in his dealings, he is true and loving to his wife,—but what is it all about, he asks himself as he watches his peasants working. " 'Why is it all being done? Why am I standing here making them work? What are they all so busy for, trying to show their zeal before me? What is that old peasant woman toiling for? I doctored her once when the beam fell on her in the fire,' he thought, looking at a thin old woman who was raking up the grain, moving painfully with her bare, sun-blackened feet over the rough, uneven floor. 'Then she recovered, but to-day or to-morrow or in ten years she won't; they'll bury her, and nothing will be left either of her or of that smart girl in the red jacket, who, with that skilful soft action, shakes the ears out of their husks. They'll bury her and that piebald horse, and very soon, too,' he thought,

gazing at the heavily moving, panting horse that kept walking up the wheel that turned under him. 'And they'll bury her and Fyodor the thresher with his curly beard full of chaff and his shirt torn on his white shoulders,—they'll bury him. He is untying the sheaves and giving orders and shouting to the women, and quickly setting straight the strap on the moving wheel. And what's more, it's not them alone,—me they'll bury, too, and nothing will be left. What for?' "

To some people all this may seem so much morbid introspection. Where ignorance is bliss, 'tis folly to be wise. But such bliss is vouchsafed only to fools and cattle. A calf may graze its way into cattlehood without being bothered by problems; but man is not a calf, although millions of men remain calf-like in their attitude toward life. Man must have a philosophy of life or perish spiritually. In addition to having hunger and thirst and other animal needs and the means of satisfying them, Levin is a man, and the problems which oppress his soul must be solved, or at least understood, else he cannot live.

A peasant accidentally enlightens him. He speaks of a friend of his who "lives for God, for his soul." To the peasant this seems perfectly clear, but the idea of it staggers Levin. What does it mean to live for God, for one's soul? Why is it that the ignorant peasant understands that which the learned landowner cannot? But an even greater puzzle confronts him: the peasant thinks that he, Levin, is also such a man, living for God, for his soul. Can it be that the peasant is right? Levin looks into his life. Could it be possible that, while his intellect has been making him miserable, deeper down in his soul truths too profound for words have been shaping the actual course of his life? "What did this mean? It meant that he had been living rightly, but thinking wrongly. He had lived, without being aware of it, on those



spiritual truths which he had sucked in with his mother's milk; but he had thought not merely without recognition of these truths, but studiously ignoring them. . . . What should I have been, and how should I have spent my life, if I had not had these beliefs," he asks himself,—“had I not known that I must live for God and not for my own desires? . . . I looked for an answer to my question. And thought could not give an answer to my question—it is incommensurable with my question. The answer has been given me by life itself, in my knowledge of what is right and what is wrong. And that knowledge I did not arrive at in any way; it is given to me as to all men, given because I could not have got it from anywhere. Where could I have got it? By reason could I have arrived at knowing that I must love my neighbor and not oppress him? I was told that in my childhood and I believed it gladly, for they told me what was already in my soul. But who discovered it? Not reason. Reason discovered the struggle for existence and the law that requires us to oppress all who hinder the satisfaction of our desires. That is the deduction of reason. But loving one's neighbor, reason could never discover, because it is irrational.”

It would of course be rank injustice to Tolstoy as an artist to say that he is preaching here. Tolstoy the novelist portrays life objectively and very often in direct opposition to what appears to be his own personal thesis. Thus the Russian critic Pisarev wrote of “War and Peace”: “The figures he has created have their own life independently of the intentions of the author; they enter into direct relations with the reader, speak for themselves, and unavoidably bring the reader to such thoughts and conclusions as the author never had in view and of which he perhaps would not approve.” Nevertheless it is quite clear to any intelligent reader of

"Anna Karenin" that here we have not merely a portrayal of men in spiritual struggles and anguish, but the portrayal of them by an author who is himself struggling and anguished spiritually. We know now what the men who read "Anna Karenin" as it was published serially in the seventies did not know—the struggle of life and death which was going on in Tolstoy's soul; a struggle between the artist and the man, a struggle of the man whose genius the world admired and approved, but who could not admire his own work because he was not sure that God approved it; indeed, was not certain there was any God to approve it, and found his life poisoned by the uncertainty. The masterpiece he was creating seemed to him paltry, futile. "Everything in it is beastly," he writes; "the whole thing ought to be rewritten, scrapped and melted down, thrown away and renounced. I ought to say I am sorry, I won't do it any more. What difficulty is there in writing how an officer fell in love with a married woman? There is no difficulty in it, and, above all, no good in it." Think of the spiritual state a man must be in if, capable of writing a masterpiece like "Anna Karenin," he is also capable of writing thus about it!

Tolstoy had gained the whole world, yet had not found his soul. He had a happy family and a well-managed estate of twenty-five hundred acres. The whole world applauded his art. Yet he contemplated suicide. There must be some way out of it. How he found this way, and how unreservedly he entered upon it, to the sacrifice of every other ideal, every other interest, is the story of the last thirty years of Tolstoy's life, and is one of the most impressive spiritual documents of modern times.

## LECTURE V

### THE GOSPEL OF TOLSTOY THE APOSTLE

AT the age of fifty Tolstoy looked back on his life and found it meaningless, a sorry jest, "a foolish and a wicked joke." "Vanity of vanities," Tolstoy repeated with Ecclesiastes and with Schopenhauer: the game is not worth the candle; life is a business that does not pay expenses; it is a tragic failure.

Yet Count Tolstoy was distinctly not a failure, as the world counts failure. On the contrary, he was a brilliant success. The descendant of a distinguished family, with an enviable military record, an honored country gentleman of excellent health, with a family of seven admiring children and a devoted wife of remarkable intelligence and efficiency, a wealthy man, admired by all the world for his literary genius,—what could he desire that was not at his disposal? Tolstoy is not to be reckoned among those who scoff at a success which they have failed to achieve; who scorn the puzzles they find too difficult to solve; who turn their backs on the world because the world has already turned its back on them. It was *after* he had won the worldly game that he found it not worth playing.

The more he saw of life, and the more he thought about life, the less satisfied he became. "What is the meaning of it all?" he kept asking himself. He had six thousand *desyatins* of land in the government of Samara, and three hundred horses. Suppose he had sixty thousand *desyatins* and as many horses,—what then? He was a famous writer. But suppose he became still more famous: "more famous than Gogol, Pushkin, Shakespeare, Molière, than all the writers

in the world—well, what then?" What was it all about? Why should he, Count Lyof Tolstoy, with his thousands of acres, healthy, rich, admired, loved, possessing all the things his heart could desire; *why* should he be living at all? "Is there any meaning in my life which will not be destroyed by the inevitable death awaiting me?" Science answered all questions but this, the most important. Experimental science refused even to entertain it; and, if abstract philosophy recognized it, it found it an insoluble puzzle.

"I could find no reply. Such questions will not wait; they demand an immediate answer; without one it is impossible to live; but answer there was none. I felt that the ground on which I stood was crumbling, that there was nothing for me to stand on, that what I had been living for was nothing, that I had no reason for living." While Russia marveled at the genius revealed in Tolstoy's portrayal of Levin's spiritual anguish in "Anna Karenin," that genius himself battled with the same problems and, despairing of finding an answer, contemplated hanging himself from the cross-beam of the very study in which he had been composing his masterpieces, and "ceased to go hunting with a gun because it offered too easy a way of getting rid of life."

And then occurred a most remarkable conversion—a Russian conversion, which robbed Russia and the world of a master-novelist, but gave us all—who can tell?—perhaps something even greater. A deep change came in Tolstoy's life. Not a sudden change, but rather the clear recognition of a truth which must have been lurking in his inner nature during his whole life, which sent him away in disgust from the University of Kazan, which made him loathe himself after his periods of dissolute living and gambling at Yasnaya Polyana, which appears in all his works and is revealed in all his great characters, in Dmitri Olyenin, in

Prince Andrei and Pierre Bezukhoi, and especially in Konstantin Levin. Let us follow Tolstoy as he awakes to the light which had long glimmered within him, as he discovers for himself and proclaims to all men the true meaning of life.

Sated with human vanity and success, rich, distinguished, he had nevertheless contemplated suicide. "Yet how do other people of my class manage to live?" Tolstoy asked himself. He found four ways out. The first way consisted in being ignorant of the fact that life is an absurdity, vanity, and evil: "Where ignorance is bliss, 'tis folly to be wise." That way was shut to Tolstoy, for he was already confronted with the problem of life. The second way out was to make the best of life as it is without thinking of the future. "But," says Tolstoy, "my imagination was too lively for that." The third way was the conclusion which the suicide draws, and this way Tolstoy understood and regarded as the worthiest, but for some reason he did not kill himself. The fourth way was to accept life as described by Ecclesiastes and Schopenhauer, and yet to live on, to wash, dress, dine, talk, and even write books. This position was revolting and painful to Tolstoy, but he adopted and maintained it.

"To see the inanity of life is a simple matter enough, and it has long been apparent to the simplest, but men have lived and still live on. Why is it that men live on?" Tolstoy asked again; and now he turned, not to his own class, but to the peasants. If life was unendurable to him, how could they bear it? They lacked the pleasures and comforts of the rich and the culture of the educated; their life was indeed a benighted and a hard life, yet they lived contentedly to a ripe old age. Tolstoy could not understand why those millions of human beings should endure their poverty when he found life in opulence intolerable. Surely, said Tolstoy, those

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peasants must possess something which I and my class, the wealthy landowners, do not possess. There must be a real meaning in life for those peasants, and in their humble, ignorant way they must be able to see that meaning, otherwise they would not live so contentedly on their bread and onions. The peasants told him they tried to follow the law of God; but what could be the meaning of that law? The skeptic unbeliever Tolstoy found that here was another question he could not answer. Was it indeed possible that the secret of life's meaning lay in the pious beliefs of the Orthodox Church, which he, early in life, had discarded as dark superstitions? His family of course were religious enough, and as a writer of Russian life he had always taken an objective interest in the faith of the masses. But now the possible truth of this religion became to him personally a matter of life and death. The peasants seemed to have the secret; he, Tolstoy, would pause at nothing, would sacrifice all, if he could only gain the peasant's peace of soul, if he could attain the sense that life is worth while and not a shallow mockery. This was the first step in his conversion. He turned his back on all his wealth, on all his aristocratic past, on all his learning, science, and philosophy; he went back to the old religion, determined to live the orthodox, pious life of the peasant and learn God's law.

One may wonder, perhaps, how it could have been possible for a deep, cultured thinker like Tolstoy to return to the crass superstitions and ritual of the Russian Orthodox Church. In spite of the superstitious character of the peasant's piety, however, Tolstoy could not help recognizing its self-forgetting character, which somehow lifted it above all his self-centered learning and modern culture.

But, though he was giving the old faith a new trial, the inspiration to follow the law of God, which he got from the

peasants, could not long blind his keen eyes to the benighted stupidity of the orthodox theology. The peasant has no intellectual demands, Tolstoy reasoned; his religion is one of unthinking devotion to God. But my devotion to God must not outrage my active mind. The peasant's theology may contain absurdities,—they are not absurdities to him, and one story is as good as another. But I cannot be contented as long as my mind is called upon to believe in absurdities. "My position was terrible," he writes. "I knew that from the knowledge which reason has given man I could get nothing but the denial of life, and from faith nothing but the denial of reason, which latter was more impossible than the denial of life. . . . If I went by faith, it resulted that, in order to understand the meaning of life, I should have to abandon reason, the very part of me that required a meaning in life!"

• But was it the peasant's belief, his theology, which gave him his peace of soul? Was it not rather his religion, his love of God, which his ignorant mind had translated in the terms of his superstitious theology? "The true office of any faith is to give to life a meaning which death cannot destroy. . . . Live to seek God, and life will not be without God." That same love of God, Tolstoy considered, which keeps the unthinking peasant orthodox, will lead me to understand perhaps more clearly the Gospel of Jesus. Thus my heart will worship God, and my intellect will honestly seek to understand His law. Accordingly Tolstoy turned with heart and soul to the critical study of the Bible, especially of the Four Gospels, and about the years 1880-81 we find him hard at work on his "Criticism of Dogmatic Theology" and on his own translation of the Gospels. His conclusion is that orthodox theology has distorted the simple, straightforward meaning of Christ's Gospel.

Now the fact that Church-Christianity misinterprets the Gospel of Christ was for Tolstoy no reflection on Christ's own teaching. The history of all religious faiths manifests the same degeneration, the same obscuring of their initial clarity in a fog of ritualism and misshapen theology; but to scorn the Bibles of humanity merely because men have distorted their message is utterly to miss the point. The Vedas, the Zend Avesta, the Old and New Testaments have given rise to superstitions because first of all they conquered the souls of men and changed their lives. But, Tolstoy maintains, Aristotle, Bacon, Comte never were and never will be subjected to superstitious distortions and excrescences precisely because they are insignificant, because they miss the truth of life, and can therefore never influence the mass of humanity. The true message of all great religions is this: "There is a God, the source of all; in man there is a particle of this divine element which he can either diminish or increase by his life; to increase this element man must suppress his passions and increase love in himself; the practical means to attain this is to act with others as one wishes others to act toward oneself." "True religion is the establishment by man of such a relation to the Infinite Life around him as, while connecting his life with this Infinitude and directing his conduct, is also in agreement with his reason and with human knowledge." Such is the true religion of Christ.

Before we can grasp this meaning, it is necessary that we recognize an idea which is plain to any one who reads the Gospels with an open mind. Jesus was not a theologian; His aim was to point out to man the way to God. It is not a dogmatic, intellectual, scientific doctrine about God which Jesus offers us; it is a new ideal of life. There is in all of us a spiritual nature, a sense of God and a love of God—we are all children of the same Father. But most of us are prodigal



sons; we have forgotten our divine origin and destiny, we have forsaken the home of our Father, and are wasting our substance in riotous living. Jesus would rouse this our dormant spiritual nature, inspire in us prodigals the desire to return to our Father; to find the meaning of life; to learn how we can live for God, for our souls. And Tolstoy devoted himself with double enthusiasm to finding what Jesus had to say about the life of the spiritually regenerated sinner, the man born anew.

Meanwhile his friends and admirers were troubled as they watched him forsaking literature and devoting himself to Bible study. His wife, the Countess, writes to her sister: "He reads and thinks till his head aches, and all to show how incompatible the Church is with the teaching of the Gospel. Hardly ten people in Russia will be interested in it; but there is nothing to be done. I only wish he would get it done quicker, and that it would pass like an illness!" And Turgenev, whose admiration for Tolstoy's literary genius was not affected by the fact that he could not get along with him personally, writes to his friend Polonsky: "It is an unpardonable sin that Lyof Tolstoy has stopped writing; he is a man who could be extraordinarily useful, but what can one do with him? He does not utter a word, and, worse than that, he has plunged into mysticism. . . . He has plunged headlong into another sphere; has surrounded himself with Bibles and Gospels in nearly all languages, and has written a whole heap of papers. He has a trunk full of these mystical ethics and of various pseudo-interpretations. He read me some of it, which I simply do not understand. . . . I told him, '*That is not the real thing*'; but he replied, '*It is just the real thing.*' . . . Very probably he will give nothing more to literature; or if he reappears, it will be with that trunk." And from his death-bed Turgenev wrote him one

last pathetic appeal not to forsake the art which he had glorified. The letter illustrates so clearly the great hopes which Tolstoy's conduct was shattering, and likewise Turgenev's own nobility of soul, that I cannot refrain from quoting it here in full:

"KIND AND DEAR LYOF NIKOLAYEVITCH:

"I have long not written to you because, to tell the truth, I have been and am on my death-bed. I cannot recover: that is out of the question. I am writing to you especially to say how glad I have been to be your contemporary, and to express my last and sincere request. My friend, return to literary activity! That gift came to you from whence comes all the rest. Ah, how happy I should be if I could think that my request would have an effect on you! I am played out—the doctors do not even know what to call my malady, *névralgie stomacale goutteuse*. I can neither walk nor eat nor sleep. It is wearisome even to repeat it all! My friend—great writer of our Russian land—listen to my request! Let me know you have received this scrap of paper, and allow me once more cordially to embrace you, your wife, and all yours. . . . I can write no more. . . . I am tired."

All these protests were futile. Some readers of Tolstoy would even add that they were superfluous. In the spiritual condition in which he found himself, Tolstoy could no longer look at life as he had looked at it when he was writing his famous novels,—objectively, as a painter or a sculptor looks at his model, trying to recreate it, whether it be a beautiful, fresh, goddess-like form or the shriveled body of a hag. To Tolstoy life and the portrayal of life meant something different now, and if he had kept on writing in violence to his convictions, his work would doubtless have shown his insincerity. As a matter of fact, he had not abandoned his art;

he had simply gained a new conception of his art, and if he did not produce another "Anna Karenin," he produced other things, in their way perhaps equally great. In a word, he was growing spiritually. He was not like his friend, the poet Fet, who, as Tolstoy puts it, wrote at the age of sixteen, "The spring bubbles, the moon shines, and she loves me!" and who went on writing and writing, and at sixty wrote: "She loves me, and the spring bubbles and the moon shines!"

Certain it is that, as far as his fame and his influence on the world are concerned, Tolstoy did not cease growing after 1880; and as to his literary art, his play "The Power of Darkness"—to mention only one example—exercised on European literature an influence quite equal to that of his novels. One does not need to make a literary apology for the creator of characters like Akim or of scenes like that of Mitritch and the ten-year-old Anyutka or the finale of the drama. A notable English critic called "The Power of Darkness" "the great modern play, the great play of the nineteenth century." But Tolstoy's chief aim now was not to portray life objectively; he had a mission, and that mission was to understand and proclaim Christ's ideal of human life. If his friends and former admirers thought that he had turned his back on life, it was because they, and not he, misunderstood life's meaning. So he writes to Fet: "I reject neither real life nor the labor necessary for its maintenance; but it seems to me that the greater part of my life and yours is taken up with satisfying, not our natural wants, but wants invented by us, or artificially inoculated by our education, and that have become habitual to us; and that nine-tenths of the work we devote to satisfying these demands is idle work."

The fallacy of human life is this, and this only: that the average man devotes all his endeavors, energy, and thought

to devising ways and means of self-gratification and self-aggrandisement. Man seeks his own interest, his own pleasures, his own power; man thinks he can never be so happy as when he can enforce his will on the will of all other men. This egoism, this lust for self-assertion and self-indulgence, sexual, economic, political, intellectual, is responsible for the evils in life; this egoism poisons the family life and the social fabric of our civilization, and it makes thought lead to cynical pessimism instead of yielding spiritual peace. "We pierce mountains, we fly round the world," Tolstoy exclaims in "Life." "Electricity, microscopes, telephones, wars, parliaments, philanthropy, the struggle of parties, universities, learned societies, museums,—is this life? The whole of men's complicated, seething activity, with their trafficking, their wars, their roads of communication, their science and their arts, is for the most part only the thronging of the unintelligent crowd about the doorway of life." Now it is precisely in pointing out and correcting this evil, this fallacy of egoism, that Christ's new conception of life consists. "Whoso saveth his life shall lose it. And he that loseth his life for my sake, the same shall find it." That is, "only by renouncing that which is destined to perish, our animal personality, shall we acquire our true life which will not and cannot perish. Our true life begins only when we cease to count as life that which was not and could not be our life—our animal existence."

Like Plato, Tolstoy exalts reason above the appetites; but, like Christ, he finds the essential activity of the higher nature of man, not in the theoretic sphere, but in the sphere of self-forgetting affection. "Life is the activity of the animal personality subjected to the law of reason. Reason is that law to which, for its own happiness, the animal personality of man must be rendered subservient." These words

from "Life" might have been quoted from the "Republic," but the conclusion of Tolstoy's paragraph is a New Testament idea: "Love is the only reasonable activity of mankind." Jesus had said, "Love thy neighbor as thyself." Confronted by the necessity of making a choice between his own and his neighbor's interests, Tolstoy would make Christ's dictum even more explicit and emphatic: "Love thy neighbor better than thyself." This is the gospel of Christ uttered in positive terms; its negative statement is the gospel of non-resistance.

To match the Decalogue of Mount Sinai, Tolstoy finds in the Sermon on the Mount five commandments which Jesus has stated with unquestionable clearness and simplicity, and which should be recognized as the foundation of the genuine Christian religion.

"Ye have heard that it was said by them of old time, Thou shalt not kill; and whosoever shall kill shall be in danger of the judgment; but I say unto you, That whosoever is angry with his brother shall be in danger of the judgment." This is a hard commandment,—so hard, indeed, that some wise theologians sought to improve on the words of Jesus by adding, after "whosoever is angry with his brother," the words, "without a cause," thus nullifying the force, and indeed the sense, of the whole passage. But Christ said simply: Anger in the heart is murder; be not angry. And when Jesus added the admonition against calling one's brother "Raca" or "Thou fool," he emphasized the moral claim which each man has upon us. We are not to excuse our anger and the evil we do to some men by saying that the object of our anger is a worthless or foolish man. "Treat every man always as an end, and never as a means only,"—to use the Kantian version of the same moral imperative.

The second commandment of Jesus stresses the spiritual

element in another department of life, and that one of the most intimate. Here again Jesus is perfectly clear: "Ye have heard that it was said by them of old time, Thou shalt not commit adultery: but I say unto you, That whosoever looketh on a woman to lust after her hath committed adultery with her already in his heart." This, Tolstoy says, is Christ's clear message; it condemns lust unreservedly. All the glorification of passion, be it veiled in never so beautiful a manner, is at heart lust and opposed to Christ's teaching. It matters little whether a union of passion is sanctified by church or society, or is in secret or in frank defiance of social and religious conventions: it is all the same so long as it is not transfigured by a motive nobler than the motive of self-gratification. If the basis of a marriage is pleasure, that marriage is adultery in God's eyes; it is surely damned. So Tolstoy writes to his son Ilya, who is about to be married: "If one marries in order to enjoy oneself, no good will ever come of it. To set up as one's main object, ousting everything else, marriage, union with the being you love, is a great mistake. . . . Object, marriage. Well, you marry; and what then? If you had no other object in life before your marriage, it will be twice as fearfully hard, almost impossible, to find one. In fact, you may be sure, if you had no common purpose before your marriage, nothing can bring you together, you will keep getting further apart. Marriage can never bring happiness unless those who marry have a common purpose."

It is doubtful whether any other Tolstoyan doctrine has suffered as much misrepresentation as this idea in which, following Jesus, Tolstoy denounces sensuality. A survey of the shelves of public libraries finds no other work of Tolstoy's so dog-eared and thumb-soiled as "The Kreutzer Sonata," a book which some self-complacent guardians of

the public weal have regarded as a menace to social morals, and which thousands of greedy readers have professed to treat as an attack on the family. "The Kreutzer Sonata" is a plain-spoken work, and on that account shares with Ibsen's "Ghosts" and Shaw's "Mrs. Warren's Profession" the features objectionable to those who tolerate and indeed find pleasure in the appeal of gorgeously veiled salacity, but for whom the undisguised portrayal of naked, hideous vice is anathema. Tolstoy shows himself to be an enemy of vice, whether commercialized or sanctioned by society; he is an enemy of divorce in any form, but of lifelong marriage loyalty he is no enemy. Those who find in "The Kreutzer Sonata" an animal conception of the marriage relation and an insult to the higher nature of man miss utterly Tolstoy's point.

In his advocacy of this idea Tolstoy pauses at no conclusion. A man who has possessed a woman only physically has killed in her and in himself the spark of divine life. That life cannot be resurrected by mere remorse or by any material restitution; moral union alone can atone for the hideous wrong inflicted. The only salvation from the sin committed in betraying a human soul into a union of lust is to win that soul back through a moral union of lifelong loyalty. This idea is the fundamental theme of the novel "Resurrection."

Prince Dmitri Nekhludov, serving on a jury, is thunderstruck when he sets eyes on the person of the prisoner Kate-  
rina Maslova, a prostitute, accused of poisoning a merchant; for she is none other than a woman whom he had betrayed years ago. Of the murder charge she is innocent, but through a technical error she is condemned to four years in Siberia. Nekhludov determines to save her, to repair the wrong he has done her, to marry her. Thus far he follows the Tolstoyan text, while the official whom he apprises of his

intentions reflects: "There is something abnormal in the young men of to-day."

But she who had lost her virtue with such tragic suddenness is not so suddenly reclaimed, now that she has dwelt in the gutters of vice until her whole soul is prostituted. When he meets her in the prison, and in an outburst of heroic repentance begs her forgiveness, she smiles luringly at him, considering how she can best use him, and ends by asking him for ten rubles. There is no immediate reconciliation; Tolstoy is too great a realist to paint any such sentimentally easy triumphs of virtue. Maslova hates Nekhludov for torturing her soul with memories which she had buried forever. "You've got pleasure out of me in this life, and want to save yourself through me in the life to come. You are disgusting to me—your spectacles and the whole of your dirty mug!" But his persistent, self-forgetting determination to atone for the wrong done her gradually melts the ice-caverns of that dreary soul; with imperceptible slowness the light of a new life begins to glimmer in the dim recesses of Maslova's being. After Nekhludov's third interview with her, the prostitute for the first time refuses to drink. "Well, shall we have a drop?" a fellow-prisoner asks her. "You have some," she answers. "I won't."

The struggle is long—it is at first a struggle between the prostitute and the long-buried woman in Maslova's soul. But as the prisoners approach Siberia a new struggle begins; a struggle in which new-born love for Nekhludov battles in her heart with a higher emotion, an emotion similar to Nekhludov's own feeling toward her, a passion of self-abnegation. For her sake, to atone for the wrong he had done her, to lift her, save her, make her life bright, Prince Nekhludov had negated his own aristocratic existence and forgotten all thoughts of self. The time will come when she will rise to



equal heights of self-forgetting devotion. Valdemar Simonson, who has fallen in love with her, asks her to marry him. She accepts him. "By going with Simonson, she thought she would be setting Nekhludov free, and felt glad that she had done what she meant to do; and yet she suffered at parting from him." Could one protest that this conclusion of the novel does not adhere literally to the second commandment of Jesus as formulated by Tolstoy?

The third commandment of Jesus affects the political sphere. "Ye have heard that it hath been said by them of old time, Thou shalt not forswear thyself, but shalt perform unto the Lord thine oaths; but I say unto you, Swear not at all. . . . But let your communication be Yea, yea; Nay, nay." This Tolstoy interprets as Christ's attitude toward government. Keep your spiritual freedom, Jesus says. God alone is your King. Do not today pledge yourself absolutely to duties and alliances of which your better conscience may not approve tomorrow. Do not surrender to another man the right to act at any future moment in accordance with your best light. Patriotic loyalty, one's oath to one's king, have led millions to kill each other in senseless wars. "The snare is in the use of God's name to sanction an imposture, and the imposture consists in promising in advance to obey the commands of one man, while I ought to obey the command of God alone. I know now that the most terrible evil in its consequences—murders in war, imprisonments, capital punishments—exists only because of the oath in virtue of which men make themselves instruments of evil, and believe that they free themselves from all responsibility. . . ."

The fourth commandment is: "Ye have heard that it hath been said, Thou shalt love thy neighbour, and hate thine enemy. But I say unto you, Love your enemies." Jesus here means to say, You have heard that love of your own

people, of your own country,—in a word, patriotism,—is good; but I tell you, love those of other nations, love all men. Elizabeth Barrett Browning has expressed her idea of true patriotism in a way quite Tolstoyan: "I confess that I dream of the day when an English statesman shall arise with a heart too large for England, having courage in the face of his countrymen to assert of some suggested policy: 'This is good for your trade; this is necessary for your domination; but it will vex a people hard by; it will hurt a people farther off; it will profit nothing to the general humanity; therefore away with it! It is not for you nor for me.' When a British minister dares speak so, and when a British public applauds him speaking, then shall the nation be glorious, and her praise, instead of exploding from within, from loud civic mouths, will come to her from without, as all worthy praise must."

But even more boldly Tolstoy writes: "I know now that my unity with others cannot be shut off by a frontier, or by a governmental decree which decides that I belong to this or that nation. I know now that all men are everywhere brothers and equals. When I think now of all the evil that I have done, that I have endured, and that I have seen about me, as the consequence of national enmities, I see clearly that it is all due to that gross imposture called patriotism and love for one's native land. . . . I understand now that true welfare is possible only on condition that I recognize my unity with the whole world. I believe this, and this belief has changed my estimate of what is right and wrong, important and despicable. What once seemed to me right and important—love for my country, love for my own nation, for my empire, services rendered at the expense of other men, military exploits—now seem to me repulsive and pitiable. What once seemed to me shameful and wrong—renunciation

of nationality and the cultivation of cosmopolitanism—now seem to me right and important.” And after the Russo-Japanese war, Tolstoy wrote a letter to a Japanese in which he signed himself: “In spite of all external differences,—your loving brother, Lyof Tolstoy.”

The fifth commandment, which I have reserved for the last, is to Tolstoy the most fundamental of all, and is the keystone of Christ's moral edifice. In his “Confession” the sudden realization of the importance of this command is compared to the finding of the central, important fragment of a broken statue about which all the other fragments can be assembled, each fitting into its proper place and all forming a unity. Thus the understanding of the fifth commandment becomes for Tolstoy the key with which he unlocks the ethics of Jesus and finds it to be a consistent, divine message of love to mankind:

“Ye have heard that it hath been said, An eye for an eye, and a tooth for a tooth: but I say unto you, That ye resist not evil: but whosoever shall smite thee on thy right cheek, turn to him the other also.” “Do not use force,” Tolstoy understands Jesus to say. Obviously no one will object to this commandment in so far as it involves using force wickedly; but what if one seeks to do good,—save a poor child from the attack of a drunken brute, or punish a criminal? Is the use of force even then to be condemned? Yes, Tolstoy maintains; that is exactly what Jesus means to say. What are his ideals? Love your neighbor as yourself; let your light shine before men. But if you have used force in compelling a bad man to desist from doing evil, have you made him less wicked? His own heart may be doubly full of hatred for you and for all men because of your use of force. Nor have you made yourself more Christlike when, in using violence, you have only allowed force and anger to

supplant love in your heart. What have you accomplished, then, by your use of force, if it has made the wicked man no better, and has made you worse? You have made the evil-doer externally safer, but can you be satisfied with this? Have you not insulted God's image in your fellow-man when, professing to save his soul, you have begun by endeavoring to make him harmless, thus treating him as a beast? But, you say, when I have once made him safe, then I can with security try to save his soul. This is mockery, Tolstoy says. An evil is an evil. You cannot get love out of hate; and where love is, there God is also. By force you can cow an evil-doer into submission, but you can get him freely to leave his evil ways and to follow God only by love. Do not under any conditions resort to violence; resist not evil. Only love can beget love. And, however we may extol justice, our law-courts and prisons and police systems do not have love as their basis: they are compounded of hatred, which is sometimes called righteous indignation; of the spirit of revenge, which is styled justice; and of the selfish desire of security for ourselves, which is collectively magnified into the virtue of public safety. But Tolstoy declares, as Dostoyevsky had declared before him, and as Jesus above all declared: "If any man would go to law with thee, and take away thy coat, let him have thy cloak also."

It would be easy for us to take this doctrine of non-resistance to pieces, to show that a literal observance of it would undo the work of our entire civilization; that in a world in which knaves and idiots abound the use of force is indispensable; that by stretching the meaning of some Gospel passages we can show that even Jesus believed in the use of force. All this would be contemptibly easy, but it would only illustrate the truth that the letter killeth. Christ is not to be refuted by being proved impracticable; for it is

precisely against the worship of the brutally practicable that Christ revolted. Certainly this modern egoistic world is practicable; certainly it is practicable to imprison and to exile to Siberia gangs of men whom we haven't reformed or who haven't reformed us. Certainly it is practicable to employ the very best years of a nation's manhood in training men to kill other men similarly trained. Certainly it is practicable to make ninety-nine persons in a hundred labor in order that the remaining one may be kept in luxurious idleness. There is nothing impracticable in assuring the innocence and safety of our own sisters and daughters and at the same time providing ready means for gratifying our passion by licensing the prostitution of the daughters and sisters of the poor, who do not count. All these things are eminently practicable; but they are not on that account the less wicked. On the other hand, Socrates was decidedly impracticable when he preferred drinking poison to renouncing his convictions. And Jesus,—what was there practicable about his allowing himself to be crucified and thus have his doubtless promising career cut short at the early age of thirty-three? Perhaps he should have adapted himself to the actual world in which he lived, fought the world with its own weapons, just as most of us would have done, and have become high rabbi of some synagogue or some weighty Roman dignitary, instead of remaining merely the Saviour of Mankind.

"We may declare that the universal practice of such a rule is very difficult; we may deny that he who follows it will find happiness; we may say with the unbelievers that it is stupid, that Christ was a dreamer, an idealist who propounded impracticable maxims which his disciples followed out of sheer stupidity: but it is impossible not to admit that Christ expressed in a manner at once clear and precise what he wished

to say; that is, that according to his doctrine a man must not resist evil, and consequently that whoever adopts this doctrine cannot resist evil. And yet neither believers nor unbelievers will admit this simple and clear interpretation of Christ's words."

Tolstoy likewise is not an efficiency expert: he is a prophet of ideals, and an ideal is not to be estimated necessarily in terms of its expediency. Man's performance is always found to fall short of his ideals; and if at least in our ideals we cannot rise in aspiration above our sordid performance, then we are still children of darkness. Tolstoy's gospel, like that of Christ, is a revolt against the merely expedient; and whether we follow him or not, we must at least understand what the good man is about. This worship of expediency Christ came to upset and to put in its place the worship of God, who is not the ideal of efficiency but of holiness, who does not prudently remain with His ninety-nine sheep that are safe in the fold, but goes to hunt for the one lost sheep.

The practical, expedient philosophy of life, on the other hand, is in the eyes of Tolstoy responsible for our nasty world. This is why a few men and women die of idle banqueting, and a good many more of starvation. Tolstoy's "slumming" experiences in Moscow abundantly proved to him the impossibility of saving men from squalor and degradation merely by giving them money. Not all who are in the gutter find life there intolerable: therein is the first problem; and the second is that those who do find gutter life and slum life wretched are experiencing a misery which is within themselves, "a misery not to be mended by any kind of bank-note." Condescending alms-giving cannot cure the ills of poverty, for it cannot save as many men from poverty as are daily made poor by the luxury of the rich, which luxury not

only impoverishes the masses, but also corrupts them, rousing in their souls greed and envy and distorted notions and dreams of happiness.

If poverty is not to be cured with bank-notes, even less does moral degradation yield to this superficial treatment. Here is a corrupt woman about to sell her thirteen-year-old daughter into a life of shame. Can this girl be saved from her fate by the police, or by kind, charitable society ladies? Tolstoy answers: "It was possible to take this girl away from her mother by force; but to convince that mother that she was doing wrong in selling her daughter was not possible. It would first be necessary to save this woman—this mother—from a condition of life approved by every one, and according to which a woman may live without marrying and without working, serving exclusively as a gratification to the passions. If I had thought about this, I should have understood that the majority of those ladies whom I wished to send here for the saving of this girl were not only themselves avoiding family duties and leading idle and sensual lives, but were consciously educating their daughters for this very same mode of existence. One mother leads her daughter to the inn, and another to the court and to balls. But the views of the world held by both mothers are the same; to wit, that a woman must gratify the lusts of men, and for that she must be fed, dressed, and taken care of. How, then, are our ladies to reform this woman and her daughter?"

These are hard words,—words which make one shrink with dismay; but is there more of error than of truth in them? The pursuit of pleasure and sensual enjoyment and idle luxury are not repellent to us only because we are intoxicated with the wine of wealth and do not realize the horror and duplicity of our lives. Consider, you good people who go to balls and brilliant receptions, Tolstoy exclaims in one

of his books; bethink yourselves—what are you about? Here are a hundred women at a royal ball. “Each of these women wearing one-hundred-and-fifty-ruble dresses has doubtless lived in the country and seen peasants, and knows her nurse and her lady’s-maid who have poor fathers and brothers for whom to earn one hundred and fifty rubles to build a hut is the aim of a long and laborious life. She knows this; then how can she make merry, knowing that at that ball she carries on her bared body the hut which was the dream of her good maid’s brother? But granting that this may not have struck her—the fact that velvets, silks, sweets, flowers, laces, and dresses do not grow of themselves, but are made by people, is one which it would seem she could not but know. One would think she must know what kind of people make these things and under what conditions they make them, and why.”

But what is to be done? For this is the very title of the book from which I am quoting,—“What Is To Be Done?” Stop thinking of yourselves, Tolstoy says, stop thinking all the time of your needs, your desires, your pleasures, your so-called cultural demands, and think of your fellow-men! Still, you persist, what am I—I personally—to do? “People will go on buying and hiring, whether I do or not, and will buy and compel others to make velvets and sweets and cigarettes; and will go on hiring people to wash shirts even if I don’t. Then why deprive myself of velvets and sweets and cigarettes and clean shirts, since things are so arranged? . . . What difference will it make if I wear my shirts a week and make my cigarettes myself or give up smoking? This difference: that some washerwoman or cigarette-maker will strain her strength less, and the money I should have paid for the washing and cigarette-making I can give to that washerwoman or even to quite other washerwomen and



workers who are weary of work, and who, instead of working beyond their strength, may then rest and get tea."

"But I hear in reply (so reluctant are the rich, luxurious people to understand their position) : Even if I did agree to wear a dirty shirt and not to smoke, but to give the money to the poor instead, it would still not save the poor from being bled of all they possess, and my drop in the ocean will not help matters. . . . If I went among savages," Tolstoy answers, "and they treated me to tasty cutlets, and the next day I learned, perhaps saw, that these tasty cutlets were made of prisoners who had been chopped up to make them; then, if I considered it bad to eat people, however tasty the cutlets might be, and however general among those with whom I am living might be the custom of eating men, and however little the prisoners kept to serve as food might gain by my refusing a cutlet, still I should not and could not eat any more of them."

The Chinese say : If there is one man idle, there is another dying of hunger. This problem is quite simple and is made complicated only by those who do not wish to solve it. We can invent more and more efficient wage-systems, and more practicable methods of organized charity; we can keep a hundred poor people employed serving rich wines and viands on our table, and then allow one or two of them to feed on the crumbs, and count ourselves philanthropic. All this is hollow mockery. "If a horseman sees that his horse is tired out, he must not remain seated on its back and hold up its head, but simply get off." Feed the horse, to be sure, Tolstoy says, but first of all get off the horse's back! Make sure, above all, that in your own personal life you are not enslaving the life of some other man. Hurt no one, but, so far as lies in your power, help! Make other men work for

you as little as possible, and work as much as possible for yourself.

The realization of this truth of life Tolstoy compares to the experience of a man who, having started on a certain errand, finds out that it is useless and turns back home. "What was at first on his right hand is now on his left, and what was on his left hand is on his right." Heretofore he had thought only of himself, of his family, his class, his nation; now he will think of others, other men's families, the other classes of society, the other nations. Life will no longer be for him a bill of fare, but a call to service. No longer will he ask himself, How much can I get out of men? but, How much can I give to them? True charity, of course, is to be the goal; but before I can willingly help any one I must first of all be sure that I am not forcing some one else to serve me unwillingly. "It is true that all our interests are interwoven, but each man's conscience tells him without much reckoning to whose credit goes the work, and to whose the idleness. And not conscience alone tells one this: it is most clearly told by one's cash-book. The more money a man spends, the more work he obliges others to do for him; and the less he spends, the more he works."

The conclusion of Tolstoy's reasoning is clear. Men suffer and are depraved because some men are in bondage to others. Therefore, until this initial cause of misery is removed, all other remedies are futile. "If I wish to help the poor—that is, to make the poor cease to be poor—I ought not to create those same poor." "I go to help the poor. But of the two who is the poorer? No one is poorer than myself. I am a weak, good-for-nothing parasite, who can exist only under very peculiar conditions, who can live only when thousands of people labor to support this life which is not

useful to any one. And I, this very caterpillar which eats up the leaves of the tree, wish to help the growth and the health of the tree and to cure it."

This is the simple truth of the matter, according to Tolstoy, much as men try to evade it by vain philosophizing. Malthus would explain the misery of the poor in terms of some unchangeable laws for which no one is to blame, unless it be the starving working-people themselves. "Why do these fools come into the world when they know they will not have enough to eat?" Comte would describe humanity as an organism of which some people are presumably the lofty head and dainty palate, and others inevitably the weary, blistered feet, trudging along, supporting the whole. Hegelianism owes its initial success, Tolstoy believes, not so much to the harmonious perfection of its system as to this: that its explanation of the world and our life allowed men the opportunity of saying, "All is reasonable, all is good; nobody is to blame for anything." The advocates of "the division of labor" write as if, in uttering the charmed phrase, they have exhausted and solved the knotty problem of life, as if the further question did not yet remain, "Whether the now existing division of labor in human society is that division which ought to be."

Those who choose "mental and spiritual labor" demand as their due that, before yielding intellectual fruit, they be given, as it were on credit, the fruits of the physical labor of others. But what if every workman should say: "Before I go to work to prepare bodily food for you, I want the fruits of the spirit. In order to have strength for laboring, I require a religious teaching, the social order of common life, application of knowledge to labor, and the joys and comforts which art gives. I have no time to work out for myself a teaching concerning the meaning of life,—give it to me!"

Is the division of labor genuine? Does the scientist, the novelist, the poet, the musician, the artist serve directly the spiritual needs of the workers who directly satisfy his physical needs? When a scientist makes a catalogue of a million beetles, when an artist paints opulence, when a poet indulges his sophisticated fancy, and all consume the product of the peasants' work, is there a real, actual exchange of labor in the process? To say nothing of the purity—*i.e.*, uselessness—of science which so frequently is its boast,—“tell a painter to draw penny pictures, tell a musician to teach country-women to sing songs, tell a poet to throw aside his poems and novels and satires, and to compose song-books for the people and stories and tales which might be intelligible to ignorant persons,—they will say you are cracked.”

The view and the estimate of art which Tolstoy holds in his essay “What is Art?” is a direct corollary of the above. Great art is measured by its capacity to communicate itself to universal humanity: not to some sophisticated coterie, not to some one class of people, but to man and woman in their simplicity and humility of soul. “Art is not a pleasure, a solace, or an amusement; art is a great matter. Art is an organ of human life, transmitting man's reasonable perception into feeling.” And a work of art which refuses to perform this chief function for a part—and that the greater part—of the human race, is bad art, even though it may enjoy the highest praises of those who, in appreciating it, aristocratically isolate themselves from the crude, unlettered millions.

Tolstoy applies his criterion mercilessly. That a Baudelaire, a Verlaine, an Ibsen, a Maeterlinck, a Burne-Jones, a Boecklin, a Richard Strauss are to be ruled out; that subtle notions, sophisticated, supersensitive, or distorted feelings, beauty-hunting, pleasure-sated prodigality athirst for new

sensations, and discontented, introspective idleness,—that these would be rejected by Tolstoy is a foregone conclusion. But Shakespeare suffers, and "Don Quixote," and the later work of Beethoven, and especially Wagner, and most of the painters accounted great. On the other hand, the great Bibles of the world endure the test, and Schiller's "Robbers," and "Les Misérables," and "The Christmas Carol," and "Adam Bede," and "Uncle Tom's Cabin," and Dostoyevsky; but, of Tolstoy's own work, discarding "War and Peace" and "Anna Karenin," barely two short stories are saved: "God Sees the Truth" ("The Long Exile"), as belonging to religious art, "transmitting feelings of love to God and one's neighbor," and "The Prisoner of the Caucasus," as an example of universal art, "transmitting the very simplest feelings common to all men." Thus, for Tolstoy, "the destiny of art in our time is to transmit from the realm of reason to the realm of feeling the truth that well-being for men consists in being united together, and to set up, in place of the existing reign of force, that kingdom of God—*i.e.*, of love—which we all recognize to be the highest aim of human life."

Tolstoy asks himself, How can I, Lyof Tolstoy, save others from being my servitors? I can take care of my own room; I can clean my boots—indeed, I can make my own boots; I can go into the fields and by honest labor produce the equivalent of the food which I consume. And only *after* I have done this shall I have a right to offer my help to my fellow-men without feeling like a robber who returns part of the booty. And the work which I do must be of a sort which will relieve some of the common people from doing that work for me, for I cannot save my fellow-man who produces and makes my bread by philosophizing in his place. Nor do I, in so doing, reject in any way the true dignity of

## The Gospel of Tolstoy the Apostle

mental work. The maximum time I can spend in really profitable mental work is five hours. I sleep eight hours. What do I do with the remaining eleven hours? Let me, during that time, relieve the peasant in his manual labor; let me allow him a chance to think at least half an hour.

Still, what was Tolstoy to do with his property, with his thousands of *desyatsins* of land, with his copyrights? Was he justified in simply giving away all his estate to the poor? That would have compelled his wife and family to abandon their rich life and follow him, contrary to their convictions, perhaps,—and compulsion is wrong, according to Tolstoy. Besides, his wife had helped increase his wealth: he could not give away her and her children's share. He himself, on the other hand, could no longer hold to his wealth and remain honest with himself. Tolstoy accordingly gave up all rights to his estate, handed it over to his wife to manage as she saw fit. In his own house he remained a guest; each day he devoted several hours to manual labor, earning his bread directly. He continued to write, but declared all his works free of copyright, free for any one to publish and circulate among men. Only when the Dukhobors faced punishment and exile because they regarded military service as contrary to the Christian religion and refused to enter the army, Tolstoy used his novel "Resurrection" to raise funds with which to enable them to emigrate to Canada, where freedom was promised them.

So we find Tolstoy writing "Popular Legends," simple stories of charity and forgiveness, and also criticisms of life, candid, penetrating, pitilessly sincere. In "Neglect a Fire and it Spreads," anger and hatred and rancor lead to the mutual destruction of two peasants' households. The shoemaker Martuin Avdyeitch, in "Where Love is, There God is Also," hears in his dreams the promise of Christ to visit

him, and his day of expectation is a Tolstoyan illustration of the verse, "Inasmuch as ye have done it unto one of the least of these my brethren, ye have done it unto me." "Master and Man" is a larger canvas: a landed proprietor of utterly selfish character is caught in a snowstorm, and attempts to save his own life by escaping on horseback, leaving his coachman to freeze to death. After long wandering, the animal brings him back to the carriage he has abandoned. He finds his coachman almost frozen. Moved by a sudden outburst of humanity, he prostrates himself over the freezing figure and thaws it back to life with the warmth of his own body. When the rescuing party finds him, he has frozen to death, but the coachman is saved.

It would be wrong to think that these are mere tracts: some of the later stories of Tolstoy are veritable literary gems. The finale of the short classic "God Sees the Truth," in which Aksenov's spirit of forgiveness conquers the criminal Makar, causing him to confess his misdeeds, exculpate Aksenov and secure his release, is a bit of genuine pathos: "When the order came to let Aksenov go home, he was dead." The spirit of the Bible is in these stories, and there is Biblical simplicity in the narratives, and a Biblical verisimilitude. Avdyeitch the shoemaker, waiting for Christ, watches the passing crowd. "Two soldiers passed by; one wore boots furnished by the crown, and the other one boots that he had made. Then the master of the next house passed by in shining goloshes . . ." and so forth, all from a veritable cobbler's viewpoint, requiring no commentary, and itself being the very essence of realism.

But there is criticism also, acute criticism, and very thinly disguised. The portrait in "A Candle," of the overseer who oppresses the peasants and compels them to break God's law by plowing on Easter Sunday, is a bold parable.

If rebel violence is not commended by Tolstoy, little doubt is left as to the identity of the overseer, and ominous is the description of the punishment which God inflicts on Michail Semyonovitch. Regicide is condemned, but tyranny is even more clearly denounced. The pithy "Skazka" requires no foot-notes. Ivan the Fool does all the work, his two brothers scorn him for his crude manners, but live and prosper at his expense. A time comes when Ivan is made Czar, and instead of laws and regulations, one simple rule suffices in his czardom: "Whoever has callous hands, comes to the table; whoever has not, gets what is left." One wonders how the censor, who mutilated "What is Art?" and forbade the publication of Tolstoy's religious writings, allowed this menacing prophecy to appear in its entirety. Perhaps we have here another illustration of the point mentioned in our first lecture: even bold fiction passes in Russia where serious exposition is forbidden: for—who knows?—perhaps the mass of readers would miss the point, perhaps the censor himself missed it. Meanwhile, literature, the novel, still remains Russia's chief channel of spiritual self-analysis.

The consciousness of death is clearly present in Tolstoy's later stories. He pictures it from various angles, and each portrayal of death is a criticism of life. Ivan Ilyitch dies as he has lived. A meaningless conclusion of an empty life, his death is merely an incident in the *tchinovnik* sphere of selfish ambition in which he has moved. Just as he had advanced in rank over the corpses of his older colleagues, so the younger men in his office anticipate the moving-up process after his own routine alphabet of life has reached its futile omega. In "Three Deaths," an aristocratic lady, an old peasant, and a tree complete the same cycle of existence and pass into the unknown.

What unknown? About the question of immortality,



Tolstoy's view passes from negatively inclined skepticism to reverent agnosticism, which tends toward the end to reach a level of devoted hopefulness. "It is impossible to receive faith from any one; it is impossible to convince oneself of immortality. In order to have faith in immortality it is necessary that the latter should exist; and in order that the latter should exist it is necessary to understand one's life in that in which it is immortal. Only he can believe in a future life who has performed his work of life, who has established in that life that new relation to the world which does not as yet find a place in the world."

Bravely Tolstoy championed his gospel of love of others, and opposed all use of force and all egoism. Thousands of men read his works, were converted to his view of life, and tried to follow him in the path of God. Holy Russia excommunicated him, the government of the Czar punished, imprisoned, exiled his followers, but dared not—literally dared not—touch this man of God, as witness his letters to the Russian Ministers of Justice and of the Interior. Still he did not feel satisfied with himself. Still he thought that he was not living as unselfishly as he ought. In the year 1897 he wrote the following letter to his wife and put it among his papers, asking that it be delivered to her after his death:

"MY DEAR SONYA:

"I have long been tormented by the incongruity between my life and my beliefs. To make you change your way of life, your habits, which I taught you myself, was impossible; to leave you has so far also been impossible, for I thought that I should be depriving the children, while they were still young, of the influence, however small, which I might have over them, and should be causing you pain. But to continue

to live as I have been living these sixteen years, at one time struggling and harassing you, at another yielding to those influences and temptations to which I was accustomed and by which I was surrounded, has also become impossible for me at last; and I have made up my mind to do what I have long wished to do,—to go away; . . . for I, who am now entering on my seventieth year, long, with all the strength of my spirit, for that tranquillity and solitude and, though not perfect accord, still something better than this crying discord between my life and my beliefs and conscience.”

Thirteen years more elapsed before Tolstoy actually fled from his house to seek peace with God. It is not our province here to recite the immediate strain and trouble which determined Tolstoy's flight, nor the tragic conditions which made his death, not one of peaceful solitude, as he desired it to be, but the most abominably published and moving-pictured event in the world. Nor is it for us to criticize as mistaken this last act of self-abnegation on the part of a man who had spent thirty years seeking to do, not his own will, but the will of God.

Was he right? Was he wrong? That no one man can decide. Tolstoy's gospel is not beyond criticism, but perhaps the Gospel of Jesus itself is not beyond criticism. Tolstoy preached, and tried to practise, the religion of consistent love of men; he tried to stop making his fellow-men his slaves, and to make himself their fellow-worker. Some of those who call themselves followers of Tolstoy have acted as if the outward expressions of his ideal of life, which suited his case, exhausted his gospel; as if the sage of Yasnaya Polyana merely taught men to wear grimy blouses, chop wood, plow the fields, and avoid the use of money. The story is told of an Englishman who became a Tol-

stoyan and would not touch money, but asked his wife to sign his checks for him, and had a secretary going about buying the things he wanted and paying for his railway tickets. And there are many more who fail to understand Tolstoy's gospel because they cannot get past their objections to his own perhaps eccentric methods of preaching that gospel. "I shall soon be dead," he sadly predicted, "and people will say that Tolstoy taught men to plow and reap and make boots; while the chief thing that I have been trying so hard to say all my life, the thing I believe in, the most important of all, they will forget." And this most important thing is the truth that selfishness which exploits others is the source of evil, and that self-sacrificing love for others is the highest good. Tolstoy advocated the denial of the powers of darkness within us: anger, lust, desire to oppress, exploitation—in a word, egoism.

This ideal of life may not be practicable, it may not be expedient, it may not be even scientific, as some people say that Nietzsche's ideal of proud self-assertion is scientific, but Tolstoy believed that it was the ideal which Jesus advocated. And upon those who call themselves Christians the necessity is imposed of determining whether Tolstoy was as intimate an acquaintance of Jesus as are those others of his professed servants who do not find it inconsistent with His gospel to sanctify the marriage of the same dissolute man to several different women, one after another, provided only that they believe that Jesus of Nazareth was conceived immaculately; who are quite able so to twist the meaning of the Book of Life that poor men and bad men and men of other races and nationalities will be kept "where they belong," provided only that they themselves continue to believe in the literal, divine inspiration of the Bible; who are able in a thousand churches and cathedrals to pray to the God of Love and

Peace to help their armies kill the armies of their enemies, so long as they assert the dogma that God is Three-in-One and created the world out of nothing.

The average man is much bolder and much more resolute in his beliefs than in his daily conduct. Most men seem quite ready to include any number of doctrines in the creed which is to obtain for them eternal happiness, if only they are allowed to retain hold of their purse-strings, if only their course of life is not interfered with. But Tolstoy heard Jesus say to him: It is not enough that you have read the law and the prophets: give all you have to the poor and follow me. He found that the religion of Jesus as well as all other great religions are religions of life, not theologies. He tried to rid his soul of anger, lust, violence, and selfishness, and to love and help his fellow-men. And the Russian Orthodox Church declared him an enemy of God because he did not believe in miracles, because he rejected the dogmas of the Trinity and the Divine Birth, and could not be dogmatic about the immortality of the soul.

Who can judge his life? The upholders of orthodoxy? But in "Three Hermits" the bishop who tries to teach the simple, untutored eremites how to pray to God in the approved manner finds that the Lord's Prayer overtaxes their memory and does not inspire religious devotion. And he is wise and pious enough to grant them the privilege of praying to God in their own simple way: "*Troe vas, troe nas, pomiluy nas!*" ("You three have mercy on us three!") This is a hint to the upholders of orthodoxy. The story "The Two Old Men" impresses me as a Tolstoyan "Apologia pro Vita Sua," brief, simple, addressed to us all. Two peasants, Yefim and Yelisei, set out on a pilgrimage to Jerusalem. But Yelisei is diverted from his holy journey by the call of mercy and, having spent on a poor family all the

money which he had saved for his pilgrimage expenses, finds himself obliged to return home. Yefim proceeds on his way, suspicious of strangers and coldly calculating from beginning to end. But when he finally stands in orthodox devotion before the Holy Sepulcher, behold! Yelisei has reached there before him. It is Yefim's vision, of course, and it is Tolstoy's parable, but the message of both is unmistakable.

And it is Tolstoy's message to the world: "God bids every one do his duty till death—in love and good deeds."

RADOSLAV ANDREA TSANOFF.





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## THE MAKING OF THE RIGHT KIND OF A LIFE<sup>1</sup>

**I**N the thirteenth chapter of the Acts of the Apostles may be found the text for our morning's message:

"David served his own generation by the will of God."

From this text is deduced the simple but vital theme: **THE MAKING OF THE RIGHT KIND OF A LIFE.**

The supreme vocation is the vocation of right living. The first question and the big question for every life is, not how to make a living, but how to make a life. Making a living is a mere incident, making a life is our business, in this world. The supreme contribution that any human being can offer to the world is to offer it the right kind of a life. One Savonarola can turn the tides of Florence. One Aristides, the just man, can perceptibly lift Athens higher. Ten righteous men would have saved Sodom. The people of Constantinople said of John Chrysostom, the golden-mouthed: "It were better for the sun to cease his shining than for John Chrysostom to cease his preaching."

In the making of a life, certain principles must be observed, and they are indicated in the text. The text itself would be a noble motto for each member of this graduating class: "David served his own generation by the will of God." "I will serve mine own generation by the will of God." There are three principles indicated in the text, and these three are necessary in the making of the right kind of a

<sup>1</sup> Baccalaureate sermon of the second commencement exercises of the Rice Institute, preached by George W. Truett, Pastor of the First Baptist Church, Dallas, Texas, in the academic court at 9:30 o'clock Sunday morning, June 10, 1917.

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life. First, the true business of life is service. That is the test of life. "David served." Service is the Great Master's test of life. "By their fruits ye shall know them." The divine emphasis is always on deeds. The most beautiful portrait ever drawn of the ideal life is in five little words: "He went about doing good." And we are called to "be imitators of Him, as beloved children." Gladstone, the imperial statesman of his time, never wearied of saying: "One example is worth a thousand arguments." What the world needs is service. Its wounds cannot be stanchd except by service. Its ignorance cannot be dispelled except by service. Its weakness cannot be met with proper reinforcement except by service. The true test of life is service. .

Only by such test can faith be vindicated. "Why call ye me, Lord, Lord, and do not the things which I say?" Faith is more than a dogma. Faith is a passion. Faith lifts. Faith achieves. Faith arrives. Great believers have always been great doers. Witness Moses and Paul and Luther and Livingstone and Grenfell and Clara Barton and Frances Willard and all the rest. Garibaldi was thrown into an Italian prison, but he managed to scribble on a little piece of paper, and get it back to his men, this message: "If fifty Garibaldis be thrown into prison, let Rome be free." Great believers are always great doers. The heroic chapters of human history are the chapters of men and women who have climbed from faith to strength on the stairway of service.

The teaching of the Great Master of men is utterly revolutionary as to all the supreme things. Jesus never gave a little answer to a big question. For example, one asked Him one day: "Master, who is my neighbor?" and He gave an answer to that question that opens up vistas of meaning and responsibility that continue to surprise and challenge

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mankind the world around. Jesus utterly revolutionized the ordinary conceptions men have of greatness.

Once men thought that the chief greatness was the greatness of brawn. Their conception of God was that He was something of a muscular giant. Their doctrine was that might makes right, and it would seem that a good many have not discarded that doctrine yet. Certainly, there is to be no disparagement of brawn. There is no glory in one's having a weak body. There is no virtue in frail health. And yet brawn is not the chief greatness.

Then there came a time when men thought that the chief greatness was the greatness of brain. They conceived of God as a great intellectual giant. Certainly brain is nowhere to be disparaged. Knowledge always has been power, and always will be. The sure foundations of states have always been laid, not in ignorance, but in knowledge. And yet, brain is not the chief greatness.

When the Master came among men He brushed aside their preconceived notions of greatness and said: "The acme of greatness, the highest greatness, is the greatness of service. If any man among you aspire to be the greatest man of all, the chief over all, let him be the servant of all." And slowly but surely the world is coming to the recognition of that doctrine of the Master concerning greatness. It is greatness through the right kind of service.

The chieftest exponent the Master has had, the highest product that Christianity has produced, the greatest single credential that the Gospel has thus far set forth, namely, the Apostle Paul, stated the true life principle for himself and for us when he said: "I am debtor to all men, to the Jews and to the Greeks, to the wise and to the unwise, to the strong and to the weak. I am debtor inasmuch as in me is possible to every human being."

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What shall we do about this debt? One of three things. How shall we pay the debt? In one of three ways. Life is always invested in one of three ways. We can play the miser with life, and you will remember, my fellow-students, that education can be just as miserly and just as selfish as gold can be. The life of the miser may be lived by any of us. There died awhile ago, in one of the larger cities, a man who was supposed to be a pauper. For years he lived alone in his little hut of squalor and wretchedness. Kindly women came, venturing now and then to leave him food, lest he should go hungry. By and by, the little cabin was strangely still for a day or two, and an investigation was made, and it was found that the old man had passed into the Silent Land. A careful investigation was made of the cottage, and they found something over four million dollars of gold and bank stocks and the like, hidden away there in that little hut of squalor and poverty. Of what value, I ask you, is such a life as that? Miserable indeed is the life of the miser.

Life may be invested in just an opposite direction. It may take the course of the spendthrift. All its power may be dragged down and debauched and prostituted. It may be recklessly flung away. The picture was recently given us in the daily press, of a young man in a large city who summoned about him a group of his comrades for an evening dinner, reminding them that when the dinner was over he expected to give them a thrill, such as they would not soon forget. A band of music entertained the gay party; and when the dinner was over he summoned the young men to follow him, preceded by his band of music, and they came to a swinging bridge across a swift-flowing stream, and then, standing on this swinging bridge, he called to them, saying, "The chief thrill is coming just now," and with a wild shriek

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he flung himself into the gurgling waters and was a suicide right before their eyes. A man can take seventy years to fling his life away, or he can do it in one hour. Life prostituted and debauched for selfish ends inexorably misses the great purpose of its Divine Creator. Pitiable beyond words is the life of the spendthrift, the prodigal.

The true conception of life is that it is a trusteeship. All power, no matter what it is, is under inescapable obligation to serve humanity. Financial power, social power, governmental power, political power, intellectual power, all power is under bonds to serve humanity. He who forgets that fact defeats the end of his being and vitiates the high plan that the Divine Maker has for human life. The poet voiced it for us in his simple poem when he said:

"I live for those who love me,  
For those who believe me true,  
For the heaven that smiles above me  
And awaits my spirit, too.  
For the cause that lacks assistance,  
For the wrongs that need resistance,  
For the bright hopes in the distance,  
And the good that I can do."

The true test of life is service. I pray you to believe it in your deepest hearts.

Our text indicates another principle that must be regarded in the building of the right kind of a life. "David served his own generation"—mark well the words—"his own generation." Some men spend much of their time sighing over what they have lost in the past. Other men sentimentally dream about the wonderful things they are going to do in the future. And between sighing over the past, which has gone forever, and sentimentally dreaming over the future,



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which has not come and which is unknown, time gets away; and the vital, responsible present, the only time that is ours, passes, and its opportunities are returnless forever. "David served his own generation." If you, my fellow-students, desire to make life that high and worthy thing that the Great Creator designs that it shall be, then I summon you to-day to lay to heart the truth that your service is to begin to-day, in the most active and worthy fashion; and the living present is worthily to be magnified by you, because to-day is the only day that is yours.

What is your sphere of service? Look about you, and if your eyes and ears are alert, you will be easily able to find your sphere. Begin with your own circle. Look at the standards and habits that obtain in your own circle. Look at the conditions that are there regnant. Should those conditions be changed? Should those habits be modified? Should those standards be different? Begin with your own circle, and every man in his own circle will have his hands full, if he would live the great and worthy life. And as you faithfully live in your own circle, meeting worthily there each duty as it comes, that circle will widen and widen and widen, and your days shall be filled with deeds of unselfish and helpful service to a waiting and needy world.

Voices clamant for helpers are all about us. Look at the ignorance that on every hand needs to be dispelled. Carlyle was right when he said: "I count this a tragedy, that any human being should live and die, capable of being taught, without that privilege and without that blessing." Listen to the world-wide call of justice. Listen to the cry that wrong provokes in every quarter. Our sphere of service is all about us. There are the poor to be helped. There are the weak to be defended. There are causes strengthful and worthy all about us crying for champions. There are

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bad laws to be rectified. Nothing can be politically right which is morally wrong. The enthronement of a law which defends immorality is a standing anachronism in our modern civilization. Every man should set himself to the rectification of every evil law that may be found, everywhere. And on and on, the circle of influence will widen and the call for service deepen as you give yourselves to the tasks of your day and generation. That Golden Rule of the Great Master, which must be the law of life, we are to seek to have enthroned everywhere. A law for nations it is to be, as well as a law for individuals. Oh, if such Golden Rule had been enthroned in the nations, we would not on this fair morning have the appalling spectacle of mighty nations battle-scarred and war-torn, the news of which daily conflicts plunges the world into its Garden of Gethsemane. And yet, what shall I say, in a passing word, about the world conflict into which our own fair land has been drawn? I must say this: That our land has been compelled, under high moral compulsion, to enter the world conflict. We enter it, not with any lust for revenge or for gain. We enter it, I believe, as touching our motives, with clean hands and pure hearts! God help us! We enter it because some things are worth dying for, and human life is a very cheap and tawdry affair if some things are disregarded and despised. The sanctity of woman is worth dying for, and the safety of childhood, and the integrity of the State, and the majesty of righteousness, and the honor and freedom of the United States of America—any and all of these are worth dying for. With conscience and courage, let the whole nation respond to the call of her nobly capable President and the National Congress and go forth in this world crisis in a battle for the rights and safety of humanity, a battle which, in its final issue, shall, please God, move the nations forward

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and upward, as they have not thus been summoned before,  
since the stars sang together in creation's morning.

*In "your own generation" is your service to be performed.*

*Without turning to the right or to the left, each one is to face faithfully his own sphere, his own circle, his own task, his own high demand, and give himself, without stint or reserve, to carrying out his duty there till the day is done. Quaint old Ben Franklin said: "Value time, for time is the stuff of which life is made." Over the gateway of many a man, his failure might be found written in just two words: "He dawdled."*

"If thou canst plan a noble deed,  
And never flag till it succeed,  
Though in the strife thy heart must bleed;  
Whatever obstacles control,  
Thine hour will come. Go on, true soul.  
Thou'lt win the prize, thou'lt reach the goal."

But, my fellow-students, in the making of a worthy life there is another supreme matter that must have the most conscientious attention at your hands, and that matter is the controlling motive for your life. That keen-minded woman, George Eliot, said: "What makes life dreary is the want of motive." Her saying points a great truth, but it may be amended: "What makes life dreary is the want of the right kind of a motive." Many lives are paltry and sordid and go groveling to the dust and the grave because they are not swayed by the right kind of motive.

One of three motives dominates life. I begin with the lowest. There is, first of all, egoism. It begins and ends with self. Everybody begins with that motive. The little child, learning to talk and to walk, puts out its hands and its voice, claiming everything within its grasp. It has no regard

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for the owner of this or that. It simply wishes it, claims it, and takes it. Alas, that human life should have men and women in it, grown and in life's middle time, and even with the gray about their temples, whose conception of life is that it begins and ends with self! All self-centered lives are doomed and defeated. History will not allow us to forget that the self-centered life, no matter whose or where, must be doomed and defeated. One of the old Hebrew prophets recites the downfall of his nation in one sharp sentence: "Israel is an empty vine, he bringeth forth fruit unto himself." Let any nation bring forth fruit simply unto herself, and she is inevitably doomed and defeated. Well may this nation sing Kipling's "Recessional," and sing it without ceasing:

"God of our fathers, known of old,  
Lord of our far-flung battle-line,  
Beneath whose awful Hand we hold  
Dominion over palm and pine—  
Lord God of Hosts, be with us yet,  
Lest we forget—lest we forget!

"The tumult and the shouting dies;  
The captains and the kings depart:  
Still stands Thine ancient sacrifice,  
An humble and a contrite heart.  
Lord God of Hosts, be with us yet,  
Lest we forget—lest we forget!

"Far-called, our navies melt away;  
On dune and headland sinks the fire:  
Lo, all our pomp of yesterday  
Is one with Nineveh and Tyre!  
Judge of the Nations, spare us yet,  
Lest we forget—lest we forget!

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"If, drunk with sight of power, we loose  
Wild tongues that have not Thee in awe,  
Such boastings as the Gentiles use,  
Or lesser breeds without the Law—  
Lord God of Hosts, be with us yet,  
Lest we forget—lest we forget!"

The nation that forgets the high purposes of a nation is doomed. The nation whose creed is the creed of the jungle is doomed. And therefore, for my part, I have no real fear about the final outcome of the world struggle that now enthalls the nations.

The self-centered organization is doomed. The self-centered college is doomed. The self-centered church is doomed. The self-centered family is doomed. I believe Gladstone was right when he said: "Napoleon had the keenest brain that was ever packed into a human skull." And yet he died like a dog in the ditch, died after he had convulsed Europe, and made the nations cower before him,—died ignominiously because life began and ended with self. What cared he to walk with ruthless heel over the heart of his beautiful Josephine? What cared he for the sacrifice of a hundred thousand brave men on the field of battle, if only he could carry out his fiendish ambition? Selfishness is the distemper of life. Selfishness is the suicide of all greatness. Selfishness is ever marked for downfall and defeat.

There is another motive, a motive incomparably higher than this first, a motive which has in it very much of praise and worthiness. That is the motive of altruism. Out of that motive have come many of the world's chiefest and highest blessings. Out of that motive—the altruistic motive—to a remarkable degree, have come our liberties and our institutions. And yet the altruistic motive, my fellow-

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students, is not high enough yet. Human nature is so forgetful and ungrateful and disappointing. Look at Moses, the first man of the long centuries of the Old Testament days, a man who was and is in himself a university for the world, a man who made the chiefest renunciation that has been made in human history to be the friend and follower of God. He turned away from a throne, with all its splendor and honor and aggrandizement, and linked his life with a down-trodden and spiritless nation of slaves, that he might recover this nation and lead it forth to its high task as a nation. And yet the nation was all along forgetful and ungrateful toward this incomparable leader. "As for this fellow Moses, we wot not ~~what~~ what has become of him,"—thus contemptuously they spoke of him when he was out of their sight. The altruistic motive is not sufficiently commanding for the highest battles of life. "Moses endured, as seeing Him who is invisible."

What is the motive sufficiently worthy and commanding for human life? Here it is in our text: "David served his own generation by the will of God."

"He always wins who sides with God.  
To him no cause is lost."

And be you well assured, my fellow-students, that he whose life purpose crosses God's purpose for him, invokes defeat for his work and destruction for his influence.

In Chicago a little while ago, where I was speaking for some days to groups of students, many were the incidents told me concerning the burning of that Iroquois Theater building, sometime in the past, with such appalling destruction of life. But the most stirring of all the incidents for my own heart was this: A young student, William McLaughlin, had just come to his hour of graduation, at the

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age of twenty-one. He was the nephew of a nobly gifted minister of Chicago. The young student waited on his uncle, and received his uncle's felicitations and counsels, in that rosy morning hour for the young man. The great minister said to him: "My son, life is not worth while if a man lives it out of harmony with his Great Saviour and King; but life, my son, is glorious, anywhere, everywhere, if only a man stands faithfully at the post where Divine Providence puts him, till the day is done." Then the minister said: "My son, I shall speak about that Sunday morning, when I come to speak to the people again, taking this for my text: 'For this cause came I unto this hour.' "

You recall the circumstances under which Jesus uttered such words. The Master of men was facing Golgotha and the cross, and He shrank back as He faced those gathering sorrows and clouds, and cried out: "Father, save me from this hour!" Then He rallied, saying: "But for this cause came I unto this hour. Father, glorify Thy name." That is to say: "Thy program shall be carried out. Thy will shall be accomplished. My task is to do the will of Him that sent me, and to finish His work."

"I shall speak on that," said the great minister, "Sunday morning." With burning heart the young man turned away from his uncle's study, pondering the great things that the uncle had said to him. Down the street he went, when presently he was confronted with a building wrapped in fire, and he heard the shoutings of men and women and children, and saw the men as, selfish and heedless, they were seeking to make their escape from the building, leaving the children and women unprotected. Somehow the heart of the young man carried him into that building, to see if haply he might rescue the helpless and weak. He began his work of rescue, and many did he rescue. Presently his clothing was aflame,

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but he flung it away. Presently he fell fainting under his last load. When he came to consciousness he was in the hospital, and bending over him were his uncle and aunt. His lips moved and his eyes betokened consciousness, and as they bent low to catch his words they heard him saying over and over again: "But for this cause came I unto this hour." They waited a few hours more, and he was conscious again, and his lips parted again, and they bent low to catch his final sentence: "Father, glorify Thy name. I have finished the work Thou gavest me to do."

Fellow-students, I had rather die at the age of twenty-one, carrying out some high purpose of life like that, than to live, like Methuselah, to be nine hundred and sixty-nine years, and live selfishly from the beginning to the end. Ponder the message of a modern poet:

"I had walked life's way with an easy tread,  
Had followed where comforts and pleasures led,  
Until one day, in a quiet place,  
The Master and I met face to face.

"With station and rank and wealth for my goal,  
Much thought for my body, but none for my soul,  
I had entered to win in life's big race,  
When I met the Master face to face.

"I had built my castles and reared them high,  
With their towers had pierced the blue of the sky.  
I had sworn to rule with an iron mace,  
When I met the Master face to face.

"I met Him and knew Him, and blushed to see  
That His eyes, full of sorrow, were fixed on me,  
And I faltered and fell at His feet that day,  
While my castles melted and vanished away,—



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*"Melted and vanished, and in their place  
Naught did I see but the Master's face,  
And I cried aloud, 'Oh, make me meet  
To follow the steps of Thy wounded feet.'*

*"My thought is now for the souls of men.  
I have lost my life, to find it again,  
E'er since one day in a quiet place  
I met the Master face to face."*

My fellow-students, on this happy baccalaureate occasion, I adjure you, choose for the commanding motive of your life, "the will of God."

GEORGE W. TRUETT.

## THE UNIVERSITY AND THE NATION<sup>1</sup>

I HAVE chosen as the subject of my address "The University and the Nation" because the Nation at this time is face to face with a crisis in its history of supreme importance, not only to itself as affecting its whole future national life, but in its relation to the politics of the world, to the great cause of human civilization, of international comity, and of the right of the individual to live in peace and in the reasonable pursuit of happiness.

And I have brought the University into intimate connection with the Nation because of the supreme part that it must play in the national consciousness, and in pointing out by its attitude toward the things of life, through the whole wide range of human intelligence, the true direction of safety and of progress. This is a time beyond almost any other in the history of the Nation when there is an immediate need, that cannot be too greatly accentuated, of taking a careful and conscientious inventory of what we have and in what we are found wanting. Our national pace has been so swift that it is high time that we should stop and take breath for the longer running that in God's good grace is before us in our national life; and the University, so far as it shall have it in its power, should be that force to set it on its certain way toward the goal of successful achievement.

I think of the University, then, in saying this, as a live and progressive force, and not as passive or inactive as it has sometimes seemed to be; and I have thought of it as not

<sup>1</sup> Address delivered by William Henry Carpenter, Provost of Columbia University, at the second commencement convocation of the Rice Institute, held Monday morning, June 11, 1917, at 9 o'clock.

belonging, in word or in deed, to a narrow or chosen teaching.

The true University, the University of the present, is already a pregnant force in the life of the community, of which, as scarcely before in its history, it is not only a part, but a participant. If there ever was a time—and there doubtless was—when the University seemed to the world without its gates to stand consciously apart upon a height of its own self-righteousness, that day has long since disappeared, and there is no valley so deep of human occupation and of human interest that does not find in it, too, the University in its manifold activities working side by side with other laborers in solving, at least in its endeavors, the complex problems of living. But in the crisis that confronts us not only as a Nation for ourselves, but as an inseparable part, for all times hereafter, of the interdependent nations of the earth, it must be even more determinate as a teacher and more dominant as a leader than it has ever been before in the whole of its history.

The conscious aim of every American University at the present time, wherever it may be, of yours and of mine, and whether it be supported by public taxation or by private endowment, is service. And by "service" I mean public service in its widest and most comprehensive sense: not merely in the generous proffer for the active and ready use of the municipality, the State, and the Nation of the teaching and research equipment of the University in materials and men; and not only in the sending out into the community competent engineers to build the country's railways and bridges, architects to design its public buildings, lawyers and judges to serve its courts of law, and physicians to heal its sick, or whatever the multitude of callings may be for which the University makes provision. Public service in the intention

of the University means much more than this. It means to have inspired these men and women who go forward to assume their part in the conduct of the material affairs of the Nation with the spirit of service—of active and intelligent and willing participation in the affairs of national life and the conscious responsibility to bear their part of its burdens.

And just here lies, in a most fundamental aspect of the matter, the inherent responsibility of the University to the Nation. Its teachings should be progressive; and not only should it lead along the whole long line of thought and action, whatever that may be, in the ideality of a new conception of the quest in search of truth or in the reality of the practical application of newly discovered fact, but it should, in its attitude toward the active affairs of the day, keep pace with the day's direct interests and relations, and promptly and fearlessly lay old ideas aside if they are unfitted to present-day conditions. It is a perfectly apparent fault of American life that we have as a national characteristic an open impressionism, but as a true concomitant a lack of stability; and the University is surely not there to increase or encourage it, but rather to preach by precept and by practice an enlightened conservatism that should have its salutary effect upon national action. There should be, however, in the University that open-mindedness to consider as its broad field of activity, not only the precious bequests of the past, however remote they may have been, if they have been weighed in the balance of experience and found worthy of keeping, but also in far increasing measure, and particularly and emphatically at this critical time in the Nation's history, the immediate interests of the present.

The great war has swept us, too, as inevitably must have been the case, into the vortex of confusion that has changed

the world. This is a new era and a new epoch of human history, and things will never be the same again; for after the storm and the murk and the clouds of the conflict have been cleared away, as they must sometime be, there will appear, as in the old-time vision of a world's renewal, a new earth to be shaped anew to the needs of men.

And to meet these needs the University must be organized to new and better service if it is to make the response that can reasonably be demanded of it in the fulfilment of its mission. More than ever before in this country's history must there be, under the new conditions that are already here, a deeper consciousness of the duty and responsibility of the citizen to the Nation. In the inaugural address last March of the President of the United States there occurs this significant sentence: "The thing I shall count upon, the thing without which neither counsel nor action will avail, is the unity of America—our America, united in feeling, in purpose, in its vision of duty, of opportunity, and of service." And these are the things in very truth to nationalize a nation, to unite it "in feeling, in purpose, in its vision of duty, of opportunity, and of service."

And is there not just here the true mission of the University set forth in terms as unmistakable as if to it they had been directly and consciously applied? The ideals of its accomplishment are as plainly the inculcation of the vision of a sense of service, and through its teachings of the opportunity to serve; and it is in these ways and along these lines that the University, more potently than almost any other human institution whatever, must be not only the support, but the conservator of the Nation, for out of the University comes and will come, not alone, as we sometimes express it, the hope of the Nation, but to no uncertain extent the reality of its existence and its perpetuation.

And the question arises: How can the University, in the legitimate exercise of its potentialities, better fulfil its mission in the presence of the new and untried conditions that confront us? The war messages of the President, that will go down to history among the great state documents of the Nation, have sent, as no such public pronouncements in our day and generation have done before them, a thrill of patriotism through the land and an awakened sense of the inherent significance to all of us of our democracy. They have left an ineffaceable mark upon the American spirit, and there is no new need at this time to characterize the convincing appeal they have sounded, that has touched all men's hearts and remade the Nation; but in all of it, in the light of the lesson of the war already learned for us, who have, nevertheless, in reality until now merely looked on from a distance, there is no appeal more trenchant and striking than the call for a new spirit and for new activities in American industry; for truly "it is evident to every thinking man," as the President states so emphatically, "that our industries on the farms, in the shipyards, in the mines, in the factories, must be made more prolific and more efficient than ever, and that they must be more economically managed and better adapted to the particular requirements of our task."

And it is directly here that I would call in, as it has not been called in before, the service of the University. I would not decry for an instant the reawakened patriotism that has come over the land with the declaration of war, for it has disclosed the soul of the Nation. But an emotional patriotism without action in the face of a world's crisis can lead nowhere but to ultimate disaster. "Patriotism," affirms a recent writer, "is useless without science"; which means, of course, that patriotism alone is impotent to conserve or to construct, if it is unsupported by the application of expert

knowledge to the active conditions of the Nation's life. I would join indissolubly together patriotism and science; and to meet this need and without any thought of destroying the paramount values of the present, I would make the idealism of the University, to a wider extent than many of us have thought possible, a *practical* idealism. The service of the University, and its opportunity to serve, as I see it in the light of the present and the prospect of the future, is in a new and extended sense, and as never before, to make science really the handmaid of industry; not in any sense to curtail or discourage the pursuit of science for science's sake, but to increase it in breadth and depth beyond any conditions at hand in our universities and institutes of research, to encourage investigation by the provision of proper opportunity to undertake the solution of new and important problems whose possibilities have been surmised but not yet realized, to give to it as an expansion of human knowledge no barriers but its own limitations of scrutiny and control.

But back of all this, and in direct contiguity with it, I would place the industries of the Nation. I would make the discoveries of science, however theoretical their value may seem in the beginning to be, immediately applicable, if it is at all possible, to the facts of industry, and I would add them as surely in this way to the industrial assets of the Nation. And to bring this about the University itself must play its part on a wider stage of interest and of action. With a new and enlightened sense of service it must establish relations more intimate and more sympathetic with the industries it shall strive to help, and in every way it must co-operate with them to an extent untried before.

But just here there lies a danger as insidious and destructive as any that confronts the University or the Nation: the danger, I mean, of a controlling materialism, of the cen-

tralization of aim and of effort that readily follows in its wake, and of an ultimate standardization not only of national activity, but of the Nation's thought and feeling, that in the end isolates it, and dehumanizes it, and makes it stand apart as a thing unto itself and different from the rest of mankind.

There is a nation that has gone precisely through this process of national decadence; a nation that beyond any other on the wide face of the globe has believed in and eagerly and intelligently put into direct and effective practice the discoveries of science, for which its keenest minds have been ever on the alert and to further which encouragement by opportunity and equipment almost without limit has been provided. It raised itself within an astonishingly short time to an extraordinary height of material prosperity, but in so doing it lost utterly, as it appears to the rest of the world, its vision of those greater things that constitute a nation if it is to take its place side by side with the enlightened nations of the earth in the forward march of human progress. To-day that nation that I have in mind is a nation of a single outlook, and that a wrong one, for it has lost its soul through a standardized materialism; and this not solely, to be sure, through the application alone of science to industry, but because that has been an impelling force that in its ultimate result of an unprecedented material prosperity, of an expansion of material influence undreamt of before, of an efficiency, to use its very word, of organization that has accrued as a concomitant to every aspect of national life. And inevitably, and as directly as cause and effect, it has obscured the nation's horizon, and more than anything else whatever has made it easy to destroy the nation's old-time vision and to bring about in the end only the thought of self-aggrandizement and selfish prosperity. And this is the



land of the "Critique of Pure Reason"; of Kant, and Fichte, and Schopenhauer, and Schleiermacher, and Schlegel, and all that long line of philosophical thinkers that once made Germany the land of contemplation and sometimes even the land of dreams! But that Germany has disappeared off the face of the earth, and it has not left, except as a heritage of history, a living trace behind it.

There is need, and an urgent need, in America for a far closer attention to the economics of national life, for thereon is dependent in the end the very fact of the Nation's existence. A country, however, without a wider vision of more things than that of material prosperity is lost indeed; and to realize it now, at this crucial time in our own Nation's life, is a factor of national preparedness much more vital and essential in its ultimate effect than any question of militancy whatever; for it must dominate as an impelling force, and more than ever before, our national consciousness, lest having in truth once found our soul, we shall lose it.

But there is no cause of pessimism as to the national attitude toward these things, or alarm for the national safety. There is, however, as truly as in the affairs of men, a tide in the affairs of nations that must be taken at its flood to lead to fortune. And this is the time to consider it, with a new understanding of the meaning that is involved in the present choice of direction as leading to desired result.

American materialism as a dominant note in our national life is still, as it always has been, a myth that our whole history from its first beginning absolutely disproves, and it has never been more plainly disproved than in the crisis that has come to us, in our turn, in the world's conflict. We have gone into the war with a true and splendid idealism; with the idea of a duty to perform and with a feeling of exaltation, of a mission to fulfil in bringing to pass those conditions of peace and stability and freedom from the misrule of

autocracy that alone can make the world a fit place to live in. An era of militant activity has begun; let us use it rationally in every way for the eternal good, and not heedlessly for the ultimate degradation of the Nation; for there are times, and this is one of them, when thinking men search their souls for a justification, if it can be found, of the things of life that in our careless self-sufficiency have seemed to us hitherto a matter of course and to be ours for the asking.

And the University in this new nationalization of the Nation has no uncertain part to play to carry out its mission. It should look more carefully and closely to the applications of science to industry, as I have already insisted; but it must also look with jealous care to those things of the spirit that are in its especial keeping to inculcate and to cherish. Never in all our history has the University been more able to respond, and certainly it has never been more willing to respond, to the call of the Nation; and the Nation as never before needs its help to fight what is not alone our cause, but the universal cause of civilization and of free government. In the President's war message to Congress, which cannot be too frequently read or quoted, he says in words that sound like a trumpet-call: "We shall fight for the things which we have always carried nearest our hearts—for democracy, for the right of those who submit to authority to have a voice in their own government, for the rights and liberties of small nations, for a universal dominion of right by such a concert of free peoples as shall bring peace and safety to all nations and make the world itself at last free." For, in that fine phrase that has already become a watch-word, "the world must be made safe for democracy."

And what of your own part, the part that you are to play who go out to-day into the real world of action? You stand to-day, as you have never stood before, upon the actual threshold of life. Shall you draw timidly backward into the

shade of your own individual environment or go consciously and proudly forward into the brighter light of national usefulness? A usefulness, I mean, that does not selfishly consider your own personal advancement as a goal of supreme attainment and desire, but one that shall further the advancement of the welfare of the community of which you are an integral part—a part intimately constituent of the whole in the true spirit of modern democracy; a part that only by its fusion with other parts can make the militant, forceful whole that with an irresistible preponderance of idea shall carry the Nation forward upon the upward path that leads to national integrity and the national perpetuity that is founded upon national truth. And the Nation, with your help, and only with it, shall in this way realize in very truth what John Milton long ago saw in that splendid vision of a nation's majesty:

“Methinks I see in my mind a noble and puissant nation rousing herself like a strong man after sleep, and shaking her invincible locks; methinks I see her as an eagle mewing her mighty youth, and kindling her undazzled eyes at the full midday beam.”

But the higher planes of national existence can be attained only by the conscious striving toward those ideals of national integrity that, like a guiding star, shall lead the Nation's vision, and whose course, if followed, alone shall determine to us and to succeeding generations the Nation's fate. And the University, too, within the Nation, must keep undiminished its vision of light to lead it onward to a deeper significance in national life and to a newer and wider national usefulness. And in no wise must this vision fail. For where there is no vision, the Nation and the University shall truly perish!

WILLIAM HENRY CARPENTER.



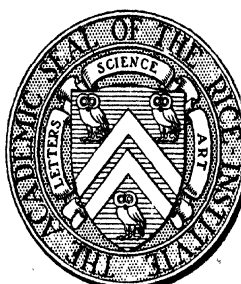


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A university of liberal and technical learning  
founded by William Marsh Rice in the City of  
Houston, Texas, and dedicated by him to  
the advancement of Letters, Science, and Art



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## MATRICULATION ADDRESS, AUTUMN OF 1917<sup>1</sup>

### I

#### IN THE WAKE OF THE WAR

---

**L**IFE in America has suddenly become worth living, death worth dying. Pessimism is perishing. Life becomes worth while because for every man it has become a hard grind with a great purpose. Death becomes worth while because it has become the shining glory of a great hope—freedom for the planet.

A theologian with some sense of humor has said that the authentic remark of Adam to Eve as they left the enchanted Garden was, "We live in an age of transition." That ours is an age of transition is the discovery from age to age alike of scholar and statesman. "Nothing that is can pause or stay." Every particle of matter in the universe is now in motion, has always been in motion, will always be in motion. From electron and ion to mountain and planet, from molecule and mountain to men and measures, this principle of ceaseless change is universally operative. Backward and forward, round and round, in waves and cycles, in whirls and spirals and rockets, from millions of swings in a second to single swings in millions of years,—and all under the written word of reason, we should like to think, rather than the spoken fiat of chance. Changes in human society and human government are neither so swift nor so slow as these extremes in modes of motion. "I have seen many changes in the last three years, but few in the last fifty," said Mr. Dooley. Nor

<sup>1</sup> Read 24th September, 1917, at the opening meeting of the sixth academic year of the Rice Institute.

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are changes in human nature swift; they are comparable, perhaps, with those in the faces of mountains and the forms of continents. However, in a general sense, every age of peace is an age of transition. An age of transformation is every age of war. Our age is specifically no exception.

For, the War is a consuming fire. In its wake a new heaven and a new earth will rise, from its wreckage a worthier world will roll, and under clearer skies. Religion, education, and patriotism will shine forth with a new light from the burning. All enterprises of the human spirit will come through purified. Neither pure gold nor pure character can be burned. Neither freedom, nor faith, nor fellowship can be burned. Superfluous and dispensable elements may be eaten away; the structural and permanent members cannot be burned. Our adoration of the good, the beautiful, and the true cannot be burned. Our aspirations for the great, the lovely, and the new cannot be burned. Education for all the people, government representative of all the people, liberty, intellectual, religious, political, industrial, for all the people—these ideas cannot be burned. Institutions founded on them will come through and carry on. The fruit-gatherers of an old civilization and the forerunners of a new will alike come through purified. You and I, whether or not we survive, will come through purified, in soul, in speech, in service. The War is a consuming fire. But after the fire, the builder. And it will be a new heaven and a new earth.

In the meantime, while the fire is raging, what of Education during the War? With the President of the United States, I have no hesitation in saying that the educational enterprises of the country should proceed without interruption. The training of intellect, the stimulating of imagination, the development of initiative, the dissemination of

intelligence, the discovery of truth, the invention and resolution of problems,—these processes must go on as though there were no war. And this to the end that we may be as economically and efficiently organized for the preservation of peace, after the War, as we shall shortly be for the waging of war, now that the War is on. The noble words of John Milton jump to mind: "I call a compleat and generous Education that which fits a man to perform justly, skilfully, and magnanimously all the offices both private and publick of Peace and War." As comprehensive and more concise was William of Wykeham's definition, "The making of a man." The business of making men must go on as usual. There must be marrying and giving in marriage. There must be mothers of men. There must be schools for the children, from kindergarten to college. The country schools, the city schools, the commonwealth's colleges and universities must be continued at concert pitch and on maximum schedules. All academic, æsthetic, and athletic activities must be maintained unimpaired in strength, that men and materials may be available in abundance for the days of reconstruction. Accordingly, the springs of educational effort and progress must be kept strong and pure at their source in the university.

For, education begins at the top, not at the bottom. Light descends from above, not from the depths. It is a question of the direction of influence, not one of the determination of importance. There is no greater or less in education. It is a question of sequence in time, not one of an estimate or scale of values. Elementary education and higher education have one and the same source, one and the same aspiration, one and the same object, and in achievement should still be one and the same. Education in a democracy would elevate the masses of men, would develop the average man

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to the limit of his possibilities, would single out and train the exceptional man for the graver responsibilities of leadership. Without the college there is no educational ladder, without the university there is no educational highway, for the people. Without the university not even an effortless educational escalator would be possible. Build the university and the schools will be built. Enlarge the university and the schools will be multiplied. No nation without universities is educating its people. The enlightenment of a people proceeds apace with the building of universities. Mexico has no universities. Turkey has no universities. Japan is multiplying her universities. America is multiplying her universities. Moreover, in America the states which have the best universities have also the best schools. And in those states any question of the greater or lesser importance of school or university drops out of consideration, for the very simple reason that in these states the relationship of school to university has become in all respects a completely reciprocal one.

The spirit of that reciprocal relationship is one of service, universal service, the manifold service of science and scholarship to state and church and society. That spirit, as the spirit of liberal and technical learning, finds its way in freedom and faith and fearlessness; and in no other way does it find its own way. It divines and directs its own service. It droops and dies under the domination of dogma or despotism. Its duty is the search for truth; its discipline, the fires of knowledge; its daring discoveries, the fruits of wisdom; its dominion, that of free spirits. In freedom it attains to life in science, art, and letters. And among freemen, fettered only by the desire of fearless hearts and open minds to serve, science cannot be sectarian, philosophy cannot be political, history cannot be heterodox, nor is there of neces-

sity either atheism or anarchy in art. Moreover, in the law and the reason thereof, the mind and the mystery thereof, the heavens and the glory thereof, there are glory and mystery and reason beyond chronicles and ~~charters~~ and creeds. Furthermore, the work of laboratories, libraries, and lecture-rooms must proceed without distraction, though the heavens fall. The scholar gives his best service when the serenity of his study is undisturbed by sounds from the street. Unless specialized, this service will be superficial. Specialization calls for concentration of effort. And it is only under conditions of quiet and uninterrupted study that such concentration becomes most effective. On the other hand, the universe of the university is not an isolated universe. It is a human institution in very human relations. Its province is the wide range of human knowledge, and within its preserves are the wider ranges of human ignorance. In many of its subjects its lecture-halls are the meeting-house, the theater, the almshouse, or the jail; its laboratories, the city or commonwealth, continent or cosmos; its libraries, the living epistles that are read of all men. It is thus, through contacts as diversified as human interests, that the service of the university becomes universal.

To attain to such service in these exacting times, both school and university must be maintained at the maximum efficiency of normal conditions in order that education may supply men and materials for constructing the new heaven and the new earth we may expect in the wake of the War.

What, then, is the normal programme of the Rice Institute? That of a university of liberal and technical learning dedicated to the advancement of letters, science, and art, by instruction and by investigation, in the individual and in the race. The preceding paragraphs are shot through and through with its spirit. Its history is not long, but it is liv-

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ing. That history has been lovely because at no point lacking in interest. It began in a vision that is now on view. What was rhetoric on the rostrum in 1912 is reality on the roster of to-day. The prospect on the prairie was not a mirage of the plains. Prophecy has become history; and promise, performance. Rice has been running its race—a race that will never be run. And that race has been paced by some such considerations as the following. Briefly, we found ourselves in a new and rapidly developing country. With all due respect to the work of institutions already established, the country had not yet produced a school of science, pure and applied, of the highest grade. The new foundation was explicitly dedicated in its very title to the advancement of letters, science, and art—the whole sweep of things—with just about money enough to take care of one of those wings well. The income, however handsome, was limited. Moreover, the Trustees wisely determined, both to build out of the income and to live within the income, keeping intact not only the endowment funds, but also those which, under the terms of the will and charter, might have been legitimately spent outright, the latter amounting to approximately five million dollars. Despite some impatience in waiting, some inconvenience in working, the wisdom of that self-imposed limitation has never been questioned. Accordingly, their choice of immediate educational endeavor in these circumstances was a comparatively simple one. They proposed that the new institution should enter upon a university programme, beginning at the science end. As regards the letters end of the threefold dedication, they proposed to characterize the institution as one both of liberal and of technical learning, and to realize the larger characterization as rapidly as circumstance might permit. With respect to the art end, it was proposed to take architecture

seriously in the preparation of all of its plans, and to see to it that the physical setting of the Institute be one of great beauty as well as of more immediate utility. This, in rough outline, was the original programme of the Rice Institute.

Such was the plan. What about the product? Another seven short sentences will suffice. We have been laying foundations, broad and deep, in the fundamental domains of pure science. We have been giving thorough training in several lines of applied science. We have been maintaining high standards and on a university plane. We have made beginnings in the other liberalizing studies of philosophy, letters, and history. We have taken architecture seriously, and housed in a home of extraordinary beauty the spirit of liberal and technical learning. We have begun to advance science, art, and letters in knowledge, cultivation, and citizenship. We have built out of our income and have lived within our income. And were I not writing under considerable restraint, I should shout here, and reverently, For all of which the name of the Lord be praised!

We felt last June we were closing a chapter. We find this September we are continuing the story, for we are not going to allow the changed conditions of our living to affect our life. I have said repeatedly that Rice is the most democratic society of which I have any knowledge. I have also been assured that the American Army is the most democratic organization on earth. Accordingly, we may anticipate an easy adjustment of our old ways to the new arrangements<sup>1</sup> by which the Rice Institute is to be converted, as far as may be consistent with its university programme, into a military camp during the War. We have been realizing in our un-

<sup>1</sup> By authority of the National Government units of the Reserve Officers' Training Corps are being established at the Rice Institute, and its courses in pure and applied science are receiving recognition at the hands of the Army and Navy Departments.



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dergraduate corporate life a measure of the Athenian ideal of liberty; the Roman, of order; the Mediæval, of unity as embodied in the Holy Roman Empire; and the French Revolutionary ideal of equality; while in local patriotism the modern ideal of nationalism has been as conspicuous as any. These things we do not intend to lose. We have said a great deal about science and scholarship and service. We have sought character and culture and citizenship as ends of education. To these we shall hold fast. Come peace, come war, we believe that in the character of the cultivated citizen lies the strength of the civilized state. To the accomplishments of that cultivated citizen we are proposing to add the soldier's art, for the very simple reason that all are subject to the country's call while the nation is at war.

While the country is at war we have no choice but to work. The time for argument and discussion is past. The time for decision and action is here. We have no time to ask, Why the War? We have barely time to win the War. And the first and final business of these United States of America is to win the War. I have worked too long at science, not to think in terms of the planet. I have worked too long at education, not to believe in cosmopolitan citizenship. I have worked too long at religion, not to be a world humanitarian. But the nation is at war. At this moment the first and final business of America, and of every individual American citizen, is to win the War. The shadow of the thing engulfs the whole planet. From the eclipse a new planet has "swum into our ken"—this planet. The roar of the planet is in all men's ears as never before in human history. We are at grips—death grips—with an enemy for the freedom of the planet. And we shall win, because we propose to hold that freedom secure, not by the sword of conquest aided by the destructive arts of war, but by the spirit of coöperation in all

the constructive arts of peace. We propose that democracy—the democracy of delegated authority to expert representatives—the democracy of representative government—shall prevail in the earth, enabling all peoples, great or small, to escape not only the “tyranny of the crown” but also the “tyranny of the crowd”; enabling all peoples, great or small, to enjoy not only freedom from man rule to-day but also freedom from mob rule to-morrow. Representative rule is at stake; with it stands or falls the possibility of a war-proof planet. Pan-planetary peace is at stake; with it comes or goes the prospect of a federation of the world. Safety first for democracy the wide world over has rung round the earth. In the spiritual urgency of his utterances, their political principles and final phrasing, the President of the United States has become “the clear and powerful spokesman” for the planet. The Pan-American Union of Republics, planned in this country for this western hemisphere, has become the immediate historical forerunner of a world-wide League of Nations to Enforce Peace, planned also in this country for the solidarity of the sphere. From the confusion of conditions too complex for compression into formulæ, and of consequences too seriously significant for superficial summary statement, at least three stubborn facts stand out in sharp relief: one stubborn fact for the present—we are at grips, death grips, with an enemy for the freedom of the planet; another stubborn fact for the future—the stake of our war against war is a war-proof planet; and this stubborn fact for all time—we fight that pity and laughter may return to the common ways of men, and the world become strong, not through force and the will to power, but through faith and the will to freedom.

## II

### THE CHARGE TO ACHILLES AND GLAUCUS

---

COMING and going about your tasks here, you will think and talk about the War. You can no more help it than I can help writing about it to-day. However, be assured of this. Hate hurts the hater. It harms no one else. The War will crush you if you curse it. It will kill you unless you laugh at it. You can wait and watch and pray only if you work and are merry at that work. It is grim business, but there must be joy as well as grit in it.

And for us at Rice there have been both joy and grit in it. Three occasions I would recall: that fair midnight in May under the flag, that rare morning in June under the tower, that twilight of evening sky at the training camp, when we sent forth our sons on their great adventure, under the Homeric rubric of Rice—the charge of aged Peleus to Achilles his son, the charge of Hippolochus to Glaucus his son—

*To win renown,  
To stand the first in worth as in command;  
To add new honours to their native land;  
Before their eyes their mighty sires to place,  
And emulate the glories of our race.<sup>1</sup>*

Charging them also in Newbolt's lines to the new men of his college, Clifton, and in tribute to old Cliftonians who went down in frontier wars,

<sup>1</sup> The longer of Alexander Pope's translations of the line,

Αἰὲν ἀριστεύειν καὶ ὑπεύροχον ἔμμεναι ἄλλων.

HOMER, *Iliad*, VI, 208; XI, 784.

*To set the cause above renown,  
To love the game beyond the prize,  
To honour while you strike him down  
The foe that comes with fearless eyes;  
To count the life of battle good,  
And dear the land that gave you birth,  
And dearer yet the brotherhood  
That binds the brave of all the earth.<sup>1</sup>*

\* \* \* \* \*

*I saw the spires of Oxford,  
As I was passing by,  
The grey spires of Oxford  
Against a pearl-grey sky;  
My heart was with the Oxford men  
Who went abroad to die.*

*They left the peaceful river,  
The cricket field, the quad,  
The shaven lawns of Oxford,  
To seek a bloody sod.  
They gave their merry youth away,  
For country and for God.*

<sup>1</sup> Sir Henry Newbolt's poem entitled "Clifton Chapel" appeared originally in the "Spectator," 10th September, 1898.

The poem recalls other lines of the author—

*"Clifton, remember these thy sons, who fell  
Fighting far over sea;  
For they in a dark hour remembered well  
Their warfare learned of thee!"—*

inscribed on the pedestal of a statue commemorating men who gave up their lives in the South African struggle. The following translation of this inscription:

*Τῶνδε τέκνων τῶν σὼν Ἀκράγας μέμνησο πέραν που  
Οἷς ἔλαχ' Ὀκεανοῦ μαρναμένοισι πεσεῖν,  
Οὔποτε γὰρ πατρίου παρὰ σοί ποτε παιδευθέντες  
Ἦμασιν ἐν δυοφεροῖς οἷδ' ἐλάθοντ' Ἀρεῶς.—*

was contributed by Sir T. Herbert Warren to the "Spectator," 10th April, 1915.

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*God rest you, happy gentlemen,  
Who laid your good lives down,  
Who took the khaki and the gun  
Instead of cap and gown.  
God bring you to a fairer place  
Than even Oxford town.*

The lines I have just recited are from some verses by Mr. W. M. Letts printed lately in the "Spectator" under the caption "The Spires of Oxford (Seen from a Train)." They will stay with you. They were ringing within me as I read on my home-bound train, a few days ago, a rumor that the enemy had offered a reward of four hundred dollars and a fortnight's furlough for the first capture of an American soldier at the front. You have said to yourselves more quickly than I could phrase the words that if that first captive haply should be a Rice man, his captor would surely need the furlough.

As we speed the parting soldier, we hail the coming student. Ladies and gentlemen of the Freshman class, men and women of 1921, this is your day. For you this meeting was called, for you these words were written. From this day forth the freedom, the fortune, the faith, and the fellowship of this place are yours. Its freedom you will not abuse; its fortune you must conserve; its faith you will uphold; its fellowship you must adorn. Its future is your future. Its past also is yours. Its traditions are few but fundamental. Your fellows and predecessors require that you hold one faith with me. You must love beautiful things and consider them important. You must believe in the power of human reason and the capacity of the human spirit for progress. Above all things, you must be enthusiastic for your fellowmen. Otherwise the spirit of this place will fail either of

## The Charge to Achilles and Glaucus 309

meaning or persuasion in your lives. But if you keep this faith, then yours becomes the inward spirit of the university, of which these beautiful buildings are but the external form. It is thus that you will honor the Founder, honor your family, honor the Faculty, honor your friends. It is in this sense that you are building the university, for I take it that you have come here, not to get an education, but to live a life.

And the building of a great university is just like the living of a great life. Each calls for the intellect, energy, courage, and independence that characterize the other. The institution outlives the individual, and the university has proved to be about the most enduring of human institutions. Think of the changes in church and state that have been weathered by the spires of Oxford, Cambridge, Paris, and Padua, Salamanca, Bologna, Harvard, and Princeton! Accordingly, we may assure you with considerable confidence that so long as men love learning, so long will the Rice Institute flourish. So long as men seek truth will the spirit and service of science endure. And in beauty and holiness, religion and art will outlast them all. These are the elements of a civilization that traces its origins to Palestine and Greece, and finds its sources in the mingling of streams from Athens and Sion through Rome. Three main currents of that civilization—the Rise of Christianity, the Revival of Learning, the Rise of Modern Science—each in its turn the new knowledge in conflict temporarily with the old, have contributed to the common knowledge of cultivated persons in all civilized communities. An education that would not draw heavily from this common stock could hardly be called liberal either in letters, in science, or in art. We earnestly hope that each of you will make sure of this sure foundation in liberal studies before seeking special or technical training.

Before concluding my remarks I venture one personal

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note. Pope Pius IX said of Pusey the Oxford theologian: "Dr. Pusey is like a bell always ringing, ringing to summon every one to church, but never going in itself." The simile is not inappropriate as an expression of the relationship to the daily life of a university experienced by an educational executive temporarily or permanently deprived of the privilege of teaching. Echoes from campus and cloister reach the loggia in yonder tower, and calls to class-room and council-chamber I may not enter. But there are compensations. Chief among these compensations during the coming year will be the personal conferences I may have with you; so I trust that long before, or certainly when, hope is reduced to the single wire of Watts's broken lyre, when faith has but a ray from some most distant star, when charity must begin at home, and right away, you will pull the rope on the first floor and come up to the belfry in the tower. For we see things, not as they are, but as we are; and I am up there to help you straighten yourself and the world out.

### III

#### THE MORNING PÆAN AT SALAMIS

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FIVE years ago the first Freshman class of the Rice Institute went from the Faculty chamber to the tasks and opportunities of the new University under an old Homeric line that I have already quoted earlier in this address. From year to year their successors have gone out under similar charges. The present Freshman class enters Rice on the anniversary of the Battle of Salamis, which was fought on or about September 20th, B.C. 480.

It was a battle for freedom in a great struggle for Western liberty. The history of patriotism records no pæan of a people more stirring than that which the Greek combatants raised when the morning sun lit up Salamis and the ships of the Persians. That morning pæan has been preserved by the Athenian poet Æschylus in "The Persians," the earliest known historical play of Europe. Æschylus himself was in the battle. He had been at Marathon on September 12th, ten years before. His younger brother, Ameinias, led the crashing charge of brazen beaks on that decisive Salamis morning. His own comrades-in-arms were in the audience when, in the eighth year after the battle, the initial performance of "The Persians" was enacted in Athens.

The drama is placed in the royal palace at Susa, where Atossa, the queen-mother, is impatiently awaiting news of the triumph of Xerxes, her son, king of a countless host. Harassed by ill-omened dreams, she comes in a chariot of state to consult the Persian counsellors, who constitute the chorus of the drama. "Where," she asks—

*"Where is this city, Athens, in the world?"*

*What shepherd rules and lords it o'er her people?"*



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And to this day we can hear the cheer from that intent Athenian audience when for answer the chorus chanted:

*"Far in the West, where sinks the sovereign sun;  
Slave to no man, and subject to no lord."*

Nor in four and twenty centuries has silence overtaken the mighty shout that rose from the theater when the messenger, describing Salamis to the queen, first spoke the lines:<sup>1</sup>

*"The squadron on the right began the fight,  
And then their whole fleet came, and from the decks  
One cry was heard, 'O sons of Hellas, rise!  
Your home-land free, your children free, your wives,  
The sacred altars of your fathers' gods,  
And your ancestral graves! Now all's at stake!'"*<sup>2</sup>

From the decks of Salamis to the halls of Rice, from the men of Hellas to the youth of Texas, from the sons of freedom to the souls of freemen, comes that conquering cheer. Ringing down the years to this anniversary season, strong as on that daybreak comes the battle-cry. It calls to courage. On a hundred fronts Western liberty is again challenged by a thousand ships and men beyond all numbering. It calls to consecration. Brave men in blood, brave women in tears,

<sup>1</sup> τὸ δεξιὸν μὲν πρῶτον εὐτάκτως κέρας  
ἤγειτο κόσμῳ, δεύτερον δ' ὁ πᾶς στόλος  
ἐπεξεχώρει, καὶ παρῆν ὁμοῦ κλύειν  
πολλὴν βοήν, ὧ παῖδες Ἑλλήνων ἴτε,  
ἐλευθεροῦτε πατρίδ', ἐλευθεροῦτε δὲ  
παῖδας, γυναῖκας, θεῶν τε πατρώων ἕδη,  
θῆκας τε προγόνων· νῦν ὑπὲρ πάντων ἀγών.'

ÆSCHYLUS, *Persians*, 399-405.

(From the Oxford text edited and revised by Arthur Sidgwick.)

<sup>2</sup> In the above English renderings of five quotations from "The Persians" free use has been made of translations by Blackie, Morshead, Nevinson, Plumptre, Potter, and Way.

are again writing the drama of freedom. In the spirit and in the letter of that call,

ὦ παῖδες Ὀρυζαίων ἔρε,  
νῦν ὑπὲρ πάντων ἀγών.

*(Rise, O sons of Rice,  
All is now at stake!)*

The making of your life is at stake: the building of your University is at stake: the saving of your country is at stake: the winning of the only noble kind of war is at stake: the conquest of the globe in freedom's cause is at stake!

IV  
THE PRAYER OF TAGORE

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AND for all of us, Trustees, Faculty, students and friends of Rice, for the college, the community, the commonwealth, the country, not only the pæan at Salamis in the West, but also the prayer of Tagore from the East:

*Where the mind is without fear and the head is held high;  
Where knowledge is free;  
Where the world has not been broken up into fragments by  
    narrow domestic walls;  
Where words come out from the depth of truth;  
Where tireless striving stretches its arms towards perfection;  
Where the clear stream of reason has not lost its way into  
    the dreary desert sand of dead habit;  
Where the mind is led forward by Thee into ever-widening  
    thought and action—  
Into that heaven of freedom, My Father, let my country  
    awake.*

EDGAR ODELL LOVETT.

## RICE BEFORE THE WAR—AND AFTER<sup>1</sup>

### I

#### THE FOUNDER

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**I**T is a common saying in drawing-room and market-place that we are living in a wonderful age. Perhaps no known period of the past towers up to it, unless it be the age of Pericles, or that in which the Roman Empire was consolidated, or that of the Reformation. No features of the age are more striking than the handsome foundations which have been provided by private donation for lengthening the days of man and enlarging the content of his spiritual life. Every child of ten years knows the names of Alfred Nobel and Cecil Rhodes, of Mr. Carnegie and Mr. Rockefeller, of Girard and Peabody, of Johns Hopkins, Leland Stanford, and Cornell: the names of these gentlemen are household words, and in thousands of American homes their bearers have become household gods.

In this charmed circle of immortal philanthropists the name of William Marsh Rice is permanently inscribed this day by the poet of Princeton, the jurist of Texas, and the bishop of Tennessee. Thanks to the inaugural lectures of those twelve prophets of the fundamental sciences, the liberal humanities, the progress of modern learning, Altamira of Madrid, Borel of Paris, Croce of Naples, De Vries of Amsterdam, Jones of Glasgow, Kikuchi of Tokyo, Mackail

<sup>1</sup> Paragraphs from an opening address, 12th October, 1912, on *The Meaning of the New Institution*, printed in full in the first number of *The Rice Institute Pamphlet*, pp. 45-132, under the headings:—*The Foundation*: I. Source, II. Site, III. History.—*The University*: IV. Studies and Standards, V. Saints and Seers, VI. Students and Staff, VII. Shades and Towers, VIII. Strength and Support, IX. Spirit and Summons.

of Oxford, Ostwald of Leipsic, the lamented Poincaré of Paris, Ramsay of London, Størmer of Christiania, and Volterra of Rome, the good-will of Mr. Rice to open new springs of inspiration and living fountains of knowledge in an institution of liberal and technical learning becomes known to the world of letters and science and art, to whose advancement he gave of his substance and of his life.

For this fair day we have worked and prayed and waited. In the faith of high adventure, in the joy of high endeavor, in the hope of high achievement, we have asked for strength, and with the strength a vision, and with the vision courage: the courage born of straight and clear thinking, the vision of enduring forms of human service, the strength in resolute and steadfast devotion to definite purpose. And to-day, by virtue of the Founder's splendid gift to the people, by virtue of the public spirit of his early advisers, by virtue of the public service of those who defended his last will and testament and thereby protected the people's rights, by virtue of the covenant which his trustees have kept in all good faith and conscience, by virtue of the constant creative work of supervising architects and the arduous labors of constructive engineers, by virtue of the cheer and the criticism and the counsel of friends in the community and throughout the commonwealth, the Rice Institute which was to be, in this its modest beginning, now has come to be—the new foundation has accomplished in its own being the miracle of all living things: it has come to life, and from this day forth takes a place, let us hope of increasing influence and usefulness, among those institutions which have made possible the civilized life of men in communities of culture and restraint—the State, the Church, and the University.

There are men and men and men. There are men of millions and men of millions. William Marsh Rice was a man

in a million, an inspired millionaire who caught the prospect of monumental service to Houston, to Texas, the South, and the Nation. With no resources other than soundness of body and strength of will, from a New England home of English and Welsh forebears, he came to Texas in his youth to make his fortune. By temperate habits of industry and thrift he made a fortune in Texas. He left his fortune in Texas. He gave his fortune—the whole of it—to Texas, for the benefit of the youth of the land in all the years to come; thus writing in the history of Texas the first conspicuous example in this commonwealth of the complete dedication of a large private fortune to the public good. Moreover, resolutely living a simple life, he denied himself even the “durable satisfaction” of seeing his philanthropy’s realization in order that he might give more abundantly of life to his fellows and their successors. Shrewd in foresight, strong in purpose, of stout courage and independent spirit, generation after generation will rise to call him blessed—“with honour, honour, honour, honour to him, eternal honour to his name.”

## II

### THE FREEDOM OF THE FOUNDATION

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To his trustees, a self-perpetuating board of seven life members, the Founder gave great freedom in the interpretation of his programme and corresponding discretion in the execution of its plans. The charter and testament under which these gentlemen discharge the obligations of their trusteeship are documents so liberal and comprehensive as to leave the institution under practically but one restriction, namely, its location must be in Houston, Texas. But therein lies what is perhaps its greatest opportunity. For men who are too busy doing the world's work to find time to talk about it would tell you that there never were more insistent challenges to constructive thinking than are confronting the South at the present time. Opportunity is written over the whole Southwest: opportunity commercial, opportunity political, opportunity educational, but educational opportunity is written larger than all the rest. We have problems to face, serious ones, that have been perplexing the South for a generation: but even to the most superficial observer it is daily becoming more and more apparent that any solution of these peculiar problems of the South calls for solutions of Southern educational problems in terms of educational opportunities for all the people. Furthermore, the agricultural and industrial transformation now in process of development offers manifold additional arguments to Southern men to prepare their sons for the possession of this land of plenty and progress. Though for nearly a generation the ambitious young Southerner may have seen larger possibilities ahead of him farther from home, to-day he finds conditions completely changed. Go South, young man! is the slogan

in one section. Stay South, young man! is the answering call of opportunity in the other.

In the South and in the West, of the South and of the West, you find yourselves in an environment whose clear-skies make men blandly or keenly observant of their powers, whose mild climate keeps men constantly human and neighborly and friendly in ways of living, whose democracy recognizes no inequalities; in an environment which will have its way with us unless we have our way with it; an environment bristling with opportunities for creative and constructive effort. You find yourselves in a State which can know no provincialism, because it has lived under seven flags. You find yourselves in a section of that State which lives under a categorical imperative of progress, for we of the plains are drawn by irresistible lure of the prairie, impelled to advance by beckoning mirage quite as wonderful as mountain prospect. You find yourselves among men who live their lives in the open, under a making sun that does not rise but jumps from the horizon full-orbed in his noonday splendor.

And how you do get into your blood and bone the wine and spirit of this country! Speedily you absorb its patriotism and pride, and as speedily come to feel the fearlessness and freedom, the frankness and the faith, that characterize the life of this Texan empire. For this reason it is that in portraying its virtues modesty is not a sin which doth so easily beset us. Houston—heavenly Houston, as it has been happily named by a distinguished local editor of more than local fame—you will find in some ways a bit too close to New York, perhaps, but here you will also find many a heartening reminder of the memories and traditions of the South, and all the moving inspiration in the promise and adventure of the West. Here, in a cosmopolitan place, in a community shaking itself from the slow step of a country village to the



self-conscious stature of a metropolitan town, completing a channel to the deep blue sea, growing a thousand acres of skyscrapers, building schools and factories and churches and homes, you will learn to talk about lumber and cotton and railroads and oil, but you will also find every ear turned ready to listen to you if you really have anything to say about literature or science or art. Of cities there are genera and species and types whose science is still to be written: cities of arms, cities of kings, cities of government, cities of commerce and industry, cities of pleasure and leisure, beautiful cities of art, holy cities of cathedrals and convents, university cities of letters and science. Houston at present may fail of qualifying for admission to certain of these classes, but there is great reason to rejoice in the commercial prosperity of the city and in the growing development of the community; for just as certainly as trade follows the flag, just so certainly does the patron of learning follow in the wake of the empire-builder. For builders of cities, great merchants and captains of industry, by the character of their work and the extent of their interests, are rendered alert, open-minded, hospitable to large ideas, accustomed to and tolerant of the widest divergencies of view. Thus it has come to be that great trading centers have often been conspicuous centers of vigorous intellectual life: Athens, Florence, Venice, and Amsterdam were cities great in commerce; but, inspired by the love of truth and beauty, they stimulated and sustained the finest aspirations of poets, scholars, and artists within their walls. It requires no prophet's eye to reach a similar vision for our own city. I have felt the spirit of greatness brooding over the city. I have heard her step at midnight, I have seen her face at dawn. I have lived under the spell of the building of the city, and under the spell of the building of the city I have come to believe in the larger

life ahead of us, in the house not made with hands which we begin this day to build. However, in the exultation of the moment in which we witness the dedication of the new university, we must not forget that the organization which William Marsh Rice incorporated has already rendered the city and State of his adoption considerable service. I need hardly remind you that during recent years the Rice Institute has contributed in a substantial manner to the upbuilding of Greater Houston. On a conservative basis—always on a conservative basis—certain of the foundation's funds have been invested in various enterprises which have sustained in no small measure the steady and continuous advance of the city in industrial and commercial prosperity.

The epoch whose beginning we observe to-day with these formal exercises marks the period in which even more powerfully that same organization is to support the intellectual and spiritual welfare of the community; and, finally, to touch again upon the material side of progress, the very machinery by which the stone age of the new university is about to be transformed into its spiritual age will distribute the income of the foundation through the several channels of Houston's business, philanthropic, social, and religious life; and thus we contemplate with some degree of satisfaction the slow but sure evolution of a threefold influence on the material, the intellectual, and the spiritual aspects of the life of the city.

*“ 'Tis not the walls that make the city, but the men” ;*

and the men in the day of Pericles were freemen who “pursued culture in a manly spirit, and beauty without extravagance.” Such freemen are the men that build the university. The strength of this foundation lies in its freedom: the

freedom to think independently of tradition; the freedom to deal directly with its problems without red tape; the freedom to plan and execute vouchsafed by the will of the Founder and the charter of his foundation; the freedom of his seven trustees, seven freemen, who approach its problems of organization, policy, and aim, without educational prejudices to stultify, without partisan bias to hinder, without sectarian authority to satisfy, with open minds accustomed to large problems, with clear heads experienced in tracking the minutest details of business; seven men always ready to reason together, steady and conscientious in reaching conclusions, quick and decisive in action when through common counsel they have come to a common mind respecting any line of action. And in their freedom these trustees are building for the Founder a university whose greatest strength likewise is in its freedom: in the freedom of its faculties of science, humanity, and technology, to teach and to search—each man a freeman to teach the truth as he finds it, each man a freeman to seek the truth wherever truth may lead: in the freedom to serve the State because entangled in no way with the government of the State, and the freedom to serve the Church because vexed by none of the sectarian differences that disturb the heart of the Church.

While we rejoice in our freedom from Church or State control, we rejoice none the less in the work of these fundamental and indispensable agencies of civilization, for we can conceive of no university in whose life there do not appear the energy and enthusiasm, the affection and the calm, that we associate in one way or another with reverence, patriotism, politics, and religion. Hence to us, quite as important as is a university's freedom from control by State or Church, are its right relations to each of these two institutions, because upon principles of order, conduct, and knowledge is

based our faith in the capacity of the human spirit for progress, and without such basic faith all theories of education become either confused or futile. As a matter of fact, the three fundamental principles I have just named—order, conduct, knowledge—find expression in the forms of three great institutions—the State, the Church, and the University. These institutions themselves are not fixed and final but fluid and forming, constantly in the flow of change, in transition from good to better, to meet new requirements of a changing world and a growing humanity. In their present mutual relations, the State, the master of the sword and peace; the Church, the guardian of the soul and purity; the University, the servant of each of them in preserving to men the mastery of their spirits. The State guaranteeing to the University intellectual freedom, to the Church religious freedom; the University in freedom of thought and research constantly enriching the State with the theory of its own greatness, constantly recalling the Church to the theories of life wherein all men are made free; the Church in its turn sustaining the Nation and supporting the University in high ideals of progress and ultimate triumph. Moreover, testing any programme for better uses of life and leisure by a double criterion: Is it based on an understanding of the ways of men and the needs of humankind? and Does it appeal to the understandings of men? the University would seek, while preserving its own freedom and independence, to assist in the advancement of humanitarian movements in State or Nation or world. This humanitarian aspect of university service, as differentiated from the more strictly scholastic and scientific activities of university life, appearing under newer forms comparatively recently in the so-called university settlements and in the university extension movement, finds its latest phase in coöperative unions for world-wide

programmes of scientific investigation on the one hand, and on the other, in the organized movements for improvement of good will and the promotion of peace among the nations. In such united efforts the new institution would participate, for in its future days it is to be a university of Texas, a university of the South, and later, let us hope, in reality as in aspiration, one among the national institutions, reflecting the national mind, one among the universities of the nations, fostering the international mind and spirit in cosmopolitan ways such as the mediæval universities enjoyed before the death of universal language and the divisions in a universal Church.

### III

#### THE FUNCTIONS OF THE UNIVERSITY

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THAT we have been making large plans is already a commonplace of our thinking and talking. In the proposed solutions of some of the problems confronting them the trustees have been moved by several considerations, which may appropriately be recapitulated at this time. In the first place, the financial resources of the institution, however handsome, are limited; for this reason it was determined to build and maintain the Institute out of the income, keeping the principal of all funds intact. In the second place, the new institution is located in a new and rapidly developing country. In the third place, the very problems pressing for resolution in the development of the environment seemed to call for a school of science, pure and applied, of the highest grade, looking, in its educational programme, quite as much to investigation as to instruction.

Accordingly, and in the spirit of the Founder's dedication of the Institute, it was proposed that the new institution should enter upon a university programme, beginning at the science end. As regards the letters end of the threefold dedication, it was proposed to characterize the institution as one both of liberal and of technical learning, and to realize the larger characterization as rapidly as circumstances might permit. With respect to the art end, it was proposed to take architecture seriously in the preparation of all of its plans, and to see to it that the physical setting of the Institute be one of great beauty as well as of more immediate utility. This in a nutshell is the programme on which we have thought with great deliberation and wrought with even greater care. Its chronology to date consists of one year of

preparatory study from England to Japan, one year in the making of preliminary plans, and two years in work of actual construction and organization.

The new institution thus aspires to university standing of the highest grade, and would achieve its earliest claims to this distinction in those regions of inquiry and investigation where the methods of modern science are more directly applicable. For the present it is proposed to assign no upper limit to its educational endeavor, and to place the lower limit no lower than the standard entrance requirements of the more conservative universities of the country. Moreover, all courses of instruction and investigation, graduate and undergraduate, will be open both to young men and to young women, and for the present, without tuition and without fees. These courses will be offered by a staff, initially organized for university and college work, ultimately to consist of three grand divisions, science, humanity, technology, each of which will break up into as many or more separate faculties. For these faculties the best available instructors and investigators are being sought wherever they may be found, in the hope of assembling a group of unusually able scientists and scholars through whose productive work the Institute should speedily take a place of considerable importance among established institutions. Friends of education in America would insist that the term "Institute" is too narrow in its connotation, friends of science in Europe would contend that it is too broad. However, in its dedication to the advancement of letters, science, and art, the educational programme of liberal and technical learning now being developed may justify the designation "Institute" as representing the functions of a teaching university of learning, and, at least in some of its departments, those of the more recent research institutions founded in this country and abroad.

The planning of universities is no new problem. The list of modern solutions under state initiative is a long one from the national universities of Japan at Tokyo and Kyoto down to the reconstruction of the University of Paris and the revival of the French provincial universities; the reorganization of the University of London and the founding of the newer English municipal universities at Durham, Manchester, Liverpool, Birmingham, Leeds, Sheffield, and Bristol; the newest members of the German system in the universities of Frankfort, Dresden, and Hamburg; and the conspicuous development of state institutions in our own country—to name but a few, in the new California under Wheeler, the new Illinois under Draper and James, the new Texas under Houston and Mezes, the new Virginia under Alderman, and the new Wisconsin under Van Hise. And at this very moment there are building two new universities in Hungary, three in Canada, and two in Japan, while plans are being formulated for new institutions in China, Australia, and South Africa. Within the memory of all of us there have arisen on the benefactions of American philanthropists the Johns Hopkins University under Gilman and Remsen, Cornell University under White and Adams and Schurman, the University of Chicago under Harper and Judson, Leland Stanford under Jordan, and Clark under Hall; while the same period of university building has witnessed equally striking evolutions in the older American private foundations, notably the new Harvard under Eliot and Lowell, the new Yale under Porter and Dwight and Hadley, the new Princeton under McCosh and Patton and Wilson and Hibben, the new Columbia under Low and Butler, and the new Pennsylvania under Harrison and Smith.

It has been remarked that an inventory of present-day universities would reveal thirteenth-century universities, fif-



teenth-century universities, nineteenth-century universities, and twentieth-century universities in formidable array and considerable confusion. There are universities that swear by Plato, others by Euclid, and others by Adam Smith. Some uphold the Thirty-nine Articles, while others worship radium and helium. From glorified engineering shops to scholastic sanctuaries, they offer the widest possible choice of type.

Nevertheless, there has been evolving a composite conception of the university in some such characterization of its functions as follows:

First, from the persistent past, in which there are no dead, to embody within its walls the learning of the world in living exponents of scholarship, who shall maintain, in letters, science, and art, standards of truth and beauty, and canons of criticism and taste.

Second, for the living present and its persistence in the future, to enlarge the boundaries of human learning and to give powerful aid to the advancement of knowledge, as such, by developing creative capacity in those disciplines through which men seek for truth and strive after beauty.

Third, on call of State or Church or University, to convey to its community and commonwealth, in popular quite as much as in permanent form, the products of its own and other men's thinking on current problems of science and society, of government and public order, of knowledge and conduct.

Fourth, in support of all institutes of civilization and all instruments of progress, to contribute to the welfare of humankind in freedom, prosperity, and health, by sending forth constant streams of liberally educated men and women to be leaders of public opinion in the service of the people, constant streams of technically trained practitioners for all

the brain-working professions of our time, not alone law, medicine, and theology, but also every department of service and learning, from engineering, architecture, commerce, and agriculture, to teaching, banking, journalism, and public administration.

As thus conceived, the university is a great storehouse of learning, a great bureau of standards, a great workshop of knowledge, a great laboratory for the training of men of thought and men of action. Under this conception of its functions the university has to do with the preservation of knowledge, with the discovery and distribution of knowledge, with the applications of knowledge, and with the making of knowledge-makers. Singling out one line of its activities, the business of a university is to teach science, to create science, to apply science, to make scientists. To be even more specific, its objects in the department of chemistry are to teach chemistry, to create chemistry, to apply chemistry in all the arts of industry and commerce, and to make more creative chemists. This conception of the manifold function of a university in scholarship, in science, in social service, and in civilization corresponds point by point to the fourfold function of the career of a scholar or scientist: in scholarship, a conservator of knowledge; in science, a creator of knowledge; in citizenship, a contributor to public opinion; in service, a controller of the destiny of the cherished institutions of civilization.

However, even to those who recognize in patriotism, education, and religion supreme enterprises of the human spirit, education itself is proverbially a dull subject whose technical details are sometimes dry as dust. For instance, I am by no means convinced that a discussion of the metaphysics of the optative mood in Greek would be especially edifying on this occasion. Then, too, mathematical studies are poems of a

variety better appreciated when read in private than when declaimed in public. Nor are you likely moved at this time by any overpowering desire for relief from the perplexity of that dear old lady who said she could readily make out how astronomers determined the distances and dimensions, masses and motions, constitution and careers of the heavenly bodies, but for the life of her she never could understand how they found out their beautiful names.

But studies and standards, students and staff are elements of a university programme quite as important as are a machine-shop, a file of journals, a lively imagination, and a printing-press, its other constituent parts. If a university should take all knowledge for its province, it becomes necessary to undertake a classification of knowledge, a problem never yet done with satisfaction to any one except perhaps the last man attempting it. Nor is the problem rendered inordinately simple when restricted to a programme in science, for, to say nothing of more recent modifications upheaving in character, the scientific thought of the nineteenth century has been made by Dr. J. Theodore Merz to align itself in a stately march of no fewer than ten views of nature: the astronomical, the atomic, the kinetic, the physical, the morphological, the genetic, the vitalistic, the psychophysical, the statistical, and the mathematical views.

Yet all would agree, I think, that in mathematics, physics, chemistry, biology, and psychology we have a logical series carefully coördinated in subject-matter and sequence, furnishing the theoretic foundations for the applied sciences of engineering, economics, eugenics, and education. Furthermore, there would also be agreement in the opinion that this coördinated series should be flanked both right and left by history and its interpretation, as a great laboratory in which to test all plans for political or social reform; by philosophy,

as a clearing-house for all theories and methods of knowledge; by letters, as the record in "thoughts that breathe and words that burn" of all human striving after sweetness and light; and by art, the creative imagination's flowering product in the ennobling and enriching of the content of life. Our studies are thus to be centered in the fundamental branches of pure science with a view to solutions of problems of applied science in engineering, whose chief business is the development of the material resources of the world; in economics, whose cardinal problem is that of the distribution of the wealth thus produced; in eugenics as the newest of the sciences, but really in idea no younger than Plato, which by taking thought would add cubits to the stature of the race; and finally in the latest of the experimental sciences, namely, education itself, in whose philosophical, psychological, and physiological foundations are now being sought the surest means of training the intellects and stimulating the imaginations of men.

## IV

### THE FAITH OF THE UNIVERSITY

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IN thus endeavoring to write about the meaning of the new institution I have at some length written about its sources in the Founder's philanthropy and its history in the public spirit of his friends; of its site, glorious in problems bristling with difficulties and joyous in possibilities of creative effort; of its scope in entering upon a university programme for the advancement of letters, science, and art, by investigation and by instruction, in the individual and in the race of all human kind; of its saints of the past and its seers of the present, pointing by exhortation and example to the highroad along which progress in these high purposes lies; of the shades and towers in which are to be undertaken the daring adventures of its life in deeds of thought and action; of its staff of professors, lecturers, and instructors, in whose personality and work of research and teaching are to be found combined the careers of citizen, scientist, scholar, and schoolmaster; of its students, through whose studies and standards in scholarship and sport constant contributions are to be made to the character, culture, and citizenship of the Republic; of its strength in its freedom from political and ecclesiastical affiliations, in its faith in the progress of the human spirit, in its faculties of science, humanity, and technology, in its self-governed student democracy, in a definite educational policy, and the driving power of ideas and ideals backed by material resources for their realization; of its support in the schools of the city, the county, and the commonwealth, in the college men and women of the community, in the captains of industry and commerce, in all organized conferences for education, welfare, and uplift, in the resolute determination of the

people who have been winning the West, now to win the best for the sons and daughters of the West. My further and final object is an attempted portrayal of the spirit which presides over the university: a presentation, more or less rough, of that breath and finer form of the spirit of learning which lends what is perhaps its chief glory to the life of reflection and gives what may be perhaps its final purpose to the life of action.<sup>1</sup>

Twenty years ago it was specialization. Ten years ago it was specialization. To-day it is specialization still, whether in academic education or in professional training, but specialization on the broadest kind of general foundation. Preparatory to attacking the practical problems of the material world, men are coming to provide themselves with the most complete theoretical training yet devised in the world of mind. On the other hand, pure scientists are continually on the outlook for applications of their discoveries either to the ideal world in which they live or to the real world in which they find their livelihood. As a result the professor's desk is nearer the market-place, closer to the counting-house, within easier call of State and Church than ever before. The university is saying to its men of letters, "You must be leaders of men"; to its men of science, "You must be also men of affairs." The world in its turn is demanding that its engineers be cultivated men, and that its skilled artisans be skilled in the liberal arts as well.

Where theory and practice thus meet there must be reason, and this reason is restoring to learning its unity, in whose spirit we read the strength and the vision of the uni-

<sup>1</sup> To bring within the time limits of the programme the reading of an address obviously too long to be read in its complete form in public on any occasion, only four sections of this address were actually delivered as a part of the formal exercises of the inauguration and dedication of the Rice Institute, and under the caption, "The Meaning of the New University: Its Source, Its Site, Its Scope, Its Spirit."

versity. This spirit appears to us under three aspects in those disciplines by which men seek for truth and strive after beauty in letters, in science, in art. Art was originally the handmaid of religion; science, at one time the servant of philosophy, has more lately become its master; letters, in the beginning the playfellow of poets and story-tellers, has grown to be humanity's recording angel. Science has its source in a sense of wonder, art in a sensitiveness to measure and proportion, while literature partakes of the substance of science and the form of art. Science consecrated to the conquest of truth would solve the universe; art would recreate it in the conservation of taste. Science progresses by inquiry, art under inspiration. Intuition dominates the artistic reason, while inference controls the scientific.

In other words, by the spirit of liberal and technical learning I understand that immortal spirit of inquiry or inspiration which has been clearing the pathway of mankind to intellectual and spiritual liberty, to the recognition of law and charm in nature, to the fearless pursuit of truth and the ceaseless worship of beauty. Its history is the history of the progress of the human spirit. Led by an instinct for knowledge, an instinct for harmony, an instinct for law, that spirit has brought the twentieth century its most precious possessions: the love of reason, the love of art, the love of freedom.

There abide these three: the spirit of science, the spirit of letters, the spirit of art, but the man has not arisen to say to us which is the greatest of the three. These are the faces of the spirit of learning, above which there hovers a halo called by the modern philosopher the spirit of service, and by the ancient seer the spirit of wisdom. Knowledge becomes power only when it is vitalized by reason; it becomes learning only when it lives in the personality of a man; it becomes

wisdom on translation into human conduct. I know as well as you that the spirits of which I speak are ghosts who will themselves not speak until they have drunk blood. We propose to give them the blood of our hearts in the service of the new institution.<sup>1</sup>

Ladies and Gentlemen of Houston: At your gates there have arisen for all time the walls and towers and men of the Rice Institute, whose life is to be an integral part of your life, whose service is to be local in the best sense, whose significance, let us hope, may be State-wide, and even national, in its reach, on a foundation builded for Houston, for Texas, the South, and the Nation. A long avenue doubly lined with trees, at one end the captains of industry and commerce in factory and counting-house, at the other a college community in academic shades dedicated to liberal and technical learning, the happy homes of Houston lying in between! A university devoted to the advancement of literature, science, and art; to the promotion of letters as the record of the achievements of the human spirit; to the promotion of science as the revealer of the laws and the conqueror of the forces of nature; to the promotion of art as the sunshine and gilding of life. A society of scholars in whose company your children, and your children's children and their children, may spend formative years of their aspiring youth under the cultivating influences of humane letters and pure science, pursuing culture with forward-looking minds and far-seeing spirit before undertaking in the Institute's professional schools special or technical training for the more sober business of life. A temple of wisdom and sanctuary of learning within whose courts and cloisters you yourselves may find an occasional retreat in which to think more quietly

<sup>1</sup> It is to Professor von Wilamowitz-Moellendorf, I believe, that I owe this figure of speech.



and more deeply; perhaps to worship more devoutly and more intelligently; certainly to contemplate the deeper things of patriotism and politics, of reverence and religion, of peace and progress; and mayhap to discover, if never before, that you may belong to the great community through which the Eternal has worked for ages, that you may have a share in the high privileges and solemn duties which belong to every member of that great community; that in the continuity of human history you may march forward, if you will, in a great pageant that moves from the living past through the living present into the living future.

Not long ago I stood on a great rock—a great living rock—within eyeshot of the birthplace of modern civilization. Upon it rose those incomparable ruins, mighty as the mind that conceived them, majestic as the mountains and sea that call to them. In their midst the gods of the Greeks still live. And of all those gods it was to her who typifies science that the Parthenon was dedicated; to that great goddess who sprang full-armed from the head of Zeus at the touch of fire and toil, to conquer the deep himself.<sup>1</sup> It is no long flight of fancy from the Parthenon above the fields of Hellas to these towers that rise on the plains of Texas. Under her ancient promise, may Pallas Athena preside over these academic groves and guide men by the spirit of science and the spirit of art and the spirit of service in their search for the great, and the lovely, and the new, for solutions of the universe in terms of the good, the beautiful, and the true!

And I recalled the words of the wise man of another chosen people:

“Except the Lord doth build the house, they labor in vain that build it.”

<sup>1</sup> The idea and experience of the first part of this paragraph I am obliged to share with Professor Sir Ronald Ross, but I am unable to supply the appropriate citation.

"I prayed, and understanding was given me; I called upon God, and the spirit of wisdom came unto me; I preferred her above sceptres and thrones, for she is unto men a treasure that never faileth."

"For wisdom is a breath of the power of God, and a pure effluence flowing from the glory of the Almighty. She is the reflection of the everlasting light, the unspotted mirror of the power of God and the image of his goodness. And in all ages, entering into holy souls, she maketh them friends of God, and prophets."

*Wisdom hath builded her house,  
She hath hewn out her seven pillars;  
She hath mingled her wine;  
She hath also furnished her table,  
She hath sent forth her maidens; she crieth  
Upon the highest places of the city,*

*"Whoso is simple, let him turn in hither";  
As for him that is void of understanding, she saith to him,  
"Come, eat ye of my bread,  
And drink of the wine which I have mingled,  
And walk in the way of understanding.*

*"Blessed is the man that heareth me,  
Watching daily at my gates,  
Waiting at the posts of my doors;  
For whoso findeth me findeth life,  
And shall obtain favor of the Lord."*<sup>1</sup>

EDGAR ODELL LOVETT.

<sup>1</sup> These several passages, from the Book of Proverbs and the Book of Wisdom, in slightly abbreviated form have been distributed in the carving on the caps of the columns which support the arches in the cloisters of the North Wing of the first Residential Hall.

